KSE | Kyiv School of Economics

**May 2026**

# Undergraduate programs of the Department of Mathematics

In the Fall of 2025 the Department of Mathematics at KSE launched two undergraduate programs: "**Applied Mathematics**" and "**Physical Mathematics**". The former was an upgraded version of the applied mathematics program which was launched at KSE in 2023. The duration of both undergraduate programs is four years, after which students can enroll in graduate programs in mathematical sciences at KSE or at other institutions.

This document presents a detailed description of the two undergraduate programs of the Department of Mathematics.

**Aims**

To foster a competitive and innovative economy, it is crucial to connect basic research with practical applications and train a large number of young graduates to an advanced level. This is especially vital for mathematics, whose importance to the economy has grown due to its role in modeling, the widespread use of data, and artificial intelligence. Recent studies show the growing importance of numeracy skills and the significant productivity of workers in mathematical science occupations (see, for example, this global analysis of evidence from PIAAC — the Programme for the International Assessment of Adult Competencies, and this report by the Academy for the Mathematical Sciences in the UK.)

The aims of the undergraduate programs of the Department of Mathematics at KSE are to:

- Offer innovative programs in mathematical sciences that maintain a core focus on foundational mathematics while providing training in its applications to other branches of science and technology, including physics, computer science and finance;
- Provide an education that is suitable for both students aiming to pursue research and those preparing for careers in other sectors;
- Deliver a flexible, integrated teaching approach that meets the individual needs of students, supported by mentorship from KSE faculty;
- Foster in students the ability to learn independently, think logically, and solve complex problems;
- Attract outstanding students;
- Present a challenging and comprehensive curriculum in mathematics and its applications that meets the needs of a diverse group of students, including some of the most talented in the country;
- Produce high-caliber graduates in mathematics, ready to embark on graduate studies at KSE and other universities, or to be sought by employers in business and public services;
- Provide an intellectually stimulating environment in which students can develop their skills and enthusiasm to their full potential.

# Undergraduate programs - Department of Mathematics at KSE

**Global scope**

Combined with intensive study of English in the first years, the undergraduate programs of the Department of Mathematics are taught by our research active and English-speaking faculty. Both programs prepare students to pursue graduate studies abroad or in the global graduate program in mathematics at our Department. Some of our graduates will advance to leadership positions in business and public services.

**Contents of this document**

- Course offerings by term
- Roadmaps showing various threads and dependencies between courses
- Recommended course contents and literature

Undergraduate programs - Department of Mathematics at KSE

## Offer of courses by term

Core courses for both undergraduate programs
"Applied Mathematics" other core courses
"Physical Mathematics" other core courses
Elective courses in mathematics
Elective courses in theoretical physics
Elective courses in other fields
CS: courses offered by the Computer Science Department
M: courses offered in our Master's program

| | Fall: September-December | Spring: January-April | Summer: May-July |
|---|---|---|---|
| Year 1 | * Foundations of Mathematics<br>* Linear Algebra I<br>* Real Analysis I<br>* General Physics I: Mechanics, Oscillations and Waves | * Linear Algebra II<br>* Real Analysis II<br>* Programming Basics (CS)<br>* Discrete Mathematics I<br>* General Physics II: Electromagnetism and Thermal Physics | * Differential Equations<br>* Multivariate Analysis<br>* General Physics III: Optics, Atoms and Particles<br>* Graph Theory (CS) |
| Year 2 | * Groups<br>* Topology and Metric Spaces<br>* Optimisation I<br>* Classical Mechanics<br><br>* Introduction to Number Theory | * Probability and Measure<br>* Complex Analysis with Applications<br>* Classical Electrodynamics<br><br>* Rings and Modules<br>* Fields and Galois Theory<br>* Geometry<br>* Continuum Mechanics<br>* Analytical Mechanics | *Analysis of Differential Equations<br>* Experimental Mathematics<br><br>* Discrete Mathematics II<br>* Analytical Electrodynamics<br>* Electrodynamics of Continuous Media |
| Years 3 & 4 | * Functional Analysis<br>* Introduction to PDEs<br>* Quantum Physics<br>* Geometry of General Relativity<br><br>* Actuarial Mathematics<br>* Logic and Set Theory | * Numerical Methods<br>* Mathematical Statistics<br>* Statistical Physics and Thermodynamics<br><br>* Stochastic Processes | |

| | | | |
|---|---|---|---|
| | * Manifolds and Differential Forms / Differential Geometry (M)<br>* Sheaves and Cohomology (M)<br>* Commutative Algebra (M)<br>* Mathematical Topics for Machine Learning (M)<br>*Algebraic Number Theory (M)<br>* Probabilistic Combinatorics (M)<br><br>* Advanced Statistical Physics<br>* Feynman Path Integral<br>* Computational Biophysics<br><br>* Developing Startup Ideas: from research to prototype<br>* Patent Law for Scientists and Innovators | * Optimization II<br>* Representation Theory (M)<br>* Lie Groups and Algebras (M)<br>* Algebraic Geometry (M)<br>* Partial Differential Equations (M)<br>* Dynamical Systems and Ergodic Theory (M)<br>* Analytic Number Theory (M)<br>* Additive Combinatorics (M)<br><br>* Advanced Quantum Physics<br>* Astrophysics and Cosmology<br>*Quantum computation<br><br>* History of Mathematics | * Automata and Formal Languages<br>* Algebraic Topology (M)<br>* Mathematical Topics in Statistical Physics (M)<br>* Modular Forms and Elliptic Curves (M)<br>* Topics in Geometry (M)<br>* Mathematical Economics<br><br>* Quantum Measurements and Correlations<br>* Superconductivity |

**Objectives**

**Year 1:** Transition to university mathematics, with a shift in learning style and pace from school mathematics. Introduction to basic concepts in higher mathematics and their applications, including the notions of proof, rigor, and axiomatic approach. Intensive English classes. Students with sufficient preliminary preparation in mathematics are allowed to take courses from Year 2 under the guidance of mentors.

**Years 2 and 3:** Generalization of familiar mathematics to unfamiliar contexts and application of mathematics to problems outside the field. Laying the foundations – knowledge and understanding – of tools, facts, and techniques necessary for advanced courses in the final year.

By the end of Year 3, students should have covered material in pure mathematics, statistics and operations research, applied mathematics, mathematical/theoretical physics, and computational mathematics, studying some of these topics in depth.

**Year 4:** Students study advanced material in the mathematical sciences, some of it in depth. They develop the ability to solve both abstract and concrete unseen problems, present concise and logical arguments, and (in some cases) use standard software to tackle mathematical problems. Capstone projects are written by students during this final year.

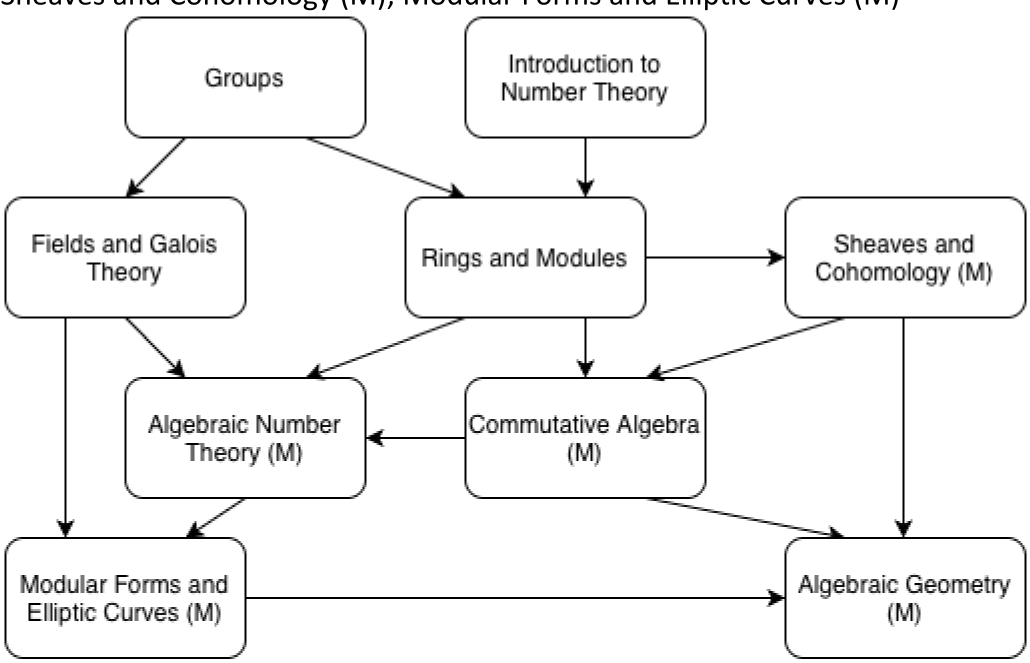
## Roadmaps

This section outlines the 'roadmaps' of course options for undergraduates interested in specific fields and applications of mathematics. Each roadmap includes both core and elective courses in the undergraduate program, as well as master's-level courses (indicated with (M)) that may be taken in the third and fourth years, provided the prerequisites are met. Undergraduates should not view themselves as specializing in any one of these fields. A much better approach is to gain experience in several of them. Students should discuss their choices with their mentors.

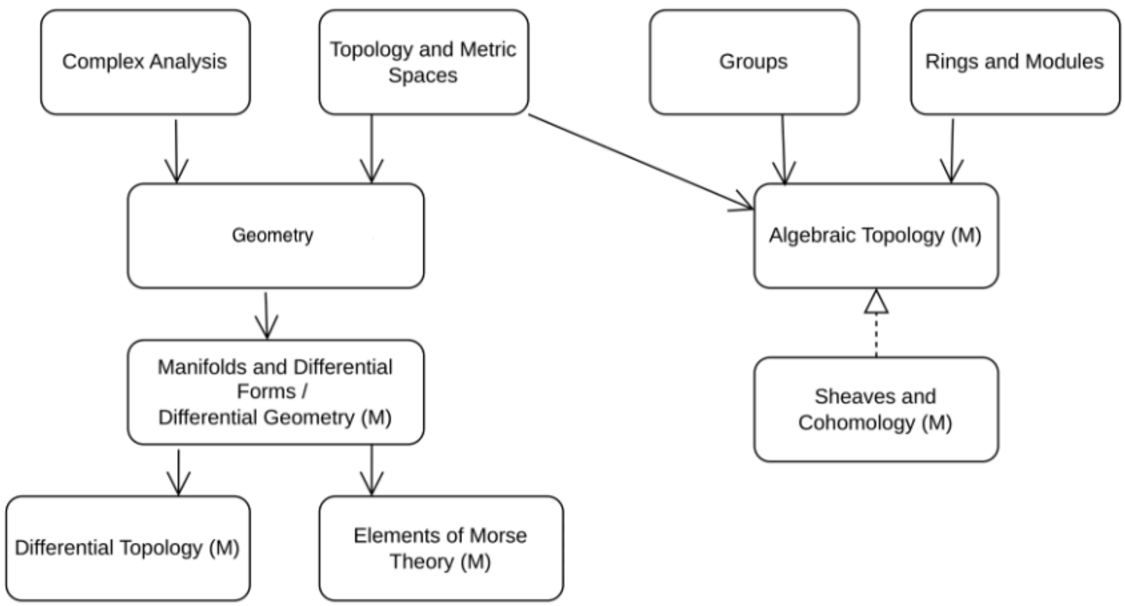
*Some of the listed elective courses may not be offered in a particular year and are subject to lecturer's availability and workload.*

*The diagrams represent logical interconnections between courses rather than a strict hierarchy of prerequisites. For the essential and desirable prerequisites of each course, please refer to its description.*

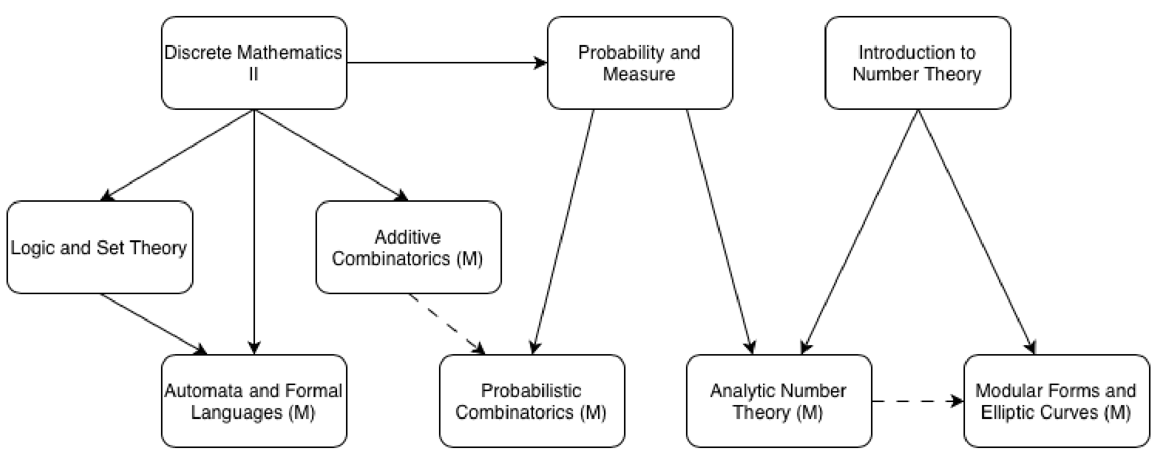
- **Algebra / Algebraic Geometry / Algebraic Number Theory**
  Groups; Introduction to Number Theory; Rings and Modules; Fields and Galois Theory; Commutative Algebra (M); Algebraic Geometry (M); Algebraic Number Theory (M); Sheaves and Cohomology (M); Modular Forms and Elliptic Curves (M)



- **Geometry / Topology**
  Topology and Metric Spaces; Geometry; Manifolds and Differential Forms (Differential Geometry (M)); Differential Topology (M) (offered as Topics in Geometry graduate course); Elements of Morse Theory (offered as Topics in Geometry graduate course) Sheaves and Cohomology (M); Algebraic Topology (M)

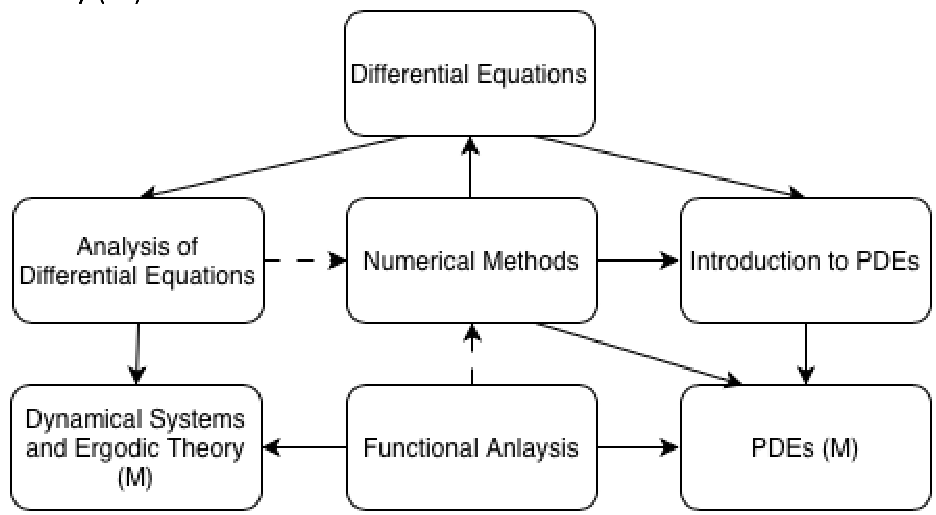



- **Combinatorics / Analytic Number Theory**
  Discrete Mathematics II; Probability and Measure; Automata and Formal Languages; Logic and Set Theory; Probabilistic Combinatorics (M); Additive Combinatorics (M) ; Analytic Number Theory (M); Modular Forms and Elliptic Curves (M).

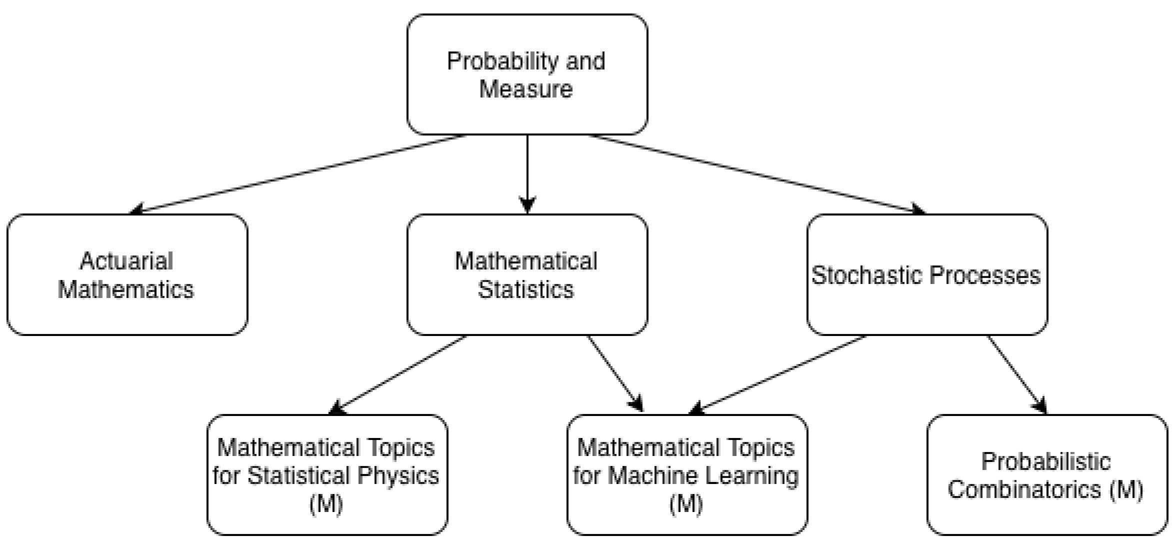


Undergraduate programs - Department of Mathematics at KSE

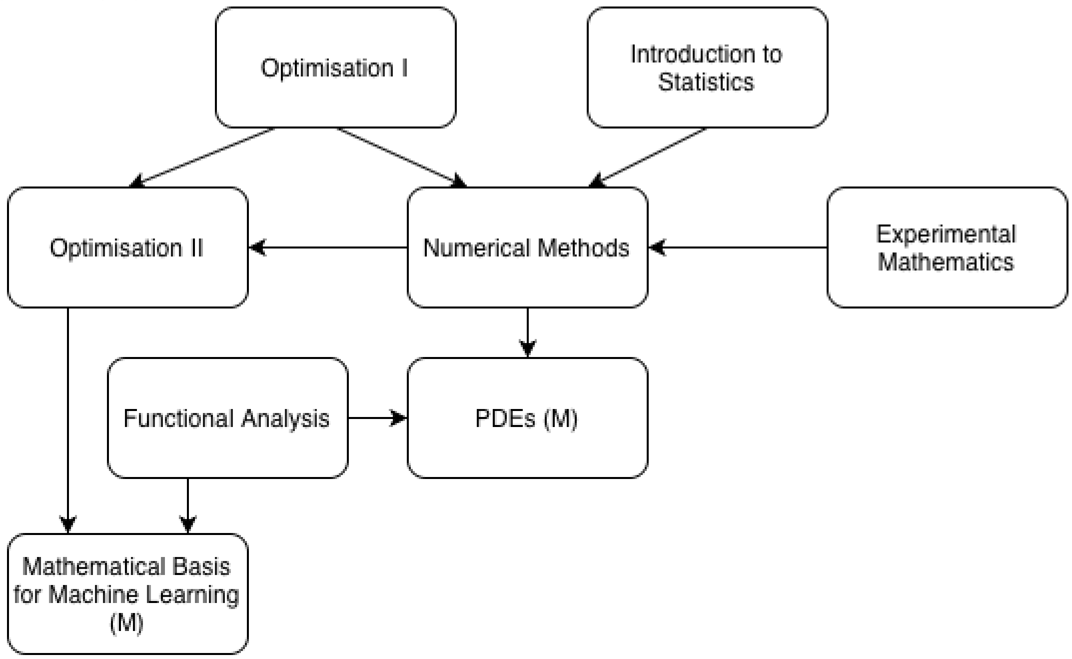
- **Analysis / Differential Equations**
  Differential Equations; Analysis of Differential Equations; Numerical Methods; Functional Analysis; Introduction to PDEs; Partial Differential Equations (M); Dynamical Systems and Ergodic Theory (M).



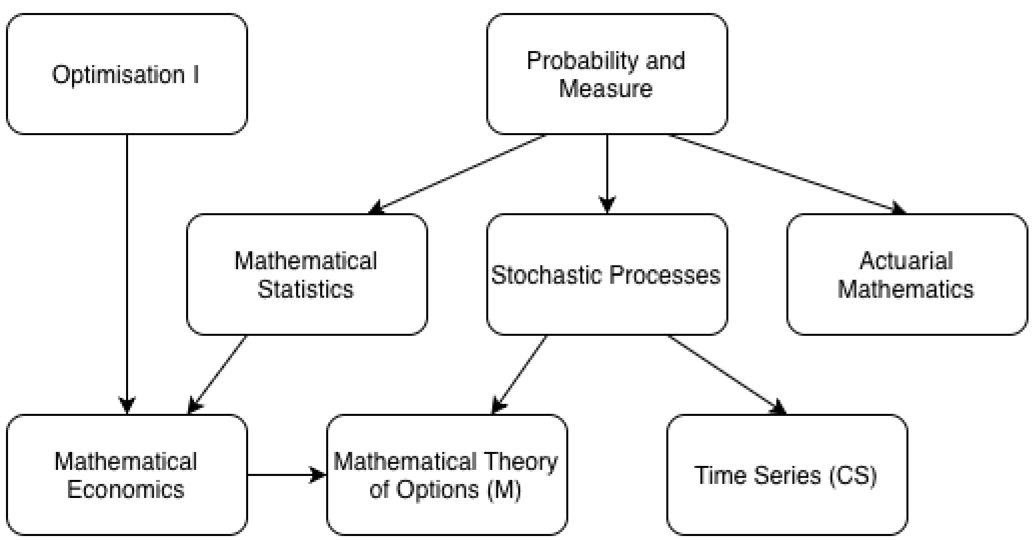

- **Probability / Statistics**
  Probability and Measure; Mathematical Statistics; Stochastic Processes; Mathematical Actuarial Mathematics, Topics for Machine Learning (M); Mathematical Topics in Statistical Physics (M); Probabilistic Combinatorics (M).



- **Optimization and Numerics**
  Optimization I; Experimental Mathematics; Introduction to Statistics; Numerical Methods; Functional Analysis; Optimization II; Mathematical Basis for Machine Learning (M); PDEs (M).

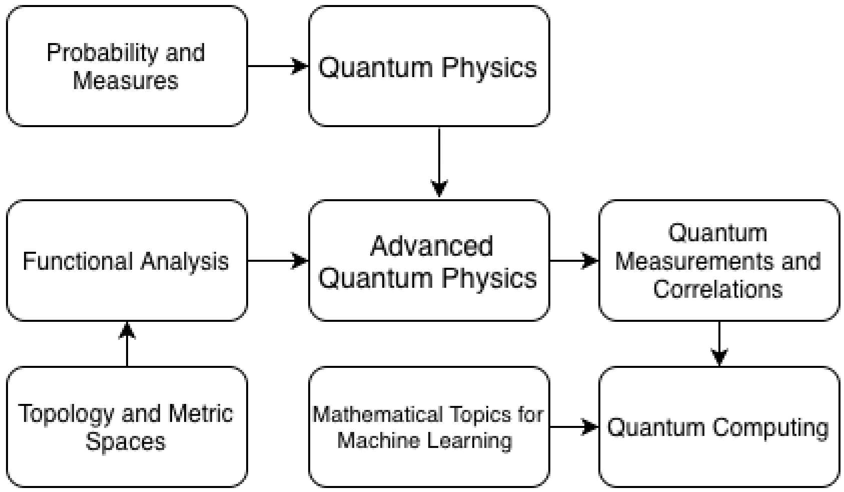



- **Economics and Finance**
  Probability and Measure; Mathematical Statistics; Stochastic Processes; Optimization I; Actuarial Mathematics; Mathematical Economics; Mathematical Theory of Options (M); Time Series (CS).

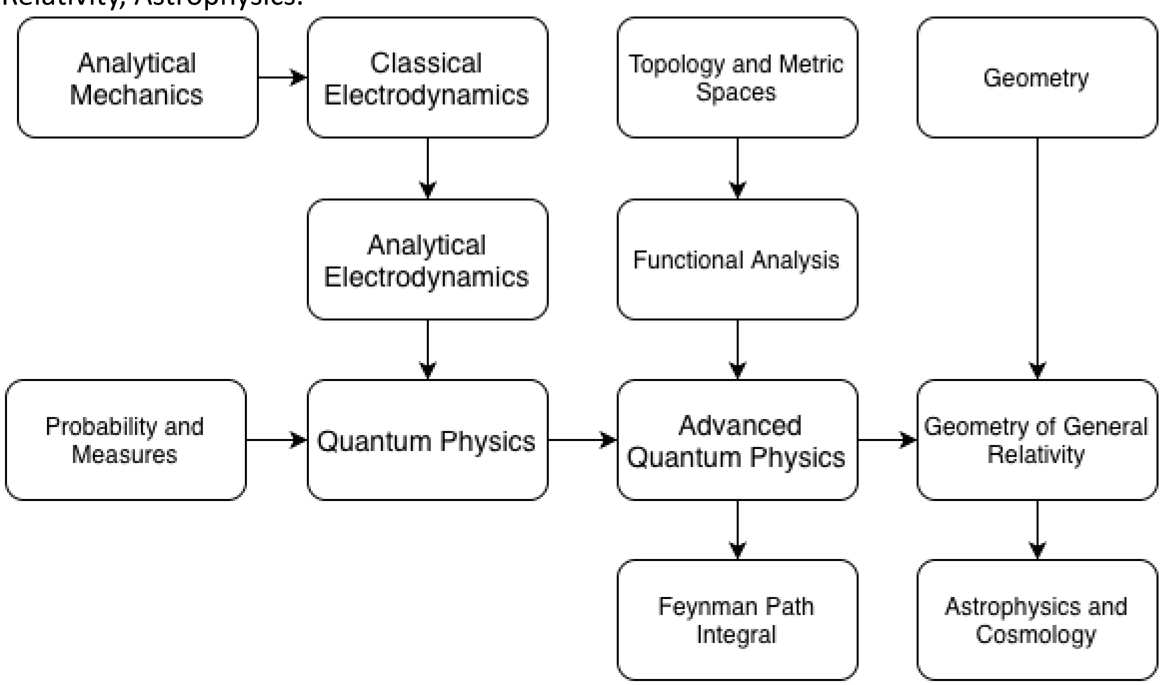



*Students may also take other elective courses offered in the following undergraduate programs at the Department of Economics: Business Economics, Economics and Big Data, Software Engineering and Business Analysis.*

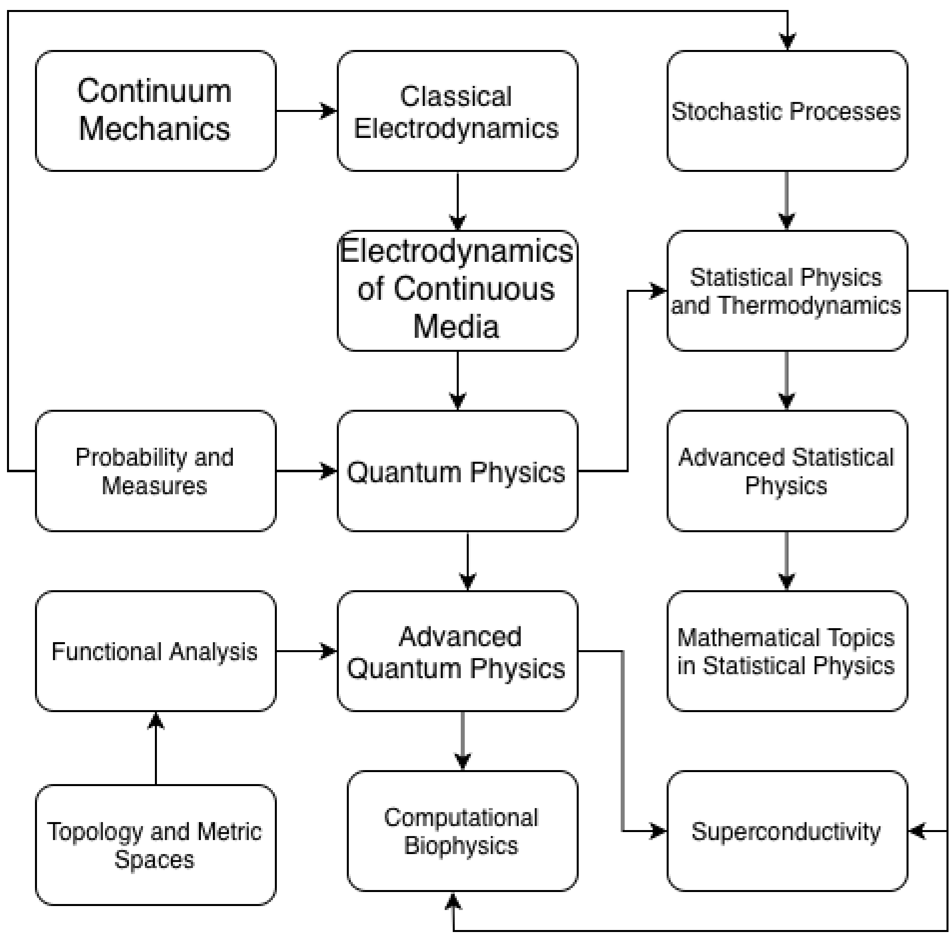
- **Theoretical Physics: Quantum Science and Quantum Technologies**
  Topology and Metric Spaces, Functional Analysis, Probability and Measure, Mathematical Topics for Machine Learning (M), Quantum Physics, Advanced Quantum Physics, Quantum Measurements and Correlations, Quantum Computing.




- **Theoretical Physics: Elementary Particles and Astrophysics**
  Topology and Metric Spaces, Functional Analysis, Geometry, Probability and Measure, Analytical Mechanics, Classical Electrodynamics, Analytical Electrodynamics, Quantum Physics, Advanced Quantum Physics, Feynman Path Integral, Geometry of Special Relativity, Astrophysics.



- **Theoretical Physics: Material Science**
  Functional Analysis, Topology and Metric Spaces, Probability and Measure, Stochastic Processes, Continuum Mechanics, Classical Electrodynamics, Electrodynamics of Continuous Media, Quantum Physics, Advanced Quantum Physics, Statistical Physics, Advanced Statistical Physics, Superconductivity, Computational Biophysics, Mathematical Topics in Statistical Physics.




*In the roadmaps below CS indicates that the course is offered by the department of Computer Sciences.*

- **Theoretical Computer Science**
  Advanced Algorithms (CS); Discrete Mathematics II; Logic and Set Theory; Automata and Formal Languages; Graph Theory (CS); Advanced Data Structures (CS); Randomised Algorithms (CS); Algorithmic Game Theory (CS); Mathematical Foundations of Cryptography (CS);

- **Data Science**
  Data Manipulation Essentials (CS); Machine Learning (CS); Databases (CS); Big Data Analytics (CS); Deep Learning (CS); Time Series Analysis (CS); NLP (CS); Computer Vision (CS); Reinforcement Learning (CS).

## Tentative contents of courses

A class at KSE lasts for 80 minutes. A course usually consists of lectures and practical sessions. In what follows only the number of ECTS credits is given. Conversion into lecture hours is done roughly as follows (standard ratio of lectures and practical sessions is 1:1):

4 ECTS: 30 classes in total (both lectures and practical sessions)
5 ECTS: 38 classes in total (both lectures and practical sessions)
6 ECTS: 45 classes in total (both lectures and practical sessions)

*The list below does not include master courses and courses offered by other departments. Their content can be found in the descriptions of the corresponding programs.*

## PART I: MATHEMATICS

**FOUNDATIONS OF MATHEMATICS** - back to the list of all courses

4 ECTS

This course is aimed to prepare students for future proof-based courses by reviewing basic logical constructions and proof techniques and illustrating them with material on numbers and polynomials. The topics listed here should necessarily be covered as other courses in the program will rely on this course. The course is taught intensively in the first month.

The lectures cover:

- **Introduction to Logic.** Statements, And, Or and negation. Implications. Quantifiers (`for all' and `there exists') including negations. Converse and proof by contradiction. Proof by induction.
- **Naive Set Theory.** Sets; set operations including union and intersection; laws of set arithmetic including De Morgan's laws, Venn diagrams and applications to counting problems. Definitions of finite, infinite, countable and uncountable sets. A countable union of countable sets is countable. Uncountability of R. Non-existence of a bijection from a set to its power set.
- **Relations and mappings.** Cartesian product of sets; relations and their properties:

symmetry, reflexivity and transitivity; equivalence relations; partial orders; mappings and their properties; injectivity, surjectivity and bijectivity; functions and inverse functions.
- **Structure of the Real Line.** Uncountability, Dedekind cuts, suprema and infimums, well-orderedness, completeness. Irrationality of $\sqrt{2}$ and e. Decimal expansions. Construction of a transcendental number. Indirect proof of existence of transcendental numbers.
- **Complex Numbers.** Algebraic, trigonometric and exponential forms of complex numbers. Complex Plane. De Moivre formula. Complex roots. Roots of unity.
- **Polynomials and modular arithmetic.** Addition, multiplication, the laws of arithmetic; rational polynomials, polynomial division and partial fractions; powers of binomials using Pascal's Triangle. Modular arithmetic. Euclidean algorithm for integers and polynomials.

**DISCRETE MATHEMATICS I** - back to the list of all courses

6 ECTS

This course provides an introduction to the core ideas and methods of discrete mathematics, with an emphasis on structures and techniques that underpin modern mathematics, computer science, and data analysis. It develops students' ability to reason rigorously about finite objects through combinatorial counting, discrete probability, relations and orders, recurrence relations, generating functions, and graph theory. By combining theoretical foundations with illustrative applications, the course aims to build problem-solving skills and prepare students to model, analyze, and solve problems involving discrete structures.

Essential prerequisite: Foundations of Mathematics

The lectures cover:

- **Combinatorial counting.** Inclusion-exclusion principle. Pigeonhole principle and its applications. Binomial and multinomial coefficients, binomial and multinomial theorems. Linear recurrence relations, Fibonacci numbers. Bell numbers, Stirling numbers.
- **Discrete probability.** Finite probability spaces. Random variables, expected value, variance. Several applications.
- **Binary relations.** Equivalence relations. Partial orders, linear orders, lattices.
- **Generating functions.** Power series, ring of sequences under convolution. Solving linear recurrence using generating functions.
- **Graph theory.** Basic definitions. Handshaking lemma. Paths, cycles and trees. Connected and bipartite graphs. Adjacency matrix. Eulerian and Hamiltonian graphs. Vertex colorings, chromatic number. Clique number and independence number.

**Recommended books:**
[1] Invitation to discrete mathematics / Jiri Matousek, Jaroslav Nesetril. – 2nd ed., repr. – New York, 2011
[2] Discrete Mathematics: Elementary and Beyond / L. Lovasz, J. Pelikan, K. Vesztergombi
Combinatorics: Set Systems etc. / B. Bollobas

**REAL ANALYSIS I** - back to the list of all courses

5 ECTS

This course introduces the fundamental concepts and techniques of mathematical analysis in one variable. It develops a rigorous understanding of limits, convergence, continuity, and differentiability for real-valued functions, emphasizing both theoretical foundations and problem-solving methods. Through central results such as the Bolzano-Weierstrass theorem, the intermediate value theorem, the mean value theorem, Taylor's theorem, and principles of uniform continuity and convexity, the course aims to build students' ability to reason precisely about functions, sequences, and approximation.

Parallel: Foundations of Mathematics

The lectures cover:

- **Limits and convergence.** Sequences and series in R and C. Sums, products and quotients. Absolute convergence; absolute convergence implies convergence. The Bolzano-Weierstrass theorem and applications (the General Principle of Convergence). Convergence of bounded monotonic sequences.
- **Continuity.** Continuity of real- and complex-valued functions defined on subsets of R. The intermediate value theorem. A continuous function on a closed bounded interval is bounded and attains its bounds. Lipschitz Functions. Uniform continuity. The sign-preserving property of continuous functions.
- **Differentiability.** Differentiability of functions from R to R. Derivative of sums and products. The chain rule. Derivative of the inverse function. Rolle's theorem; the mean value theorem. One-dimensional version of the inverse function theorem. Taylor's theorem from R to R; Lagrange's form of the remainder. Extremal values of real functions. Fermat's theorem and sufficient conditions of a local extreme. Convex functions.

-*Extra topic(s) (if time permits and up to a lecturer's preference):*
The Toeplitz and the Stolz-Cesaro theorems. The contraction theorem for Lipschitz functions.

**Recommended books:**
[1] Abbott, S. Understanding Analysis. 2nd ed. Springer, 2015.
[2] Apostol, T. M. Mathematical Analysis. 2nd ed. Addison–Wesley, 1974.
[3] Bartle, R. G., and Sherbert, D. R. Introduction to Real Analysis. 4th ed. Wiley, 2011.
[4] Дороговцев, А. Я. Математичний аналіз. Частина I, К.: Либідь, 1993.

**REAL ANALYSIS II** - back to the list of all courses

6 ECTS

This course continues the study of mathematical analysis by developing the theory and applications of infinite series and integration. It focuses on series, power series, functional sequences, uniform convergence, Riemann integration, improper integrals, parameter-dependent integrals, and special functions such as the gamma and beta functions. The course also introduces Fourier series and the Fourier transform as tools for representing and analyzing functions. Emphasis is placed on rigorous reasoning, convergence criteria, termwise operations, and the connection between analytic theory and methods of approximation.

Essential prerequisite: Foundations of Mathematics, Real Analysis I

The lectures cover:

- **Series and power series.** Comparison and ratio tests, alternating series test. Functional sequences and series. Uniform vs. pointwise convergence. Dirichlet and Abel tests for convergence. Termwise integration and differentiation of functional series. Complex power series and radius of convergence. Taylor series. Exponential, trigonometric and hyperbolic functions, and relations between them. Direct proof of the differentiability of a power series within its circle of convergence.
- **Integration.** Definite and indefinite integration, antiderivatives. Definition and basic properties of the Riemann integral. A non-integrable function. Integrability of monotonic functions. Integrability of piecewise-continuous functions. The fundamental theorem of calculus. Differentiation of indefinite integrals. Integration by parts. The integral form of the remainder in Taylor's theorem. Improper integrals. Integrals depending on a parameter. Gamma- and beta-functions.
- **Fourier Series and Fourier Transform.** Formal derivation of Fourier coefficients and formal Fourier series. Pointwise and uniform convergence of Fourier series. Fourier transform, sine and cosine transforms.

*Extra topic(s) (if time permits and up to a lecturer's preference):*
Infinite products. Mean value theorems for integrals and estimation of integrals.

**Recommended books:**

[1] Abbott, S. Understanding Analysis. 2nd ed. Springer, 2015.
[2] Apostol, T. M. Mathematical Analysis. 2nd ed. Addison–Wesley, 1974.
[3] Bartle, R. G., and Sherbert, D. R. Introduction to Real Analysis. 4th ed. Wiley, 2011.
[4] Дороговцев, А. Я. Математичний аналіз. Частина I, К.: Либідь, 1993.

**LINEAR ALGEBRA I** - back to the list of all courses

5 ECTS

This course introduces the fundamental concepts and methods of linear algebra, with an emphasis on both computational techniques and abstract structure. It develops students' understanding of systems of linear equations, Gaussian elimination, vector spaces, bases and dimension, linear maps, matrices, determinants, and polynomial rings. Alongside core theoretical results such as the Steinitz lemma, Kronecker-Capelli theorem, change of basis theorem, and properties of determinants, the course highlights applications and algorithmic perspectives, including PageRank and LU-decomposition. The course aims to build rigorous reasoning, algebraic problem-solving skills, and a foundation for further study in mathematics, computer science, and data analysis.

Parallel: Foundations of Mathematics

The lectures cover:

- **Systems of linear equations.** Systems of linear equations over number fields. Matrix of SLE. Gaussian elimination. Time Estimate for Gaussian Algorithm. *PageRank Algorithm* (about 2 lectures). A concept of a field.
- **Vector spaces.** Arithmetic vector space. Linear combination of vectors, span, linear dependence and independence. Basis and dimension. Steinitz lemma. Matrix rank. Abstract vector space. Examples. Homogeneous SLEs and their solution spaces. Kronecker-Capelli theorems. Sum and intersection of vector spaces. Grassmann formula.
- **Linear maps and their matrices.** Linear maps. Kernel and image. Isomorphism of finite-dimensional spaces of the same dimension. *****Quotient spaces and homomorphism theorem*. Matrix representation of a linear map. Four fundamental matrix spaces. Operations on linear maps and operations with matrices. Elementary matrices and their connection with row operations. Invertible matrices. An idea of a group. Change of basis theorem. *LU-decomposition*. The dual space (space of functionals). Adjoint maps.
- **Determinants.** Oriented area and volume. Determinants and their properties.
- **Polynomial ring.** Polynomials (one variable): roots, Bezout theorem, complex numbers are algebraically closed (no proof).

**Appropriate books**:
[1] G. Strang, Linear Algebra and Its Applications, 4th Edition
[2] P. Olver, Ch. Shakiban, Applied Linear Algebra, 2018
See also lecture notes by Alexandre Eremenko which follow this book and include applications:
https://www.math.purdue.edu/~eremenko/lecturenotes.html
and lecture notes by Yaroslav Vorobets which include applications of linear algebra:
https://people.tamu.edu/~yvorobets/MATH304-2011C/MATH304-502.html

**LINEAR ALGEBRA II** - back to the list of all courses

6 ECTS

This course continues the study of linear algebra by focusing on the structure of linear operators and their applications. It develops the theory of eigenvalues and eigenvectors, characteristic polynomials, diagonalization, Jordan normal form, matrix exponentials, and spectral methods. The course also introduces Euclidean and unitary spaces, orthogonality, projections, QR-decomposition, least squares, self-adjoint and unitary operators, positive definite matrices, and singular value decomposition.

Essential Prerequisite: Linear Algebra I

The lectures cover:

- **Eigenvectors and eigenvalues.** Review on linear maps. Eigenvectors and eigenvalues. Characteristic polynomial. Algebraic and geometric multiplicity. Characteristic polynomial of an endomorphism and of a matrix. Cayley-Hamilton theorem. PageRank: revisited. Stochastic matrices. Diagonalization and its applications.
- **Jordan normal form.** Nilpotent operators and matrices. Jordan normal form (nilpotent and general case). Matrix exponents. Applications.
- **Euclidean and unitary spaces.** Real scalar product. Orthogonality: orthogonal bases, orthogonal projections, Gram-Schmidt orthogonalization. Applications: QR-decomposition and least square method. Linear operators in Euclidean and unitary spaces. Isometries in Euclidean space. Orthogonal matrices. Hermitian scalar product. Unitary transformations. Self adjoint operators over reals and their matrices. Spectral theorem for real symmetric matrices. Positive definite matrices. Gram matrix and Gram determinant. Hermitian matrices. Diagonalization of unitary transformations. *Singular value decomposition and its applications.*
- **Bilinear and quadratic forms.** Bilinear and quadratic forms. Their matrices. Symmetric and skew-symmetric forms. Sylvester's Inertia law. Classification of quadratics form over real and complex numbers. Scalar product: revisited.

If time permits:

- Affine Euclidean space: an introduction.
- Linear groups. Orthogonal, symplectic, unitary, Lorenz. Applications in special relativity: Rotations in 3-space. Lorentz transformations as transformation groups.
- Applications in probability (if time permits, up to lecturers's choice). Definition and basic properties of Markov chains, the transition matrix. Calculation of n-step transition probabilities. Invariant distribution. Recurrence, irreducibility, ergodicity.

**Recommended books**:
[1] G. Strang, Linear Algebra and Its Applications, 4th Edition
[2] P. Olver, Ch. Shakiban, Applied Linear Algebra, 2018
See also lecture notes by Alexandre Eremenko which follow this book:
https://www.math.purdue.edu/~eremenko/lecturenotes.html
and lecture notes by Yaroslav Vorobets which include applications of linear algebra:
https://people.tamu.edu/~yvorobets/MATH304-2011C/MATH304-502.html

**INTRODUCTION TO NUMBER THEORY** - back to the list of all courses

4 ECTS

Essential Prerequisites: Foundations of Mathematics, Real Analysis I, Linear Algebra I
Useful complementary courses: Groups

*This course gives an overview of topics in number theory with almost no prerequisites. It serves as a motivation and a gentle introduction to the abstract algebra courses coming in the following years.*

The lectures cover:

- **Divisibility.** Review of prime numbers: existence and uniqueness of factorisation into primes; greatest common divisors and least common multiples. Euclid's proof of the infinity of primes. Euclid's algorithm. Solution in integers of ax+by = c.
- **Congruences.** Review of modular arithmetic; Wilson's and Fermat-Euler congruences. Chinese Remainder Theorem. Units modulo n. Public key cryptography and the RSA algorithm. Residues modulo prime powers and Hensel's lemma. Existence of primitive elements modulo n. Fields with p elements, quadratic residues and proof of quadratic reciprocity law.
- **Polynomials.** Review of Euclid's algorithm and factorisation of polynomials. Uniqueness of factorisation and analogy with integer numbers. Greatest common divisors and least common multiples of polynomials. Gauss' lemma.
- **Quadratic fields and non-uniqueness of factorisation.** Integer elements; prime elements; examples of non-unique factorisation into primes; examples of fields with unique factorisation. Continued fractions and Pell's equation.
- **Distribution of primes.** Bertrand's postulate (Chebychev's proof). Prime number theorem (without proof). Primes in arithmetic progressions.
- **Further topics.** Depending on the taste of the lecturer further topics may be covered, such as rational points on curves, partitions, basic arithmetic functions, introduction to algebraic number fields.

**Recommended books:**
[1] I. Niven, H. S. Zuckerman and H. L. Montgomery, An Introduction to the Theory of Numbers
[2] N. Koblitz, A Course in Number Theory and Cryptography

[3] K. Ireland, M. Rosen, A Classical Introduction to Modern Number Theory

Similar course: https://ocw.mit.edu/courses/18-781-theory-of-numbers-spring-2012/

**MULTIVARIATE ANALYSIS** - back to the list of all courses

5 ECTS

This course extends mathematical analysis to functions of several variables and vector-valued mappings. It develops the theory of partial derivatives, differentials, gradients, Jacobians, Taylor expansions, constrained optimization, and the inverse function theorem. The course also introduces multivariable integration, change of variables, line and surface integrals, curves and surfaces, and the main operators of vector calculus.

Prerequisites: Real Analysis II, Linear Algebra II

The lectures cover:

- **Multivariable functions and partial derivatives.** Level curves, partial derivatives. Differentials, chain rule. Directional derivatives and the gradient vector, tangent plane. Second derivatives; symmetry of mixed partial derivatives. Taylor series for functions on R^n. Local extrema of real multivariable functions, classification using the Hessian matrix. Lagrange multipliers.
- **Multivariable vector-functions.** Definition of derivative as a linear map. Jacobian. Elementary properties; the chain rule. The inverse function theorem (proof of continuity of inverse function; statement of differentiability).
- **Multivariate Riemann Integration in R^2, R^3, and R^n.** Volume integrals: definitions, examples using Cartesian, cylindrical and spherical coordinates; change of variables.
- **Curves in R^2, R^3 and surfaces in R^3.** Parameterised curves and surfaces, arc length, tangents and normals to curves and surfaces in R^3, curvature and torsion. Line integrals. Surface integrals.
- **Vector operators.** Vector fields. Gradient curl and divergence in Cartesian coordinates, examples; formulae for these operators (statement only) in cylindrical, spherical and general orthogonal curvilinear coordinates. Solenoidal fields, irrotational fields and conservative fields; scalar potentials. Vector derivative identities.
- **Integration theorems.** Divergence theorem, Green's theorem, Stokes' theorem, Green's second theorem: statements; examples, sketches of the proofs.

**Similar courses:** https://ocw.mit.edu/courses/18-022-calculus-of-several-variables-fall-2010/
https://ocw.mit.edu/courses/18-02-multivariable-calculus-fall-2007/pages/calendar/

Recommended books:

[1] Marsden, J. E., and Tromba, A. J. *Vector Calculus*. 6th ed. W. H. Freeman, 2012.

[2] Apostol, T. M. *Calculus, Volume II: Multi-Variable Calculus and Linear Algebra, with Applications to Differential Equations and Probability*. 2nd ed. Wiley, 1969.

[3] Дороговцев, А. Я. Математичний аналіз. Частина II, К.: Либідь, 1994.

**DIFFERENTIAL EQUATIONS** - back to the list of all courses

5 ECTS

This course introduces differential equations as a language for describing change, motion, growth, and interaction in mathematical models. It emphasizes the transition from explicit calculation to qualitative reasoning: understanding solution behavior through direction fields, phase portraits, stability, approximation, and existence-uniqueness principles. Students learn how analytic, numerical, and geometric methods complement one another in studying both single equations and systems. The course aims to develop the ability to model real processes, interpret solutions, and analyze the long-term behavior of dynamical systems.

Essential Prerequisites: Real Analysis II, Linear Algebra II
Desirable Prerequisites: Multivariate Analysis

The lectures cover:

- **First-Order Differential Equations.** Informal review of differentiation and integration; geometric and physical interpretations. Definition and examples of differential equations. Basic models and radioactive decay; numerical experiments. General and particular solutions. Classification of differential equations. Geometric approach: direction fields, integral trajectories, and isoclines.
- **Solution Methods for First-Order ODEs.** Linear first-order equations. Separable equations. Bernoulli and Riccati equations. Method of variation of constants. Autonomous equations and population dynamics. Exact equations and integrating factors. Equations implicitly involving derivatives: Lagrange and Clairaut equations. Solution by parameter introduction.
- **Existence, Uniqueness, and Discrete Equations.** The Existence and Uniqueness Theorem (statement and idea of proof only). Initial value problems and Picard iterations. The Lipschitz condition and convergence of successive approximations. First-order difference equations.
- **Higher-Order Linear Equations.** Second-order homogeneous equations with constant coefficients. Fundamental solutions, linear independence, and the Wronskian formula. Abel's theorem. Nonhomogeneous equations: method of undetermined coefficients and variation of parameters. Extension to linear differential equations of order n.
- **Variable Coefficients and Transform Methods.** Initial value problems for linear equations with variable coefficients. Reduction of order for homogeneous equations. The Laplace transform method.
- **Systems of Equations and Stability.** Systems of first-order linear equations: homogeneous systems, complex eigenvalues, repeated eigenvalues, and fundamental matrices.

Nonhomogeneous systems. Existence and uniqueness for systems. The phase plane: autonomous systems and stability. Linearization of almost linear systems. Mathematical biology models: competing species and predator-prey dynamics.

**Useful (but larger) course**:
https://ocw.mit.edu/courses/18-03-differential-equations-spring-2010/

**Recommended books:**

[1] William E. Boyce, Richard C. DiPrima, and Douglas B. Meade, Elementary Differential Equations and Boundary Value Problems. Wiley, 2021.

[2]Gerald Tesch, Ordinary Differential Equations and Dynamical Systems, AMS: Graduate Studies in Mathematics, Vol. 140; 2012.

[3] J. L. Brenner, Problems in Differential Equations, Dover Publications, 2013.

**GROUPS** - back to the list of all courses

5 ECTS

This course introduces group theory as the study of symmetry, structure, and transformation. Its central aim is to show how abstract algebraic rules arise naturally from geometric symmetries, permutations, matrix transformations, and Möbius maps. Students learn to recognize groups through their actions, classify them using invariants such as cosets, conjugacy, orbits, and characters, and understand how quotient constructions reveal hidden structure. The course builds a conceptual bridge between concrete examples and abstract algebraic reasoning, preparing students to use symmetry as a unifying tool across mathematics.

Essential prerequisites: Foundations of Mathematics, Linear Algebra I
Desirable prerequisites: Linear Algebra II
*This course can be taken by stronger 1st year students in parallel with Linear Algebra I.*

- **Examples of groups.** Axioms for groups. Examples from geometry: symmetry groups of regular polygons, cube, tetrahedron. Cyclic groups. Permutations on a set; the symmetric group. Subgroups and homomorphisms. Symmetry groups as subgroups of general permutation groups. The Möbius group; cross-ratios, preservation of circles, the point at infinity. Conjugation. Fixed points of Möbius maps and iteration. Abelian groups. Direct sums and fundamental theorem (no proof).
- **Lagrange's theorem.** Cosets. Lagrange's theorem. Groups of small order (up to order 8). Fermat-Euler theorem from the group-theoretic point of view.
- **Group actions.** Group actions; orbits and stabilizers. Orbit-stabilizer theorem. Cayley's theorem (every group is isomorphic to a subgroup of a permutation group). Conjugacy classes. Cauchy's theorem. Characteristic polynomial as an invariant of conjugacy action.

- **Quotient groups.** Normal subgroups, quotient groups and the isomorphism theorem.
- **Matrix groups.** The general and special linear groups; relation with the Möbius group. The orthogonal and special orthogonal groups, invariance of norm and inner product. Proof (in R^3 ) that every element of the orthogonal group is the product of reflections and every rotation in R^3 has an axis. Basis change as an example of conjugation.
- **Permutations.** Permutations, cycles and transpositions. The sign of a permutation. Conjugacy in S_n and in A_n. Simple groups; simplicity of A_5 .
- **Representations and characters.** Group representations, unitary representations, characters, one-dimensional characters the regular representation.

**Recommended books:**

[1] M.A. Armstrong, Groups and Symmetry. Springer–Verlag 1988
[2] S. Lang, Undergraduate algebra
[3] Alan F Beardon, Algebra and Geometry. CUP 2005
[4] R.P. Burn Groups, a Path to Geometry. Cambridge University Press 1987
[5] J.A. Green Sets and Groups: a first course in Algebra. Chapman and Hall/CRC 1988
[6] W. Lederman Introduction to Group Theory. Longman 1976
[7] John B. Fraleigh. A first course in Abstract Algebra

**TOPOLOGY AND METRIC SPACES** - back to the list of all courses

5 ECTS

This course introduces topological and metric spaces as frameworks for understanding continuity, convergence, compactness, and structure beyond Euclidean space. It emphasizes the ideas that remain invariant under continuous deformation, the role of connectedness and compactness in controlling global behavior, and the passage from metric spaces to normed and Hilbert spaces. Students develop a conceptual understanding of completeness, fixed point principles, weak and strong convergence, duality, and orthogonality, preparing them to work with abstract spaces that arise throughout modern analysis, geometry, and applied mathematics.

Prerequisite: Multivariate Analysis, Linear Algebra II

The lectures cover:

- **Topological spaces.** Definition and examples. Further examples. Neighborhoods, closed sets, convergence and continuity. Hausdorff spaces. Homeomorphisms. Topological and non-topological properties. Subspace, product and quotient topologies.
- **Connectedness.** Definition using open sets and integer-valued functions. Examples. Components. The continuous image of a connected space is connected. Path-connectedness and its relation to connectedness. Connected open sets in Euclidean space are path-connected. Connectedness of products.

- **Compactness.** Definition using open covers. Examples including [0, 1]. Closed subsets of compact spaces are compact. Compact subsets of Hausdorff spaces are closed. The compact subsets in the Euclidean space. Continuous images of compact sets are compact. Continuous real-valued functions on a compact space are bounded and attain their bounds. Compactness of products. Tychonoff's theorem. Sequential compactness.
- **Metric spaces**. Definition and examples. Limits and continuity. Uniform convergence theorem. Open sets and neighborhoods. Characterizing limits and continuity using neighborhoods and open sets. Completeness. Cantor's intersection theorem. The contraction mapping theorem. Examples of closed bounded sets which are not compact.
- **Normed Vector Spaces.** Various examples of norms. Equivalence of norms in finite dimension. The unit ball is compact iff the space is finite-dimensional. Linear Functionals. Dual space. Strong convergence versus weak convergence. Examples. Continuous maps, continuous linear maps, Lipschitz maps between normed vector spaces. Differentiable map, examples. Local inversion theorem, implicit function theorem.
- **Introduction to Hilbert Spaces.** Orthogonal systems. Bessel inequality. Subspaces and orthogonal complements. Riesz-Fisher theorem (no proof). Isomorphism of Hilbert spaces. Uniqueness up to isomorphism of the infinite-dimensional separable Hilbert space.

*Extra topics (if time permits and up to a lecturer's preference)***:**
Compactness of quotient spaces. Applications of the contraction theorem including Picard's solution of differential equations. Compactness in metric spaces via epsilon-nets.

**Recommended books:**

[1] Kelley, John L. General Topology. New York: Springer, 1975.
[2] Rudin, Walter. Principles of Mathematical Analysis. 3rd ed. New York: McGraw-Hill, 1976.
[3] Simmons, George F. Introduction to Topology and Modern Analysis. New York: McGraw-Hill, 1963.
[4] Kreyszig, Erwin. Introductory Functional Analysis with Applications. New York: Wiley, 1978.

**COMPLEX ANALYSIS WITH APPLICATIONS** - back to the list of all courses

5 ECTS

This course introduces complex analysis as a rigorous theory of complex-valued functions and their analytic, geometric, and structural properties. It emphasizes the central role of analyticity, contour integration, and Cauchy's theorem in deriving powerful consequences such as integral formulas, maximum principles, residue calculus, and analytic continuation. Students learn how local behavior near singularities determines global information, how conformal maps preserve geometry, and how complex methods can solve problems in integration, geometry, fluid flow, and electrostatics.

Prerequisites: Multivariate Analysis, Topology and Metric Spaces

The lectures cover:

- **Analytic functions.** Complex differentiation and the Cauchy-Riemann equations. Examples. Conformal mappings. Informal discussion of branch points, examples of log z and z^c.
- **Contour integration and Cauchy's theorem.** Contour integration (for piecewise continuously differentiable curves). Statement and proof of Cauchy's theorem for star domains. Cauchy's integral formula, maximum modulus theorem, Liouville's theorem, fundamental theorem of algebra. Morera's theorem.
- **Expansions and singularities.** Uniform convergence of analytic functions; local uniform convergence. Differentiability of a power series. Taylor and Laurent expansions. Principle of isolated zeros. Residue at an isolated singularity. Classification of isolated singularities.
- **The residue theorem.** Winding numbers. Residue theorem. Jordan's lemma. Evaluation of definite integrals by contour integration. Rouche's theorem, principle of the argument. Open mapping theorem.
- **Conformal mappings with applications.** Geometric definition of conformal mappings. Proof that analytic functions are conformal. Riemann mapping theorem (statement only). Fractional linear transformations, reflections and symmetries. Flows around cylinders. Applications in electrostatics.
- **Analytic continuation and the gamma function.** Uniqueness of analytic continuation; definition and properties of the Gamma function

**Recommended books**:
[1] Ahlfors, Lars V., Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable

**Similar course**:
https://ocw.mit.edu/courses/18-04-complex-variables-with-applications-spring-2018/
(see also lecture notes and literature given there)

**PROBABILITY AND MEASURE** - back to the list of all courses

6 ECTS

This course develops probability theory from elementary models to its modern measure-theoretic foundation. It emphasizes probability as a rigorous framework for quantifying uncertainty, describing random variables and distributions, and analyzing limiting behavior. Students learn how measure and integration clarify the structure of probability spaces, expectations, convergence, and independence, while classical inequalities, laws of large numbers, and central limit theorems explain why stable statistical patterns emerge from randomness. The course aims to connect intuitive probabilistic reasoning with the analytical tools needed for advanced study in probability, statistics, and stochastic modeling.

Prerequisites: Multivariate Analysis, Discrete Mathematics I
Desirable Prerequisites: Topology and Metric Spaces

The lectures cover:

- **Introduction to Probability.** Discrete probability spaces. Classic definition of probability. Combinatorics, counting and the inclusion-exclusion formula. Conditional probability, Bayes's formula. The law of total probabilities. Geometric probability. Bertrand paradox and Buffon's needle. Independence. Binomial, Poisson, geometric and hypergeometric distributions. Poisson limit theorem. De Moivre–Laplace theorem.
- **Introduction to Measure theory (with not that many proofs).** Scheme of the construction of Lebesgue measure on $R^2$. Semi-rings, rings, algebras and sigma-algebras of sets. Premeasure, outer measure and measure. Continuity of measures. Uniqueness of a measure extension. Finite and sigma-finite measures. Lebesgue-Stieltjes measures on the line and in $R^d$. Decomposition of a Lebesgue-Stieltjes measure on the real line into discrete and continuous parts.
- **Lebesgue integration (with not that many proofs).** Measurable functions. Equivalence (up to a set of measure zero). Convergence in measure and convergence almost everywhere. Egorov's theorem. Relation between various modes of convergence. Simple functions. Lebesgue integral of simple functions. Lebesgue integral for measurable functions and its basic properties. Sigma-additivity and absolute continuity of the Lebesgue integral. Absolutely continuous measures and the Radon-Nikodym theorem. Interchanging integration and limit. Monotone convergence theorem, Fatou's lemma, the dominated convergence theorem. Product measures. The Fubini theorem.
- **Probability spaces and random variables.** General definition of a probability space. Continuity and subadditivity of probability measures. Limsup and liminf of a sequence of events. The Borel-Cantelli lemma. Infinite Monkey Theorem. Definition of a random variable (random vector) and its distribution. Expectation and its properties. Distribution functions. Functions of a random variable, indicator function. Variance, covariance and the correlation coefficient. Convolution of probability measures.
- **Discrete random variables.** Probability mass function. Generating functions: sums of independent random variables, random sum formula, moments. Standard discrete distributions including multinomial distribution.
- **Continuous random variables.** Distributions and probability density functions. Expectation. Uniform, normal and exponential random variables. Memoryless property of exponential distribution. Joint distributions: transformation of random variables (including Jacobians), examples. Multivariate normal distribution and Box–Muller transform.
- **Inequalities and limits.** Markov's inequality, Chebyshev's inequality. Almost sure convergence. Convergence in probability. Weak law of large numbers. Strong law of large numbers under 4-th moment condition. Convexity: Jensen's inequality for general random variables. The Cauchy-Schwartz inequality and the Hölder inequality. Moment generating function and characteristic function. Levy's continuity theorem.
- **Central limit theorem.** Convergence in distribution. Central limit theorem for iid random

variables. Triangular arrays of independent variables. The Linderberg condition. The Lindeberg-Feller theorem. Lyapunov's central limit theorem.

*Extra topics (if time permits and up to a lecturer's preference):* Basics of random walks and Poisson process.

**Recommended books:**

[1] Ross, Sheldon M. A First Course in Probability. 10th ed. Boston: Pearson, 2019.
[2] Resnick, Sidney I. Adventures in Stochastic Processes. Boston: Birkhäuser, 1992.
[3] Resnick, Sidney I. A Probability Path. Boston: Birkhäuser, 1999.
[4] Pitman, Jim. Probability. New York: Springer, 1993.
[5] Billingsley, Patrick. Probability and Measure. 3rd ed. New York: Wiley, 1995.

**GEOMETRY** - back to the list of all courses

4 ECTS

This course develops the geometry and topology of surfaces, emphasizing the interplay between local measurements and global structure. Students learn how charts, parametrizations, metrics, curvature, geodesics, and orientability describe surfaces both intrinsically and as objects embedded in three-dimensional space. Central ideas include the relation between curvature and area, the variational meaning of geodesics, and the role of symmetry in Euclidean, spherical, and hyperbolic geometries. The course aims to build a conceptual bridge between differential geometry, topology, and geometric structures on surfaces.

Prerequisites: Multivariate Analysis, Topology and Metric Spaces,
Parallel: Complex Analysis with Applications

*This course is an introduction to Differential Geometry.*

The lectures cover:
- **Surfaces**. Topological surfaces via charts and atlases. Examples including the sphere via stereographic projection, the real projective plane and polygons with side identifications. Smooth surfaces and smooth parametrizations. Orientability. The implicit-function theorem. Riemann surfaces. Informal discussion of triangulations; Euler characteristic and genus.
- **Surfaces in 3-space**. The first fundamental form of an embedded surface in R^3. Length and area. Examples including surfaces of revolution. Change of parametrisation. The second fundamental form. Gauss curvature of an embedded surface. The Gauss map; Gauss curvature and area. Statement of theorema egregium.
- **Geodesics**. Length and energy. Geodesics as critical points for the energy functional; Euler–Lagrange equations for energy. Review of Picard's theorem and existence of geodesics. Examples: geodesics on spheres, flat tori, surfaces of revolution.

- **Hyperbolic surfaces.** Abstract Riemannian metrics on a disc; isometries. Moebius group of the sphere, disc and half-plane. The hyperbolic metric on the disc and half-plane; geodesics and isometries. Gauss–Bonnet theorem for hyperbolic triangles. Hyperbolic hexagons; hyperbolic structures on closed surfaces.
- **Further topics.** Statement of Gauss–Bonnet for geodesic polygons and closed surfaces. Action of SL(2, Z) on the torus; elliptic, parabolic and hyperbolic elements. Moduli space of flat metrics on the torus. Moving frames, Serret-Frenet equations; Klein geometries/symmetric spaces.

**Recommended book:**
[1] Manfredo P. do Carmo, Differential Geometry of Curves and Surfaces. Englewood Cliffs, NJ: Prentice-Hall, 1976

**RINGS AND MODULES** - back to the list of all courses

4 ECTS

This course introduces rings and modules as natural extensions of familiar algebraic systems such as integers, polynomial rings, vector spaces, and abelian groups. It emphasizes how algebraic structures can be studied through ideals, homomorphisms, quotients, factorization, and decomposition. Students learn to distinguish unique from non-unique factorization, use prime and maximal ideals to understand rings, and apply module theory to canonical forms, finitely generated abelian groups, and Jordan normal form. The course aims to build a rigorous foundation for further study in algebra, number theory, and geometry.

Essential prerequisites: Linear Algebra II, Groups;

Desirable prerequisites: Introduction to Number Theory

The lectures cover:

- **Rings**. Definition and examples of rings (commutative, with 1). Ideals, homomorphisms, quotient rings, isomorphism theorems. Prime and maximal ideals. Integral domains. Fields. The characteristic of a field. Field of fractions of an integral domain. Factorization in rings; units, primes and irreducibles. Unique factorization in principal ideal domains, and in polynomial rings. Gauss' Lemma and Eisenstein's irreducibility criterion. Rings Z[α] of algebraic integers as subsets of C and quotients of Z[x]. Examples of Euclidean domains and uniqueness and non-uniqueness of factorization. Factorization in the ring of Gaussian integers; representation of integers as sums of two squares. Ideals in polynomial rings. Hilbert basis theorem.
- **Modules.** Definitions, examples of vector spaces, abelian groups and vector spaces with an endomorphism. Submodules, homomorphisms, quotient modules and direct sums. Tensor products. Equivalence of matrices, canonical form. Structure of finitely generated modules over Euclidean domains, applications to abelian groups and Jordan normal form.

**Recommended books:**
[1] Serge Lang, Undergraduate Algebra

**FIELDS AND GALOIS THEORY** - back to the list of all courses

4 ECTS

This course develops field theory and Galois theory as tools for understanding polynomial equations and their symmetries. It emphasizes how field extensions encode the process of adjoining roots, how automorphism groups reveal the structure of those extensions, and how this structure explains classical questions about constructibility and solvability by radicals. Students learn to connect algebraic properties such as separability, normality, trace, norm, and finite-field structure with concrete applications, including equations of low degree, the impossibility of solving the general quintic by radicals, and arithmetic over finite fields. The course also highlights how finite fields support modern computational and cryptographic methods.

Essential prerequisites: Linear Algebra II, Groups;
Desirable prerequisites: Introduction to Number Theory, Rings and Modules.

The lectures cover:

- **Fields.** Characteristics of a field. Field extensions; the degree of a field extension and the tower theorem. Algebraic elements. Constructions with ruler and compass. Symbolic adjunction of roots of polynomials, multiple roots. Finite fields: existence, uniqueness, primitive elements, number of irreducible polynomials over the field of size p, subfields of finite fields.
- **Field extensions.** Existence and uniqueness of algebraic closure. Separability. Theorem of primitive element. Trace and norm.
- **Galois theory.** Normal and Galois extensions, automorphism groups. Fundamental theorem of Galois theory. Galois theory of finite fields. Reduction mod p. Cyclotomic polynomials, Kummer theory, cyclic extensions. Symmetric functions. Galois theory of cubics and quartics. Solubility by radicals. Insolubility of general quintic equations and other classical problems. Artin's theorem on the subfield fixed by a finite group of automorphisms. Polynomial invariants of a finite group; examples.
- **Applications of finite fields.** Algorithms for constructing finite fields, efficient arithmetic in finite fields, cryptographic applications: the discrete logarithm problem, algorithms for solving the discrete logarithm problem, the Diffie–Hellman algorithm, elliptic curve over finite fields (optional).

* Sylow subgroups and Sylow theorems can be considered in practical sessions.

**Recommended books:**
[1] P. Morandi, Field and Galois Theory (Graduate Texts in Mathematics 167)
[2] J.S.Milne, Fields and Galois Theory
[3] R. Lidl, H. Niederreiter, Finite Fields.
[4] Alfred Menezes (ed), Applications of Finite Fields.

**COMMUTATIVE ALGEBRA** - back to the list of all courses

5 ECTS

Commutative Algebra develops the algebraic language used to study systems of polynomial equations, geometric spaces, and arithmetic structures. The course focuses on commutative rings and modules through the key ideas of prime ideals, localization, tensor products, differentials, finiteness conditions, dimension, valuations, and completion. Students learn how local properties of rings relate to global structure, and how results such as Nakayama's lemma, Noether normalization, the Nullstellensatz, and integral dependence provide tools for modern algebraic geometry and number theory.

Essential Prerequisites: Rings and Modules, Fields and Galois Theory

*This is a graduate course with a lighter option for undergraduate students.*

The lectures cover:
- **Preliminaries about commutative rings**. Principal ideal domains, quotient rings. Basic category theory. Review of Galois theory.
- **Spectral theory.** The spectrum of a commutative ring. Localizations and spectra. Nilradicals.
- **Modules.** Localization of modules. Nakayama's lemma. Tensor products of rings and modules. Flatness. Modules of differentials.
- **Chain conditions**. Noetherian and Artinian rings.
- **Graded rings and modules.** Noether normalization, Nullstellensatz, and their corollaries.
- **Homomorphisms and valuations.** Finite and integral ring homomorphisms. Normal rings. Finiteness of integral closure. Valuation rings and valuations.
- **Krull dimension.** Dimension and transcendence degree.
- **Discrete valuation rings and Dedekind domains.** Modules over PIDs. Ideal class group.
- **Completion**. Complete local rings.

**Recommended books:**
[1] M. F. Atiyah, I. G. Macdonald Introduction to Commutative Algebra, Addison–Wesley 1969
[2] D. Eisenbud Commutative Algebra with a View Toward Algebraic Geometry, Graduate Texts in Mathematics 150, Springer 2004

**REPRESENTATION THEORY** - back to the list of all courses

4 ECTS

Representation Theory studies groups by realizing their elements as linear transformations of vector spaces. The course emphasizes how abstract symmetries become more concrete through matrices, invariant subspaces, irreducible representations, and characters. Students learn how character theory, group algebras, tensor products, induced representations, and compact-group examples reveal the internal structure of groups and their representations. The course aims to develop a conceptual and computational framework for understanding symmetry in algebra, geometry, and mathematical physics.

Essential Prerequisites: Linear Algebra II, Groups, Rings and Modules

The lectures cover:

- **Representations of finite groups.** Representations of groups on vector spaces, matrix representations. Equivalence of representations. Invariant subspaces and submodules. Irreducibility and Schur's Lemma. Complete reducibility for finite groups. Irreducible representations of Abelian groups.
- **Character theory.** Determination of a representation by its character. The group algebra, conjugacy classes, and orthogonality relations. Regular representation. Permutation representations and their characters. Induced representations and the Frobenius reciprocity theorem. Mackey's theorem. Frobenius's Theorem.
- **Arithmetic properties of characters.** Divisibility of the order of the group by the degrees of its irreducible characters. Burnside's pa q b theorem.
- **Tensor products.** Tensor products of representations and products of characters. The character ring. Tensor, symmetric and exterior algebras.
- **Representations of S^1 and SU2.** The groups S^1 , SU2 and SO(3), their irreducible representations, complete reducibility. The Clebsch-Gordan formula. Compact groups.
- **Further worked examples.** The characters of one of GL2 (Fq ), S_n or the Heisenberg group.

[1] Jean-Pierre Serre, *Linear Representations of Finite Groups.*
[2] Gordon James & Martin Liebeck, *Representations and Characters of Groups.*
[3] William Fulton & Joe Harris, *Representation Theory: A First Course.*
[4] I. Martin Isaacs, *Character Theory of Finite Groups.*

**SHEAVES AND COHOMOLOGY** - back to the list of all courses

6 ECTS

*This is a graduate course with a lighter option for undergraduate students. The contents below was proposed by Pierre Schapira, a visiting professor from Sorbonne, who taught the course in the Fall of 2025.*

Prerequisites: Rings and Modules; Topology and Metric Spaces;

The lectures cover:
- **The language of categories.** Sets and maps. Modules and linear maps. Categories and functors. Yoneda, representable functors, adjoint functors.
- **Limits and colimits.** Products and coproducts, kernels and cokernels. Limits and colimits. Filtered colimits.
- **Additive categories.** Additive categories. Complexes. The homotopy category.
- **Abelian categories.** Abelian categories. Right/left exact functors. Cohomology. Koszul complexes.
- **Triangulated categories.** Triangulated categories. Application to the homotopy category. Localization of triangulated categories.
- **Derived categories.** Derived categories. Resolutions. Derived (bi-)functors. The Brown representability theorem.
- **Abelian sheaves.** Introduction: sheaves on topological spaces. Grothendieck topologies. Sheaves. Sheaf associated with a presheaf. Injective and flat sheaves. The derived category of sheaves. internal operations. External operations. Cech complexes. Proper direct images on locally compact spaces.
- **Cohomology of some classical manifolds.** Proper direct images. Sheaves on C^0-manifolds. Spheres and Tori. De Rham cohomology. Dolbeault–Grothendieck cohomology. Leray–Grothendieck integration. Constructible sheaves. Sheaves on subanalytic spaces.

**Recommended books:**
[1] Masaki Kashiwara and Pierre Schapira, An Introduction to Categories and Homological Algebra (2024), available at http://webusers.imj-prg.fr/~pierre.schapira/
[2] Masaki Kashiwara and Pierre Schapira, An Introduction to Sheaves on Grothendieck Topologies (2024), available at http://webusers.imj-prg.fr/~pierre.schapira/

**FUNCTIONAL ANALYSIS** - back to the list of all courses

5 ECTS

The course develops the analytic framework for studying functions, operators, and equations in infinite-dimensional spaces. It emphasizes the structure of L^p, Banach, Hilbert, Sobolev, and distribution spaces, together with the topological and compactness principles that make analysis possible beyond finite dimensions. The course aims to connect abstract functional-analytic methods with applications to partial differential equations, the calculus of variations, and mathematical physics.

Essential Prerequisites: Topology and Metric Spaces, Probability and Measure, Complex Analysis with Applications

The lectures cover:

- **Spaces of integrable functions.** Review of integration: simple functions, monotone and dominated convergence; existence of Lebesgue measure; definition of L^p spaces and their completeness. The Lebesgue differentiation theorem. Egorov's theorem, Lusin's theorem. Mollification by convolution, continuity of translation and separability of L^p.
- **Banach and Hilbert space analysis.** Strong, weak and weak-* topologies; reflexive spaces. Review of the Riesz representation theorem for Hilbert spaces; the dual of L^p . Compactness: review of the Ascoli–Arzelà theorem; weak-* compactness of the unit ball for separable Banach spaces. The Riesz representation theorem for spaces of continuous functions. The Hahn–Banach theorem and its consequences: separation theorems; Mazur's theorem.
- **Fourier analysis.** Definition of Fourier transform in L^1 ; the Riemann–Lebesgue lemma. Fourier inversion theorem. Extension to L^2 by density and Plancherel's isometry. Duality between function's regularity and decay of its Fourier transform.
- **Generalized functions, derivatives and function spaces.** Definition of generalized derivatives and of the basic spaces in the theory of distributions: D/D′ and S/S′ . The Fourier transform on S′ . Periodic distributions; Fourier series; the Poisson summation formula. Definition of the Sobolev spaces H^s in R^d . Sobolev embedding. The Rellich–Kondrashov theorem. The trace theorem.
- **Linear operators in infinite-dimensional spaces.** Continuity and boundedness of linear operators. Operator norm. Invertible operators. Spectrum of an operator. Self-adjoint operators and the spectral theorem. Resolvents. Operator algebras. Compact operators.
- **Applications.** Construction and regularity of solutions for elliptic PDEs with constant coefficients on R^n . Construction and regularity of solutions for the Dirichlet problem of Laplace's equation. The spectral theorem for the Laplacian on a bounded domain. The direct method of the Calculus of Variations.

**Recommended course:**
https://ocw.mit.edu/courses/18-102-introduction-to-functional-analysis-spring-2021/

**Recommended Books:**
[1] Dunford, Nelson, and Jacob T. Schwartz. Linear Operators. Part I: General Theory. New York: Wiley, 1958.
[2] Folland, Gerald B. Real Analysis: Modern Techniques and Their Applications. 2nd ed. New York: Wiley, 1999.
[3] Rudin, Walter. Functional Analysis. 2nd ed. New York: McGraw-Hill, 1991.
[4] Stein, Elias M., and Rami Shakarchi. Fourier Analysis: An Introduction. Princeton, NJ: Princeton University Press, 2003.
[5] Evans, Lawrence C. Partial Differential Equations. 2nd ed. Providence, RI: American Mathematical Society, 2010.

**STOCHASTIC PROCESSES** - back to the list of all courses

6 ECTS

Stochastic Processes develops the mathematical framework for studying random phenomena that evolve over time or space. The course emphasizes how dependence, memory, recurrence, stationarity, and limiting behavior shape probabilistic models such as Markov chains, renewal processes, queues, martingales, point processes, Gaussian processes, and Brownian motion. Students learn to analyze both discrete and continuous random systems using transition structures, stopping arguments, convergence theorems, and probability measures on infinite-dimensional spaces. The course aims to build the conceptual and technical foundations for applications in statistics, finance, physics, operations research, and stochastic modeling.

Essential Prerequisites: Probability and Measure, Introduction to Statistics
Desirable Prerequisites: Topology and Metric Spaces, Functional Analysis

The lectures cover:

- **Discrete-time Markov Chains.** Transition operator. Kolmogorov-Chapman equations. Classification of states. Aperiodicity and Irreducibility. Transience and recurrence. Invariant measures and stationary distributions. Coupling technique and Doeblin's theorem. Null-recurrence. Ergodicity.
- **Renewal Theory.** Renewal function and its properties. Elementary Renewal Theorem. SLLN and CLT for the renewal counting function. Renewal equation. Stationary renewal process. Blackwell's theorem. Direct Riemann integrability. Key Renewal Theorem.
- **Queuing theory.** Kendall's notation. M/M/1 queue. M/G/1 queuing system and embedded Markov chains. GI/M/s queuing system.
- **Discrete-time martingales.** Filtrations. Uniform integrability. Supermartingales and their a.s. convergence. Martingales and submartingales. Stopping times. Optional stopping theorem.
- **Probability measures on infinite-dimensional spaces.** Cylindrical sigma-algebras. Finite-dimensional distributions. Separability. Kolmogorov's extension theorem. Tightness of probability measures. Prokhorov's theorem.
- **Point processes and Poisson Processes.** Dirac and point measures. Random point measures. Definition of an abstract Poisson process. Intensity measure. Existence theorem. Mapping theorem. Superposition theorem for Poisson processes. Marking theorem. Poisson process on the real line.
- **Gaussian Processes and Brownian Motion.** Definition. Mean and covariance functions. Mean square and sample paths continuity. Brownian motion: independent increments, continuity and non-differentiability properties. Reflection principle. Donsker's Invariance Principle. Geometric Brownian motion.

**Recommended books:**

[1] Karlin, S., & Taylor, H. M. (1975). *A First Course in Stochastic Processes*. Academic Press.

[2] Resnick, S. I. (1992). *Adventures in Stochastic Processes*. Birkhäuser.
[3] Kallenberg, O. (2002). *Foundations of Modern Probability* (2nd ed.). Springer.
[4] Asmussen, S. (2003). *Applied Probability and Queues* (2nd ed.). Springer.
[5] Mörters, P., & Peres, Y. (2010). *Brownian Motion*. Cambridge University Press.
[6] Скороход, А. В. (1990). *Лекції з теорії випадкових процесів: навчальний посібник*. Київ: Либідь.

**OPTIMISATION I** - back to the list of all courses

4 ECTS

The course studies how to make optimal choices under mathematical constraints. It emphasizes the geometric and economic meaning of optimization: convexity as a source of global optimality, duality as a way to interpret constraints and prices, and algorithms as systematic procedures for finding solutions. Students learn to connect theoretical optimality conditions with practical methods for constrained decision-making, including problems involving allocation, competition, flows, and networks.

Prerequisites: Multivariate Analysis, Linear Algebra I

The lectures cover:

- **Elements of convex optimisation.** Convex sets and functions in R^n, global and constrained optimality. Algorithms for unconstrained convex optimisation: gradient descent, Newton's algorithm. Introduction to convex optimisation on a convex set, the barrier method. Examples.
- **Lagrangian methods and duality.** General formulation of constrained problems; the Lagrangian sufficiency theorem. Interpretation of Lagrange multipliers as shadow prices. The dual linear problem, duality theorem in a standardized case, complementary slackness, dual variables and their interpretation as shadow prices. Relationship of the primal simplex algorithm to dual problem. Examples.
- **Linear programming in the nondegenerate case**. Convexity of feasible region; sufficiency of extreme points. Standardization of problems, slack variables, equivalence of extreme points and basic solutions. The primal simplex algorithm and the tableau. Examples.
- **Applications of linear programming.** Two person zero-sum games. Network flows; the max-flow min-cut theorem; the Ford–Fulkerson algorithm, the rational case. Network flows with costs, the transportation algorithm, relationship of dual variables with nodes. Examples. Conditions for optimality in more general networks. The formulation of simple practical and combinatorial problems as linear programming or network problems.

**Recommended books**:

[1] Boyd, Stephen, and Lieven Vandenberghe. Convex Optimization. Cambridge: Cambridge University Press, 2004.

[2] Rockafellar, R. Tyrrell. Convex Analysis. Princeton, NJ: Princeton University Press, 1970.
[3] Introduction to Linear Optimization — Bertsimas, Dimitris, and John N. Tsitsiklis. Introduction to Linear Optimization. Belmont, MA: Athena Scientific, 1997.

**MANIFOLDS AND DIFFERENTIAL FORMS** - back to the list of all courses

4 ECTS

The core idea of the course is to develop the language of calculus on curved spaces. The course emphasizes how local coordinate computations can be organized into coordinate-free objects such as vector fields, tensors, densities, and differential forms. Students learn how integration, change of variables, degree, and Stokes' theorem fit into a unified geometric framework, and how de Rham theory connects analysis on manifolds with topological information. The course aims to build a conceptual foundation for modern geometry, topology, and mathematical physics.

Essential Prerequisites: Multivariate Analysis, Topology and Metric Spaces;
Desired Prerequisites: Groups, Rings and Modules, Complex Analysis with Applications; Geometry

*This course is taught as a part of the graduate course Differential Geometry (M).*

Introduction to the theory of manifolds: vector fields and densities on manifolds, integral calculus in the manifold setting and the manifold version of the divergence theorem. Multilinear algebra: tensors and exterior forms. Differential forms on R^n: exterior differentiation, the pull-back operation and the Poincaré lemma. Applications to physics: Maxwell's equations from the differential form perspective. Integration of forms on open sets of R^n. The change of variables formula revisited. The degree of a differentiable mapping. Differential forms on manifolds and De Rham theory. Integration of forms on manifolds and Stokes' theorem.

**EXPERIMENTAL MATHEMATICS** - back to the list of all courses

3 ECTS

A course in numerical and symbolic methods of experimental mathematics involving extensive practical sessions using such software as Maple, Sage, MATLAB. In this course, students perform computational projects under the supervision of the course assistants. Every year in the fall a list of projects is collected by the faculty of the Department of Mathematics. The author of a project may be involved as a consultant in the course of work to the extent their time permits.

Prerequisites: Programming Basics (CS)

**DISCRETE MATHEMATICS II** - back to the list of all courses

6 ECTS

This is a more advanced course that deepens students' understanding of discrete structures and methods beyond the introductory level. It emphasizes how combinatorial problems can be approached through optimization, generating functions, probabilistic reasoning, additive structure, and Ramsey-type phenomena. Students learn to combine algorithmic, algebraic, analytic, and probabilistic tools to study graphs, partitions, set systems, sumsets, and patterns in finite structures. The course aims to develop flexible techniques for advanced work in discrete mathematics, theoretical computer science, and related areas.

Prerequisites: Linear algebra II , Discrete Mathematics I
Desirable Prerequisites: Optimization I

Study of illustrative topics in discrete applied mathematics, including probability theory, information theory, coding theory, secret codes, generating functions, and linear programming. It is recommended to practice written communication with this course. This component requires teaching assistance; it may take the form of writing assignments (often proofs) in homework problems; one of the assignments may be a relatively long term paper whose writing would take a few iterations.

The lectures cover:

- **Combinatorial Optimization.** Minimum spanning trees. Dijkstra's algorithm. Matchings, Konig's theorem and Hall's theorem. Max-Flow and Min-Cut, Manger's theorem.
- **Generating Functions.** Binomial series and Catalan numbers. Exponential generating functions and Stirling numbers. Integer partitions. Euler product. Using generating functions to prove identities.
- **Probabilistic Combinatorics.** Random variables; expected value and variance. Markov's inequality. Chebyshev's inequality. Chernoff's inequality. First and second moment method. Estimates on Ramsey numbers. Lovasz Local Lemma (if time permits).
- **Introduction to additive Combinatorics.** Sumsets and doubling constant. Ruzsza distance. Cauchy-Davenport theorem. Freiman-Ruzsza theorem. Discrete Fourier transform. Plancharel identity. Fourier pseudorandomness. Roth's theorem (if time permits).
- **Ramsey theory** (expository lectures). Ramsey number for graphs, small values. Hypergraphs. Rado's theorem. Van der Waerden theorem. Szemeredi's theorem. Gowers norms and higher order Fourier analysis.

**Recommended books:**

[1] Noga Alon & Joel H. Spencer — *The Probabilistic Method*
[2] Béla Bollobás — *Modern Graph Theory*
[3] Terence Tao & Van H. Vu — *Additive Combinatorics*
[4] Richard P. Stanley — *Enumerative Combinatorics, Volumes 1 and 2*

**AUTOMATA AND FORMAL LANGUAGES** - back to the list of all courses

4 ECTS

The course develops the theory of formal models of computation, from finite-state machines to Turing machines and beyond. It emphasizes how different computational models recognize patterns, define languages, express algorithms, and reveal the limits of decidability. It also explores automatic sequences as a bridge between automata theory, number representation, logic, and algebra. Students learn to connect formal languages, computability, numeration systems, and structural classification results, building a foundation for theoretical computer science and theory of algorithms.

Essential prerequisites: Discrete Mathematics I
Desired prerequisites: Discrete Mathematics II

The lectures cover:

- **Finite automata and regular languages.** Deterministic and nondeterministic finite automata (DFA, NFA). Regular languages and regular expressions. Equivalence of models. Closure properties. Minimisation of automata. Pumping lemma for regular languages.
- **Context-free languages.** Push-down automata. Context-free grammars. Ambiguity. Normal forms. Closure properties. Pumping lemma for context-free languages.
- **Computability theory.** Turing machines. Variants and robustness of the model. The Church–Turing thesis. Decidable and undecidable problems. The halting problem. Reducibility. The recursion theorem.
- **Automatic sequences and numeration systems.** Automatic sequences via finite automata (DFAO). Examples: Thue–Morse, Rudin–Shapiro, paperfolding sequences. k-kernel and closure properties. Finite-state transducers. Morphic and k-regular sequences.
- **Structure and classification results.** Cobham's theorem. Christol's theorem (statement and basic consequences). Connections with algebraic power series over finite fields. Subword complexity and frequency of letters.
- **Logical and algorithmic aspects.** Logical definability of automatic sequences. Decidability and automatic theorem proving (e.g. Walnut). Automatic sets of integers and rationals.
- **Non-standard numeration systems.** Representations beyond base-k (e.g. Zeckendorf representation). Automata over alternative numeration systems. Extensions of automaticity.

**Recommended books:**

[1] Michael Sipser — Introduction to the Theory of Computation
[2] John E. Hopcroft, Rajeev Motwani & Jeffrey D. Ullman — Introduction to Automata Theory, Languages, and Computation
[3] Jean Berstel & Dominique Perrin — Theory of Codes
[4] Jean-Paul Allouche & Jeffrey Shallit — Automatic Sequences: Theory, Applications, Generalizations

**NUMERICAL METHODS** - back to the list of all courses

4 ECTS

This course introduces numerical techniques for solving mathematical problems that may be difficult or impossible to address analytically. Emphasis is placed on the development, analysis, and implementation of algorithms to approximate solutions.

Prerequisites: Linear Algebra II, Multivariate Analysis, Differential Equations;

The lectures cover:

- **Error Analysis.** Understanding approximation errors and their propagation.
- **Root-Finding Methods**. Techniques such as the bisection method, Newton's method, and secant method.
- **Interpolation and Polynomial Approximation**. Lagrange and spline interpolation, least-squares approximation.
- **Numerical Differentiation and Integration**. Techniques for approximating derivatives and integrals, including trapezoidal and Simpson's rules.
- **Solving Systems of Linear Equations**. Gaussian elimination, LU decomposition, iterative methods.
- **Numerical Solutions to Differential Equations**. Euler's method, Runge-Kutta methods, stability and convergence.

**Recommended books:**
[1] Burden, Richard L., and J. Douglas Faires. Numerical Analysis. 10th ed. Boston: Cengage Learning, 2015.
[2] Atkinson, Kendall E. An Introduction to Numerical Analysis. 2nd ed. New York: Wiley, 1989.

**INTRODUCTION TO STATISTICS** - back to the list of all courses

4 ECTS

Statistics introduces the principles and methods used to collect, summarize, model, and interpret data. The course emphasizes the transition from descriptive analysis to statistical inference, showing how probability and sampling theory allow conclusions about populations to be drawn from limited observations. Students learn to quantify uncertainty, test hypotheses, estimate unknown parameters, and analyze relationships between variables through regression and comparison of groups.

Essential Prerequisites: Linear Algebra I, Probability and Measure

The lectures cover:

- **Introduction. Organization and Description of Data.** Introduction to fundamental statistical concepts, including populations and samples. Methods for organizing and summarizing data using tables and graphical representations. Measures of central tendency (mean, median, mode) and measures of variability (range, variance, standard deviation).
- **Probability Distributions**. Review of basic probability concepts. Study of continuous random variables and their distributions, with emphasis on the normal distribution. Introduction to the normal approximation to the binomial distribution.
- **Sampling Distributions**. Concept of sampling distributions and their role in statistical inference. Distribution of the sample mean and statement and applications of the Central Limit Theorem.
- **Statistical Inference.** Methods for drawing conclusions about populations from samples. Point and interval estimation of a population mean. Hypothesis testing procedures for a population mean. Inference for small samples from normally distributed populations.
- **Regression and Correlation Analysis.** Introduction to relationships between variables. Simple linear regression, estimation using the method of least squares, and inference in regression models. Basic concepts of correlation.
- **Multiple Linear Regression.** Extension of linear regression to multiple predictors. Interpretation of coefficients and inference in multiple regression models.
- **Analysis of Variance (ANOVA).** Methods for comparing means across multiple groups using one-way ANOVA. Assumptions, computations, and interpretation of results.

**Recommended books:**

[1] Montgomery, D. C., & Runger, G. C. (2018). *Applied statistics and probability for engineers* (7th ed.). Wiley.
[2] Devore, J. L. (2015). *Probability and statistics for engineering and the sciences* (9th ed.). Cengage Learning.

**OPTIMISATION II** - back to the list of all courses

6 ECTS

Optimization II is an advanced course that develops a deeper theoretical and algorithmic understanding of convex optimization and its extensions. It emphasizes the structural role of convexity, separation, duality, smoothness, strong convexity, and generalized optimality conditions in designing and analyzing optimization methods. Students learn how modern iterative algorithms arise from geometric and variational principles, and how regularization, monotone operator methods, primal–dual schemes, and bilevel formulations address complex or ill-posed problems. The course aims to prepare students for advanced work in optimization, applied mathematics, data science, and systems with distributed parameters.

Essential Prerequisites: Multivariable Analysis, Linear Algebra II, Probability and Measure, Optimization I

Desirable Prerequisites: Topology and Metric Spaces

The lectures cover:

- **Convex sets: separability , duality, smoothness and strong convexity.** Convex Sets and Functions, Separation Theorems, Subgradients, Optimality Conditions, Duality, Smoothness and Strong Convexity, Proximal Operator, Convex Feasibility Problem
- **Algorithms.** Von Neumann Alternating Method, Gradient and Conditional Gradient Method, Gradient Flows, Subgradient Methods, Proximal Gradient Method, Primal–Dual Methods, Alternating Direction Method of Multipliers.
- **Applications**. Variational Inequalities, Methods for Solving Variational Inequalities with Monotone Operators, Ill-Posed Optimization Problems, Tikhonov Regularization Method for Optimization Problems. Bakushinskii Iterative Regularization Method, Bilevel Optimization Problems, Moore–Penrose Pseudoinverse Operator, Optimization Problems for Systems with Distributed Parameters

**Recommended books:**

[1] Polyak R.A. Introduction to Continuous Optimization. Springer Cham, 2021.
[2] Beck A. First-Order Methods in Optimization. Philadelphia: Society for Industrial and Applied Mathematics, 2017.
[3] Bauschke H.H., Combettes P.L. Convex Analysis and Monotone Operator Theory in Hilbert Spaces. Berlin, Heidelberg, NY: Springer, 2017.
[4] Ryu E.K., Yin W. Large-Scale Convex Optimization. Algorithms and Analysis via Monotone Operators. Cambridge: Cambridge University Press, 2023.

**MATHEMATICAL STATISTICS** - back to the list of all courses

6 ECTS

Mathematical Statistics develops the theoretical foundations of statistical inference. The course emphasizes how probability, sampling, and asymptotic principles justify methods for estimating unknown quantities, testing hypotheses, and comparing competing models. Students learn to evaluate statistical procedures through bias, variance, consistency, efficiency, sufficiency, and information, while also contrasting frequentist and Bayesian approaches to uncertainty. The course aims to build a rigorous basis for advanced work in statistics, econometrics, data science, and applied research. This is an advanced version of the “Introduction to Statistics” course.

Prerequisites: Linear Algebra II, Multivariable Analysis, Differential Equations, Probability and Measure,

The lectures cover:

- **Probability & Sampling Foundations**: Random variables, distributions, expectation, variance. Joint distributions, covariance, independence. Law of Large Numbers, Central Limit Theorem. Sampling distributions: Sample mean and variance. Normal case, introduction to t-distribution. Empirical distribution function (EDF) and its properties. Glivenko–Cantelli theorem and uniform convergence of the EDF.
- **Point Estimation:** Statistical models, estimators, bias, MSE. Consistency and asymptotic normality (informal). Method of moments. Maximum likelihood estimation (MLE): definition and properties, examples (normal, binomial, exponential).
- **Sufficiency & Efficiency:** Likelihood function, sufficient statistics, Fisher–Neyman factorization theorem (with examples), Fisher information, Cramér–Rao bound (basic form), Rao–Blackwell theorem (statement + main idea).
- **Confidence Intervals:** Concept of confidence intervals. CI for mean: known variance (normal), unknown variance (t-Student). Asymptotic confidence intervals (CLT-based).
- **Hypothesis Testing:** Testing framework, errors, power, p-values. Neyman–Pearson lemma (basic form + intuition). Likelihood ratio tests. Tests for mean (z-test, t-test). Chi-square tests (goodness-of-fit, independence). Nonparametric methods: Kolmogorov–Smirnov test (idea), brief mention of distribution-free methods.
- **Bayesian Inference:** Bayesian framework and prior distributions. Posterior and predictive distributions. Maximum a posteriori (MAP) estimation. Conjugate priors (basic examples). Credible intervals and comparison with confidence intervals.

*Extra topics (if time permits):*

**Order statistics:** Definition and basic properties, Distribution functions of order statistics, Order statistics for the uniform distribution. Estimators based on order statistics.
**Lehmann–Scheffé theorem** and uniformly minimum variance unbiased estimators (UMVUE).

**Recommended books:**

[1] E.L. Lehmann, George Casella. Theory of Point Estimation. Springer, 1998.
[2] P.K. Bhattacharya and Prabir Burman. Theory and Methods of Statistics. Academic Press, 2016.
[3] V.G. Voinov , M.S. Nikulin. Unbiased Estimators and Their Applications. Springer, 1993.

**ANALYSIS OF DIFFERENTIAL EQUATIONS** - back to the list of all courses

6 ECTS

Analysis of Differential Equations develops advanced methods for understanding differential equations beyond explicit solution techniques. The course emphasizes how analytic, qualitative, spectral, and transform methods reveal the structure of solutions, their stability, asymptotic behavior, and response to forcing or boundary conditions. Students learn to analyze singularities, boundary value problems, oscillation and orthogonality phenomena, responses to impulses and discontinuous forcing, and the emergence of periodic or chaotic behavior. The course aims to provide a conceptual and technical foundation for modern analysis, mathematical physics, and applied differential equations.

Prerequisites: Differential Equations, Multivariable Analysis, Linear Algebra II, Topology and Metric Spaces, Complex Analysis

The lectures cover:

- **Power Series Solutions and Special Functions.** Power series solutions about ordinary and regular singular points; the method of Frobenius. Cauchy-Euler equations. Theory of special functions: Bessel's equation, solutions of the first and second kind, recurrence relations. Introduction to Jacobi's theta functions and their role in mathematical physics.
- **Qualitative Theory and Dynamical Systems.** Foundations of existence and uniqueness: the Cauchy-Lipschitz (Picard–Lindelöf) theorem and Grönwall's inequality. Stability analysis for nonlinear systems: equilibrium points and linearization. Liapunov's second (direct) method. Periodic solutions, limit cycles, and the Poincaré–Bendixson theorem. Introduction to deterministic chaos: the Lorenz equations and strange attractors.
- **Sturm–Liouville Theory and Eigenfunction Expansions.** Self-adjoint differential operators and Hilbert spaces. The Sturm–Liouville problem: eigenvalues and eigenfunctions. Reality of eigenvalues and orthogonality. Completeness and mean square approximation. Fourier series as a prototype for eigenfunction expansions; Parseval's theorem.
- **Green's Functions and Distribution Theory.** Theory of generalized functions (distributions): the Dirac delta function and its properties. Construction of Green's functions for initial and boundary value problems. Eigenfunction expansions of the delta function. Physical interpretation: impulse functions and response to discontinuous forcing.
- **Integral Transforms and Applications.** Fourier transforms: definition, inversion, and convolution theorems. The Discrete Fourier Transform (DFT). Relationship between the transfer function and the Green's function. Applications to linear systems and the Central Force (Kepler) problem.
- **Integral Equations.** Classification of integral equations: Fredholm and Volterra types. Relationship between differential and integral equations. Methods of solution: resolvent kernels and transform techniques.

Extra topics (if time permits):

**Symmetry Analysis**
Introduction to Lie point symmetries; the concept of invariance and infinitesimal generators. Application of symmetry methods for order reduction and integration. Examples: scaling symmetries, translational invariance in autonomous systems, and rotational symmetry.

**Recommended books:**
[1] Mathematical Methods for Physicists / G.B. Arfken, H.J. Weber & F.E. Harris, Elsevier 2013
[2] Mathematical Methods in the Physical Sciences / M.L. Boas, Wiley 2005
[3] Elementary Differential Equations and Boundary Value Problems, Boyce, W. E., DiPrima, R. C., Meade, D. B.,Wiley, 2021.

[4] Nonlinear Dynamics and Chaos, Strogatz, S. H., Boca Raton: CRC Press, 2014.
[5] The Fourier Transform and Its Applications, Osgood, B. G., Stanford University Lecture Notes, 2019 (online).
[6] Ordinary Differential Equations and Dynamical Systems, Teschl, G., Providence, RI: AMS, 2012.
[7] Introduction to the Theory of Distributions, Friedlander, F. G., Joshi, M., Cambridge University Press, 1998.
[8] Applications of Lie Groups to Differential Equations, Olver, P. J.,2nd ed. New York: Springer, 1993.
[9] Integral Equations, Tricomi, F. G., New York: Dover Publications, 1985.
[10] Fourier Analysis and Its Applications, Folland, G. B., Providence, RI: AMS, 2009.

Parts of this course are also relevant:
https://legacy-www.math.harvard.edu/archive/115_fall_06/index.html

**INTRODUCTION TO PDEs** - back to the list of all courses

6 ECTS

Introduction to PDEs introduces the main classes of partial differential equations and the methods used to study them. The course explains how elliptic equations model equilibrium, parabolic equations model diffusion and smoothing, and hyperbolic equations model waves, propagation, and finite-speed effects. Students learn how classical solution formulas, weak formulations, Sobolev spaces, variational principles, maximum principles, and energy estimates are used to prove existence, uniqueness, and qualitative properties of solutions. The course also shows why nonlinear equations require new ideas, such as shocks, entropy conditions, and viscosity solutions, and introduces computational approaches for solving PDEs in applications.

Essential Prerequisites: Differential Equations, Multivariable Analysis, Linear Algebra II, Topology and Metric Spaces, Complex Analysis

Desirable Prerequisites: Functional Analysis

The lectures cover:
- **Foundations and First-Order Equations.** Definition and classification of PDEs: elliptic, parabolic, and hyperbolic. Linear and quasilinear first-order equations. The method of characteristics. Introduction to Hamilton–Jacobi equations and the concept of viscosity solutions.
- **Classic Linear PDEs.** Laplace Equation: Harmonic functions, mean value property, and the maximum principle. Heat Equation: Fundamental solution, the heat kernel, and smoothing effects. Wave Equation: d’Alembert’s formula, domain of dependence, and causality. Method of separation of variables
- **Functional Framework: Sobolev Spaces.** Motivation for weak solutions. Distributions and weak derivatives. Sobolev spaces $W^{k,p}$ and $H^s$. The Lax-Milgram Theorem and its

application to the existence of weak solutions for the Poisson equation. Sobolev Embedding Theorems (statement and intuition).
- **Variational and Energy Methods.** Dirichlet's principle and energy functional. Existence and uniqueness for elliptic boundary value problems. Maximum principles (weak and strong forms) for elliptic and parabolic equations.
- **Nonlinear PDEs and Conservation Laws.** Introduction to non-linear phenomena. Conservation laws, Burgers' equation, and the formation of shocks. Entropy conditions and Rankine-Hugoniot jump conditions.
- **Modern Numerical and Spectral Methods.** Green's functions and eigenfunction expansions. Spectral methods via Fourier and Chebyshev polynomials. Solving PDEs with machine learning.

**Recommended books:**
[1] Lawrence C. Evans, Partial Differential Equations. AMS, 2022.
[2] S. Salsa, Partial Differential Equations in Action: From Modelling to Theory. Springer, 2016.
[3] H. Brezis, Functional Analysis, Sobolev Spaces and PDE. Springer, 2011.
[4] J.N. Kutz, Deep Learning in Science and Engineering. CRC Press, 2023.

**LOGIC AND SET THEORY** - back to the list of all courses

4 ECTS

The course explains how infinite objects are organized through ordinals, cardinals, well-orderings, and set-theoretic hierarchies, and how principles such as induction, recursion, and Zorn's lemma support constructions across algebra, analysis, and topology. It also develops the syntax and semantics of formal logic, showing how mathematical statements can be proved, interpreted, and studied through models. Students learn to understand both the power and the limitations of formal systems, including questions of completeness, compactness, consistency, and independence.

Prerequisites: Discrete Mathematics II

The lectures cover:
- **Ordinals and cardinals.** Well-orderings and order-types. Examples of countable ordinals. Uncountable ordinals and Hartogs' lemma. Induction and recursion for ordinals. Ordinal arithmetic. Cardinals; the hierarchy of alephs. Cardinal arithmetic.
- **Posets and Zorn's lemma.** Partially ordered sets; Hasse diagrams, chains, maximal elements. Lattices and Boolean algebras. Complete and chain-complete posets; fixed-point theorems. The axiom of choice and Zorn's lemma. Applications of Zorn's lemma in mathematics. The well-ordering principle.
- **Propositional logic.** The propositional calculus. Semantic and syntactic entailment. The deduction and completeness theorems. Applications: compactness and decidability.
- **Predicate logic.** The predicate calculus with equality. Examples of first-order languages and theories. Statement of the completeness theorem; sketch of proof. The compactness

theorem and the Löwenheim-Skolem theorems. Limitations of first-order logic. Model theory.
- **Set theory.** Set theory as a first-order theory; the axioms of ZF set theory. Transitive closures, epsilon-induction and epsilon-recursion. Well-founded relations. Mostowski's collapsing theorem. The rank function and the von Neumann hierarchy.
- **Consistency.** Problems of consistency and independence.

**Recommended books:**
[1] Thomas Jech — *Set Theory*
[2] Herbert B. Enderton — *A Mathematical Introduction to Logic*
[3] Joseph R. Shoenfield — *Mathematical Logic*
[4] Kenneth Kunen — *Set Theory*

**ACTUARIAL MATHEMATICS** - back to the list of all courses

4 ECTS

This mini-course introduces the mathematical foundations of actuarial modeling, with emphasis on life-contingent risks, premium calculation, reserves, aggregate claims, and ruin theory. Students learn how probabilistic models are used to value insurance products, quantify risk, and analyze solvency.

Prerequisites: Multivariate Analysis, Probability & Measure

Desirable prerequisite*:* Stochastic Processes

The lectures cover:

- **Introduction to actuarial mathematics.** Insurance cash flows, time value of money, deterministic interest, discounting and accumulation, actuarial notation.
- **Survival models and lifetime distributions.** Future lifetime, curtate lifetime, survival and distribution functions, force of mortality. Life tables, survival and death probabilities, select and ultimate tables, Gompertz and Makeham laws.
- **Life insurance models.** Whole life, term life, deferred and endowment insurance; present-value random variables; actuarial present values. Life annuities**.** Continuous and discrete annuities, annuities-due and immediate annuities, temporary and deferred annuities.
- **Premium calculation principles.** Equivalence principle, net and gross premiums, expense loadings, level premiums, single premiums.
- **Collective risk model.** Aggregate claims, frequency-severity models, compound distributions, expectation and variance of aggregate loss, compound Poisson model. Cramér–Lundberg risk model. claim arrival process, aggregate claims process.
- **Ruin theory.** Ruin probability, finite- and infinite-time ruin. Lundberg inequality, adjustment coefficient, exponential claim sizes, integro-differential equations for ruin probability, interpretation for solvency.

- **Risk measures, reinsurance, and modern applications.** Value-at-Risk, Tail Value-at-Risk, solvency capital, proportional and excess-of-loss reinsurance, overview of actuarial modeling in practice.

**Recommended books:**

[1] Bowers, N. L., Gerber, H. U., Hickman, J. C., Jones, D. A., and Nesbitt, C. J. Actuarial Mathematics. 2nd ed. Society of Actuaries, 1997.
[2] Dickson, D. C. M., Hardy, M. R., and Waters, H. R. Actuarial Mathematics for Life Contingent Risks. 3rd ed. Cambridge University Press, 2020.
[3] Dickson, D. C. M. Insurance Risk and Ruin. Cambridge University Press, 2005.

**MATHEMATICAL ECONOMICS** - back to the list of all courses

6 ECTS

*The course introduces mathematical models and methods used in modern economic theory. Emphasis is placed on rigorous formulation, optimization, equilibrium analysis, dynamic models, uncertainty, and selected applications to markets, games, finance, and mechanism design. The course is focused on developing the mathematical background.*

Essential prerequisites: Multivariate Analysis , Linear algebra I, Optimisation I, Probability & Measure

Desirable prerequisite*:* Stochastic Processes

The lectures cover:

- **Introduction to mathematical economics.** Economic models, agents, preferences, constraints, equilibrium, optimization, comparative statics.
- **Preference relations and utility representation.** Rational preferences, completeness, transitivity, continuity, monotonicity, convexity, utility functions.
- **Consumer choice theory.** Budget sets, utility maximization, Marshallian demand, indirect utility, expenditure minimization. Hicksian demand, expenditure function, Roy's identity, Shephard's lemma, Slutsky equation.
- **Production theory.** Production sets, production functions, returns to scale, convexity, cost minimization, profit maximization.
- **Optimization in economics.** Lagrange multipliers, Kuhn–Tucker conditions, envelope theorem, comparative statics. Edgeworth box, Pareto efficiency, competitive equilibrium, welfare theorems. Existence of equilibrium**.** Fixed point theorems, Kakutani theorem, excess demand, existence of Walrasian equilibrium.
- **Game theory.** Strategies, best responses, dominant strategies, mixed strategies, Nash equilibrium. Brouwer/Kakutani fixed point approach, zero-sum games, minimax theorem, correlated equilibrium.

- **Dynamic optimization.** Intertemporal choice, Bellman equation, contraction mapping, value function iteration. Continuous-time models, Hamiltonian, Pontryagin maximum principle, economic growth examples.
- **Uncertainty and expected utility.** Lotteries, von Neumann–Morgenstern utility, risk aversion, certainty equivalent, stochastic dominance.
- **Social choice and welfare.** Social welfare functions, Arrow impossibility theorem, voting rules, Pareto optimality.
- **Mathematical finance.** Arbitrage, state-price vectors, risk-neutral probabilities, complete markets. Mean-variance optimization, CAPM, utility-based portfolio selection.

**Recommended books:**

[1] Mas-Colell, A., Whinston, M. D., and Green, J. R. Microeconomic Theory. Oxford University Press, 1995.
[2] Simon, C. P., and Blume, L. Mathematics for Economists. W. W. Norton, 1994.
[3] Sundaram, R. K. A First Course in Optimization Theory. Cambridge University Press, 1996.
[4] Osborne, M. J., and Rubinstein, A. A Course in Game Theory. MIT Press, 1994.
[5] Föllmer, H., and Schied, A. Stochastic Finance: An Introduction in Discrete Time. De Gruyter, 2016.

**MATHEMATICAL TOPICS FOR MACHINE LEARNING** - back to the list of all courses

6 ECTS

Not a machine learning class, but an introduction to a selection of mathematical topics relevant for contemporary machine learning.

Essential prerequisites: Multivariate Analysis , Linear algebra I, Probability & Measure, Optimisation I

Desirable prerequisites: Functional Analysis, Introduction to PDEs.

The lectures cover:

- **Approximation and expressiveness**. Universal approximation theorems. Approximation rates for smooth function classes, curse of dimensionality for linear methods.
- **Optimization.** Gradient descent for convex problems: convergence rates, role of strong convexity and smoothness. Stochastic gradient descent. Non-convex optimization: Łojasiewicz inequalities.
- **Concentration and high-dimensional probability**. Sub-Gaussian and sub-exponential random variables, Hoeffding and Bernstein inequalities. Gaussian concentration, measure concentration on the sphere. Johnson-Lindenstrauss lemma.
- **Random matrix theory**. Marchenko-Pastur law, Wigner semicircle law. The BBP phase transition.

- **Optimal transport and sampling**. Monge and Kantorovich formulations, existence of optimal plans, Wasserstein distances. Duality theory, Brenier's theorem, structure of optimal maps. Entropic regularization and the Sinkhorn algorithm. Langevin dynamics (overdamped), Fokker-Planck equation, convergence.

Extra topics (if time permits): **Vapnik-Chervonenkis Theory.**

**Recommended books:**

[1] Roman Vershynin — High-Dimensional Probability: An Introduction with Applications in Data Science.
[2] Stéphane Mallat — Understanding Deep Learning
[3] Cédric Villani — Optimal Transport: Old and New
[4] Martin J. Wainwright — High-Dimensional Statistics: A Non-Asymptotic Viewpoint

**MATHEMATICAL TOPICS IN STATISTICAL PHYSICS** - back to the list of all courses

5 ECTS

The goal is introducing some of the main statistical physics models that are studied by mathematicians, and the core techniques.

Essential prerequisites: Probability and Measure, Statistical Physics and Thermodynamics

The lectures cover:

- **Mathematical foundations:** ensembles, Gibbs measures, ergodicity, concentration of measure.
- **Elements of percolation theory.**
- **The Ising model and other lattice spin models.**
- **Large deviation theory.**
- **Discrete versus continuous symmetry breaking.**
- **Interacting particles in the continuum.**

**Recommended books:**

An excellent free textbook for most of this https://www.unige.ch/math/folks/velenik/smbook/

**PROBABILISTIC COMBINATORICS** - back to the list of all courses

5 ECTS

The course emphasizes the use of expectation, variance, correlation, concentration, and local dependence to prove existence results, estimate typical behavior, and understand sharp transitions in combinatorial systems. Students learn how randomness can be used both as a proof technique and as an object of study, especially in random graphs, where questions about connectivity, components, cliques, coloring, and phase transitions become central. The course also introduces modern ideas such as quasirandomness and property testing, showing how large discrete structures can be analyzed through approximate and probabilistic information.

Essential prerequisite: [Discrete Mathematics II], [Probability and Measure]

The lectures cover:

- **Basics and revision.** Markov's inequality and Chebyshev's inequality. The first moment method. The second moment method. Deletion arguments. Lovasz Local lemma.
- **Correlation inequalities.** Monotone properties. Harris's lemma. Fortuin–Kasteleyn–Ginibre (FKG) inequality.
- **Concentration inequalities.** Chernoff bound. Janson's inequality.
- **Random graphs.** Erdos-Rényi random model. Threshold functions and phase transitions. Component structure and emergence of the giant component. Galton–Watson branching process. Clique number and chromatic number of random graphs.
- **Property testing.** Testing colourability. Szemeredi regularity lemma. Testing triangle-freeness. Quasirandom graphs.
- **Selected topics** (depending on available time and interest of students).

Similar course: https://courses.maths.ox.ac.uk/course/view.php?id=5050

**ADDITIVE COMBINATORICS** - back to the list of all courses

5 ECTS

Additive Combinatorics studies finite sets of numbers or group elements through the arithmetic patterns created by adding and subtracting their elements. The course focuses on questions such as when a set has a small sumset, when it must resemble an arithmetic progression or approximate group, and when it must contain arithmetic progressions. Students learn concrete tools for comparing sumsets, measuring additive structure through additive energy, applying Fourier analysis on finite abelian groups, and using density-increment and higher-order uniformity methods. The course connects these methods to results such as Freiman's theorem, Roth's theorem, Szemerédi's theorem, and sum-product estimates.

Essential Prerequisites: Discrete Mathematics II, Real Analysis II,
Desirable Prerequisites: Groups

The lectures cover:

- **Basic concepts and examples.** Sumsets and difference sets. Restricted sumsets. Doubling constant and elementary bounds.

- **Combinatorial tools.** Ruzsa triangle inequality. Plünnecke inequalities. Covering lemmas. Additive energy and its properties. Balog–Szemerédi–Gowers theorem.
- **Structure theory.** Sets with small doubling. Generalised arithmetic progressions. Approximate groups. Freiman's theorem.
- **Sum-product phenomena and combinatorial geometry.** Sum-product estimates. Szemeredi-Trotter theorem.
- **Fourier methods.** Fourier transform on finite abelian groups. Convolution and basic identities. Parseval identity. Applications to additive problems. Roth's theorem on 3-term arithmetic progressions. Density increment strategy.
- **Higher order Fourier methods.** Gowers norms. Generalised von Neumann theorem. Szemeredi's theorem.
- **Selected topics (**depending on available time and interest of students).

Related courses:
http://thomasbloom.org/teaching/AC2021.pdf
https://people.math.ethz.ch/~kowalski/additive-combinatorics.pdf
https://courses.maths.ox.ac.uk/pluginfile.php/27304/mod_resource/content/2/C3.10-notes.pdf

# PART II: PHYSICS

## GENERAL PHYSICS I: MECHANICS, OSCILLATIONS, AND WAVES

5 ECTS

This course is designed to provide students with foundational knowledge and physical intuition, introducing physics as a rigorous discipline that explains natural phenomena through mathematical modeling. It serves as a crucial bridge between mathematical formalism and the observable world, demonstrating how mathematical tools describe and predict the behavior of physical systems. The course focuses on classical mechanics, oscillations, and mechanical waves. Students will explore both intuitive and counterintuitive physical phenomena through simple experiments that illustrate fundamental laws of mechanics. These phenomena will be analyzed mathematically, providing a practical and conceptually clear introduction to key mathematical tools such as differential and integral calculus and differential equations.

The lectures cover:

• **Methodology of physics:** theoretical and experimental methods in physics, physical quantities, measurements and measurement errors;
• **History of physics:** natural philosophy in the ancient world, celestial mechanics and classical physics, aether theory and special relativity;

• **Mechanics:** reference frames, kinematics, forces, Newton's laws, non-inertial reference frames, conservation laws, simple machines, Kepler motion, oscillations, motion of rigid bodies, variable-mass systems (rocket equation);
• **Special relativity:** Michelson–Morley experiment, Galilei and Lorentz transformations, invariance of speed of light, relativity principle, space-time, four-vectors, proper time, relativistic equations of motion, relativity of simultaneity, time dilatation, length contraction, velocity-addition formula, relativistic energy;
• **Mechanics of fluids and gases:** Pressure in fluids and gases, Pascal's law, Archimedes' principle, Bernoulli's equation, viscosity, laminar and turbulent flows; lift force of an airplane wing;
• **Oscillations and waves:** coupled oscillators, eigenfrequencies, damping, resonance, mechanical waves, transverse and longitudinal waves, acoustic waves.

**Recommended books:**
[1] R.Ph. Feynman, R.B. Leighton, M.L. Sands. The Feynman Lectures on Physics. Pearson/Addison-Wesley, 2006.
[2] H.D. Young, R.A. Freedman. University Physics with Modern Physics. Pearson Education, 2015.
[3] J. Walker, D. Halliday, R. Resnick. Fundamentals of Physics. Wiley, 2011.

[4] R.A. Serway, J.W. Jewett, Jr. Physics for Scientists and Engineers with Modern Physics. Cengage Learning, 2013.
[5] P.A. Tipler, R. Llewellyn. Modern Physics. W.H. Freeman, 2003.

**GENERAL PHYSICS II: ELECTROMAGNETISM AND THERMAL PHYSICS**

5 ECTS

Essential prerequisites: General Physics I: Mechanics, Oscillations, and Waves; Real Analysis I; Linear algebra I.

Physics of electricity, magnetism, molecular physics, and thermodynamics forms central pillars of classical physical science, explaining how matter and fields interact across microscopic and macroscopic scales. These disciplines investigate electric charge, current, and electromagnetic fields, the behavior of molecules and gases, and the flow and transformation of thermal energy. Just as in mechanics, their methodology relies on a close interplay between theoretical models and experimental observations. Experimental studies measure electrical, magnetic, and thermal properties of matter, reveal new forms of ordering and phase transitions, and test the limits of established laws. Theoretical approaches express these laws in mathematical form, predict new physical effects, and provide the description of electromagnetic and thermal phenomena—from Maxwell's equations and Coulomb interactions to kinetic theory and the laws of thermodynamics. Experimental and theoretical approaches remain inseparably linked, forming the conceptual foundation of modern physical science.

The lectures cover:

• **History of physics:** heat machines and their impact on thermodynamics, molecules and atoms, sub-atom particles, history of electromagnetism;
• **Kinetics and thermodynamics:** thermodynamic systems, thermodynamic states, equation of states, temperature, work, heat, internal energy, laws of thermodynamics, entropy, heat capacity, Clausius theorem, Carnot cycle, microstates and macrostates, ideal and real gases, gases in external fields, diffusion.
• **Electromagnetism:** electrostatics, Coulomb law, conductor and dielectric, electric field, scalar potential, mechanical motion in electric fields, capacitors, direct current, Ohm law, magnetic forces and fields, magnetostatics, vector potential, mechanical motion in magnetic field, diamagnetic, paramagnetic, ferromagnetic, Faraday's law of induction, transformers, RLC circuit, alternating current, electrical circuits, Kirchhoff's circuit laws, transistors, electronic amplifiers.

**Recommended books:**
[1] R.Ph. Feynman, R.B. Leighton, M.L. Sands. The Feynman Lectures on Physics. Pearson/Addison-Wesley, 2006.
[2] H.D. Young, R.A. Freedman. University Physics with Modern Physics. Pearson Education, 2015.
[3] J. Walker, D. Halliday, R. Resnick. Fundamentals of Physics. Wiley, 2011.
[4] R.A. Serway, J.W. Jewett, Jr. Physics for Scientists and Engineers with Modern Physics. Cengage Learning, 2013.
[5] P.A. Tipler, R. Llewellyn. Modern Physics. W.H. Freeman, 2003.

**GENERAL PHYSICS III: OPTICS, ATOMS, AND PARTICLES**

2 ECTS

Essential prerequisites: General Physics I: Mechanics, Oscillations, and Waves; General Physics II: Electromagnetism and Thermal Physics; Real Analysis II; Linear algebra II.

Light and wave optics together with the introduction to quantum physics constitute fundamental areas of modern physical science, describing the behavior of light and its interaction with matter according to classical and quantum approaches. The proposed discipline explores the wave nature of light, including interference, diffraction, and polarization, as well as the limits of classical descriptions that led to the emergence of quantum theory. It examines how electromagnetic radiation propagates through space and media, how optical phenomena arise from wave superposition, and how quantization governs the interaction between radiation and matter at the atomic scale. As in other branches of physics, methodology of course is based on the close interplay between experimental observation and theoretical modeling. Experimental investigations reveal optical patterns, spectral distributions, and quantum effects such as the photoelectric effect, while theoretical approaches describe these phenomena through wave equations, Maxwell’s theory, and the principles of quantum mechanics. Together, they establish the conceptual transition from classical wave optics to quantum descriptions of physical reality.

The lectures cover:

• **History of physics:** the structure of matter (molecules, atoms, sub-atom particles), nature of light, history of quantum physics, general relativity and cosmology, frontiers of modern physics;
• **Electromagnetic waves:** Maxwell's equations; interference and diffraction of electromagnetic waves, polarization, thermal radiation, ;
• **Introduction to quanta:** atomic spectra, Balmer series, ultraviolet catastrophe, Planck formula, Rutherford scattering experiments, Bohr model of the atom, double-slit experiment, de Broglie waves, wave function, Compton effect, Schrödinger equation, Born's rule.

**Recommended books:**
[1] R.Ph. Feynman, R.B. Leighton, M.L. Sands. The Feynman Lectures on Physics. Pearson/Addison-Wesley, 2006.
[2] H.D. Young, R.A. Freedman. University Physics with Modern Physics. Pearson Education, 2015.
[3] J. Walker, D. Halliday, R. Resnick. Fundamentals of Physics. Wiley, 2011.
[4] R.A. Serway, J.W. Jewett, Jr. Physics for Scientists and Engineers with Modern Physics. Cengage Learning, 2013.
[5] P.A. Tipler, R. Llewellyn. Modern Physics. W.H. Freeman, 2003.

**CLASSICAL MECHANICS** - back to the list of all courses

5 ECTS

Essential prerequisites: Differential Equations
Desirable prerequisites: General Physics I: Mechanics, Oscillations, and Waves

Classical mechanics deals with the mathematical description of the motion of bodies, or point-like objects. By understanding the forces that are exerted on a body we can construct Newton's equation that describes the motion of the object in question. There are, however, other mathematical approaches to this class of problems, known as the Lagrangian and Hamiltonian descriptions of classical mechanics. This course will introduce these various perspectives, and in the process cover the subject of the calculus of variations. Furthermore this course is the first in a series of mathematical physics courses, such as Classical Electrodynamics, Quantum Physics, Statistical Physics and Thermodynamics, and Geometry of General Relativity.

This course is an introduction to the subject of classical mechanics. It will cover Newton's equation, the motion of point particles, including planetary motion, and an introduction to the notion of variational calculus for point particles. In particular the course will cover Hamilton's principle of least action, Lagrangians for systems with conservative forces, and Noether's theorem. The latter provides a conserved quantity whenever there exists a continuous symmetry. Finally, an introduction to the Hamiltonian formalism will be given which prepares the ground for the follow-up course on quantum mechanics for mathematicians. The classical mechanics for mathematicians course is a great opportunity to learn about many classical differential equations and physical problems that helped shape many developments in mathematics. It is also a nice arena to practice one's knowledge of several variable calculus and differential equations.

The lectures cover:

- **Newtonian mechanics:** Inertial and gravitational mass, Newton's laws of motion, kinetic and potential energy, momentum, angular momentum, Noether's theorem and continuous symmetries, damping, Lorentz force, Lagrange–d'Alembert principle;
- **Examples:** one-dimensional motion, motion in electromagnetic field, harmonic oscillator, damped harmonic oscillator, resonance, two-body problem, Kepler problem, celestial mechanics;
- **Holonomic constraints:** systems with holonomic constraints, generalized coordinates and momenta, kinetic energy for systems with holonomic constraints, equations of motion, Lagrange equations, simple pendulum, spherical pendulum, double pendulum;
- **Lagrangian mechanics:** least action principle, Lagrangians, Euler-Lagrange equations, harmonic oscillator in Lagrangian formulation, coupled oscillators, anharmonic oscillator, Lagrangian of a particle in electromagnetic field, Lagrangian approach to electrical circuits;
- **Hamiltonian mechanics:** Legendre transformations, Hamiltonians, Hamilton equations, harmonic and anharmonic oscillators in Hamiltonian formulation, Hamiltonian of a particle in electromagnetic field, generalized Hamilton equations, Poisson brackets, integrals of motion in Hamiltonian formalism, Liouville's theorem, Liouville's function, Liouville's equation.

**Recommended books:**

[1] H. Goldstein, Ch.P. Poole, and J.L. Safko. Classical Mechanics. Addison-Wesley, 2002.

[2] T.W.B. Kibble and F.H. Berkshire. Classical Mechanics. Imperial College Press, 2004.

[3] D. Gregory. Classical Mechanics. Cambridge University Press, 2006.

[4] M.A. Lunn. A First Course in Mechanics. Oxford University Press, 1991.

**ANALYTICAL MECHANICS** - back to the list of all courses

5 ECTS

Essential prerequisites: Differential Equations, Linear Algebra II, Classical Mechanics

Desirable prerequisites: Topology and Metric Spaces, General Physics I: Mechanics, Oscillations, and Waves, Groups

This optional course is a natural continuation of the Classical Mechanics course. Its aim is to provide both a deeper insight into the mathematical formalism underlying classical mechanics and practical methods for analyzing non-trivial mechanical systems. Symplectic geometry and related phase-space techniques form the foundation for a rigorous mathematical treatment of Hamiltonian systems. These methods find applications far beyond standard mechanical systems, extending into classical and quantum electrodynamics, general relativity, and various engineering problems.

The course also offers a deeper understanding of integrability in classical mechanics and its relation to chaotic motion. Students will explore advanced perturbation theory techniques and their use in approximating solutions to important problems, such as parametric resonance and the three-body problem. An important topic covered in this course is the theory of constrained systems, where the Lagrangian formulation may not directly translate to the Hamiltonian one. This provides an elegant introduction to electrodynamics and a mathematically rigorous view of gauge invariance—knowledge that is also valuable for the study of quantum electrodynamics. Finally, the course includes the mechanics of rigid bodies, presented as an example of mathematically rich systems with subtle physical behavior.

The lectures cover:

- **Phase space:** symplectic manifolds, canonical transformations and its generating functions, linear canonical transformations, Hamilton-Jacobi equation, action-angle variables, integrability, chaotic motion, example: three-body problem;
- **Perturbation methods:** perturbation methods based on Hamilton-Jacobi equation, pendulum with oscillating support, perturbation methods in celestial mechanics;
- **Constrained systems:** Lagrangians not inverting into Hamiltonians (singular theories), Dirac brackets, second-class constraints, first-class constraints, gauge invariance;
- **Mechanics of rigid bodies:** rigid system of particles, inertia tensor, Euler angles, Euler equations, symmetric and asymmetric top.

**Recommended books:**

[1] H. Goldstein, Ch.P. Poole, and J.L. Safko. Classical Mechanics. Addison-Wesley, 2002.

[2] V.I. Arnold. Mathematical Methods of Classical Mechanics. Springer, 2013.

[3] A.J. Lichtenberg, M.A. Lieberman. Regular and Chaotic Dynamics. Springer, 1992.

[4] M. Henneaux, C. Teitelboim. Quantization of Gauge Systems. Princeton University Press, 1992.

[5] J.V. José, E.J. Saletan. Classical Dynamics: A Contemporary Approach. Cambridge University Press, 2002.

[6] M. Tabor. Chaos and Integrability in Nonlinear Dynamics: An Introduction. Willey, 1989.

**CONTINUUM MECHANICS** - back to the list of all courses

5 ECTS

Essential prerequisites: Differential Equations,Multivariate Analysis, Classical Mechanics

Desirable prerequisites: General Physics I: Mechanics, Oscillations, and Waves

This optional lecture course complements the Classical Mechanics course. Continuum mechanics focuses on physical systems whose behavior can be effectively described without considering their discrete particle structure, as their macroscopic continuum properties are of primary interest. The field broadly encompasses three interconnected areas (solid mechanics, fluid mechanics, and aeromechanics) all of which are grounded in the same fundamental mathematical and physical principles.

Solid mechanics is widely applied in mechanical and civil engineering, where elasticity theory plays a central role. Fluid mechanics and aerodynamics are essential for both engineering applications, such as in aircraft and ship design, and for fundamental scientific research. The study of acoustic phenomena in solids, fluids, and gases also falls within the scope of this discipline.

In addition to its wide range of practical applications, continuum mechanics involves deep theoretical challenges. Its foundational equation, the Navier–Stokes equation, is the subject of one of the Millennium Prize Problems. The question of whether smooth solutions always exist in the general case remains unsolved, making this an area of active and profound mathematical inquiry. This lecture course also provides an excellent context for the application of tensor analysis, offering students a deep and intuitive understanding of this important area of mathematics.

The lectures cover:

- **Backgrounds of Continuum Mechanics:** concept of continuum, infinitesimal elements, velocity field, material, local, and convective derivatives, Cauchy stress tensor, surface and body forces, Cauchy's equations of motion (balance of momentum and balance of angular momentum, continuity equation (balance of mass), balance equation for energy, conservation form for the equations of continuum mechanics; strain and strain-rate tensors; relation of Cauchy stress tensor to strain and strain-rate tensors;
- **Solid mechanics:** Hooke's law, linear deformations, stationary deformations; non-stationary deformations; sound waves in solids;
- **Fluid mechanics:** equations of fluid mechanics, Navier-Stokes equation, perfect fluid, Bernoulli's principle, incompressible fluid, Hagen–Poiseuille equation, Reynolds number, flow around a sphere, Stokes formula, sound waves in fluids and gases.

**Recommended books:**

[1] L.E. Malvern. Introduction to the Mechanics of a Continuous Medium. Prentice-Hall, 1969.

[2] M.E. Gurtin, E. Fried, L. Anand. The Mechanics and Thermodynamics of Continua. Cambridge University Press, 2009.

[3] A.J.M. Spencer. Continuum Mechanics. Dover Publications, 2015.

[4] P. Chadwick. Continuum Mechanics: Concise Theory and Problems. Dover Publications, 2012.

[5] Y.C. Fung. Foundations of Solid Mechanics. Prentice-Hall, 1965.

[6] W.M. Lai, D. Rubin, E. Krempl. Introduction to Continuum Mechanics. Elsevier, 2010.

[7] G.K. Batchelor. An Introduction to Fluid Dynamics. Cambridge University Press, 2000.

[8] R.L. Panton. Incompressible Flow. Willey, 2013.

[9] J.E. Marsden, Th.J.R. Hughes. Mathematical Foundations of Elasticity. Dover Publications, 1994.

[10] C.Truesdell, W.Noll. The Non-Linear Field Theories of Mechanics. Springer, 2004.

**CLASSICAL ELECTRODYNAMICS** - back to the list of all courses

5 ECTS

Essential prerequisites: Differential Equations, Multivariate Analysis, Classical Mechanics

Desirable prerequisites: General Physics II: Electromagnetism and Thermal Physics

Electrodynamics studies the properties of the electromagnetic field, which mediates one of the four fundamental interactions between particles. From a mathematical perspective, it is a theory of a massless vector field. It serves as a primary example of a constrained system, where the Lagrangian formulation cannot be directly transformed into the Hamiltonian one. This leads to the crucial concept of gauge invariance, which plays a fundamental role in both physics and mathematics. Special relativity naturally emerges from the symmetry properties of Maxwell's equations, which describe the behavior of the electromagnetic field. This foundational theory is also explored as part of the course.

Classical electrodynamics encompasses a wide range of applications, including electrostatics, magnetostatics, electromagnetic radiation from charged particles, and classical optics. These topics are not only practically significant but also provide a rich context for introducing key mathematical methods. In particular, students will encounter harmonic functions and classical orthogonal polynomials such as the Legendre, Hermite, and Laguerre polynomials within the framework of this course.

The lectures cover:

- **Electromagnetic field:** Maxwell’s equations in integral and differential forms, physical interpretation of Maxwell’s equations, Coulomb’s law, Biot-Savart's law, Faraday’s law, electrostatic and magnetostatic, harmonic functions.
- **Special relativity:** Lorentz and Galilei transforms, Lorentz invariance of Maxwell's equations, Minkowski space, four-vectors and four-tensors, electromagnetic field tensor, field invariants, relativistic mechanics, least action principle for a free particle and for a particle in electromagnetic field.
- **Gauge invariance:** Scalar potential and vector potential, vector-potential in the four-dimensional form, Coulomb and Lorentz gauge conditions;
- **Charged particles in electromagnetic field:** Electric field for systems of charged particles, multipole expansions, Legendre polynomials, multipole expansions for the system of currents on large distances, retarded potential, Liénard-Wiechert potential, dipole radiation, magnetic dipole and quadruple radiation, linear current radiation (dipolar antenna), Bremsstrahlung;
- **Electromagnetic waves:** Wave equation, plane and spherical waves, interference and diffraction of electromagnetic waves, Fresnel, Fraunhofer, and Kirchhoff diffraction equations, paraxial approximation and paraxial Helmholtz equation, Gaussian beams, collimated and focused beams, Hermite-Gaussian and Lager-Gaussian modes, polarized light, linear and circular polarization, Stokes parameters.

**Recommended books:**

[1] D.J. Griffiths. Introduction to Electrodynamics. Cambridge University Press, 2017.

[2] E.M. Purcell, D.J. Morin. Electricity and Magnetism. Cambridge University Press, 2013.

[3] A. Zangwill. Modern Electrodynamics. Cambridge University Press, 2013.

[4] J.D. Jackson. Classical Electrodynamics. Wiley, 1999.

[5] P. Lorrain. D. Corson. Electromagnetism, Principles and Applications. Freeman, 1990.

[6] R. Feynman, R. Leighton, M. Sands. The Feynman Lectures on Physics, Vol 2. Basic Books, 2011.

[7] A. E. Siegman. Lasers. University Science Books, 1986.

[8] M. Born, E. Wolf. Principles of Optics. Cambridge University Press, 1999.

**ANALYTICAL ELECTRODYNAMICS** - back to the list of all courses

4 ECTS

Essential prerequisites: Differential Equations, Multivariate Analysis, Classical Mechanics, Analytical Mechanics

Desirable prerequisites: General Physics II: Electromagnetism and Thermal Physics

This optional course is intended for students who wish to explore the deeper mathematical and theoretical foundations of classical electrodynamics. The course presents electrodynamics from the perspectives of classical field theory, differential geometry, and analysis, including Lagrangian and Hamiltonian formulations, gauge symmetry, and the language of differential forms. It also provides a physically motivated introduction to partial differential equations, boundary value problems, Green's functions, harmonic analysis, and elements of potential theory, illustrating how modern mathematical methods arise naturally in electromagnetic theory and related areas of theoretical physics.

The lectures cover:

- **Elements of classical field theory:** Action principle, Lagrangian and Hamiltonian formulations of electrodynamics, canonical structure of the electromagnetic field, constrained dynamical systems, gauge symmetry and gauge invariance, conservation laws;
- **Differential forms in electromagnetism:** Exterior algebra and differential forms, exterior derivative and Hodge duality, electromagnetic field and current forms, Maxwell's equations in the language of differential forms, covariant formulation of electrodynamics, electromagnetism in curved spacetime;
- **Elements of potential theory:** Harmonic and subharmonic functions, Green's functions, boundary value problems for Laplace and Poisson equations, electrostatic potentials, Dirichlet and Neumann problems, connections with geometric analysis and measure theory.

**Recommended books:**

[1] A. Zangwill. Modern Electrodynamics. Cambridge University Press, 2013.

[2] V. Guillemin, S. Sternberg. Symplectic Techniques in Physics. Cambridge University Press,1984.

[3] J.D. Jackson. Classical Electrodynamics. Wiley, 1999.

[4] F. W. Hehl, Y.N. Obukhov. Foundations of Classical Electrodynamics: Charge, Flux, and Metric. Birkhäuser, 2003.

[5] T. Frankel. The Geometry of Physics: An Introduction. Cambridge University Press, 2017.

[6] S.J. Axler, P. Bourdon, W. Ramey. Harmonic Function Theory. Springer, 2001.

**ELECTRODYNAMICS OF CONTINUOUS MEDIA** - back to the list of all courses

4ECTS

Essential prerequisites: Differential Equations, Multiavariate Analysis, Classical Mechanics, Classical Electrodynamics

Desirable prerequisites: General physics II: Electromagnetism and Thermal Physics

This lecture course extends the foundations of classical electrodynamics to the study of electromagnetic phenomena in material media. It explores how the presence of matter, whether dielectric, conductive, isotropic, anisotropic, or dispersive, modifies the behavior of electromagnetic fields and waves. The propagation of electromagnetic waves in media is studied in various regimes: in gases, plasmas, anisotropic crystals, and conductors. The course addresses practical and fundamental aspects of wave behavior, such as dispersion, absorption, reflection, birefringence, and magneto-optical effects. The interaction between fields and media leads to a variety of new physical phenomena and raises important mathematical challenges, making this subject a rich interface between physics and applied mathematics.

Wave propagation in media is studied from both physical and mathematical perspectives. Topics such as phase and group velocity, wave reflection, and transmission involve solving Maxwell's equations under various approximations, often leading to eigenvalue problems, asymptotic methods, and complex-variable techniques. Special attention is given to anisotropic and gyrotropic media, where tensorial permittivity leads to nontrivial wave solutions.

The lectures cover:

- **Fundamentals of electromagnetic fields in media:** Free and bound charges, Maxwell's equations in media, boundary conditions for Maxwell's equations, local field, Lorentz–Lorenz formula, frequency dependence of the dielectric permittivity, Kramers–Kronig relations, dielectric permittivity for gases of isotropic charged oscillators, dielectric permittivity of plasma;
- **Electromagnetic waves in media:** General theory of electromagnetic waves in media, phase and group velocity, electromagnetic waves propagation in anisotropic crystals, magneto-optic effect, gyrotropic permittivity, electromagnetic waves propagation in conductors, differential form of Ohm's law, Maxwell equations in quasistatic approximation, Skin effect;
- **Wave-matter interaction and optical phenomena:** Electromagnetic wave scattering by free and bound charges, electromagnetic wave reflection at the boundary between media, Fresnel equations, wave plates, beam splitters, geometric optics, eikonal equation, Cherenkov radiation;
- **Circuit theory:** Electromotive force across an electrical conductor, electromagnetic induction, complex resistance, capacitance in circuits with quasistatic current, LC and RC circuits, electrical circuits, Kirchhoff's circuit laws, Lagrangian and Hamiltonian circuit theory, electromechanical systems.

**Recommended books:**

[1] A. Zangwill. Modern Electrodynamics. Cambridge University Press, 2013.

[2] L.D. Landau, E.M. Lifshitz. Electrodynamics of Continuous Media. Pergamon Press, 1960.

[3] M. Dressel, G. Grüner. Electrodynamics of Solids. Optical Properties of Electrons in Matter. Cambridge University Press, 2010.

[4] J.A. Kong. Electromagnetic Wave Theory. EMW Publishing, 2008.

[5] M. Born, E. Wolf. Principles of Optics. Cambridge University Press, 1999.

[6] V. Guillemin, S. Sternberg. Symplectic Techniques in Physics. Cambridge University Press,1984.

**QUANTUM PHYSICS** - back to the list of all courses

4 ECTS

Essential prerequisites: Differential Equations, Analysis of Differential Equations, Probability and Measure, Linear Algebra II, Multivariate Analysis, Complex Analysis with Applications, Classical Mechanics, Classical Electrodynamics

Desirable prerequisites: Groups, Topology and Metric Spaces, General Physics III: Optics, Atoms, and Particles

Modern physics is inseparable from quantum mechanics – one of the most successful and conceptually rich physical theories. Many contemporary technologies, from semiconductors to quantum computing, trace their origins to quantum principles. At the same time, quantum theory has inspired ongoing philosophical debates since its inception. One of its most striking features is that it often defies classical physical intuition. In contrast, mathematical reasoning and formal structure provide the most effective tools for understanding this fundamental theory, which is primarily applied to atomic and subatomic phenomena.

This lecture course introduces the foundational physical principles of quantum mechanics, beginning with Schrödinger wave mechanics and Born's rule. Through the study of simple quantum systems, students will explore the mathematical framework of the theory, including key concepts from Hilbert space theory and operator analysis. The second part of the course formalizes quantum mechanics using the abstract language of functional analysis. Topics include observables as self-adjoint operators, the Dirac bra–ket notation, the density operator formalism for pure and mixed states, and the unitary time evolution of closed systems. The Dyson series and the various pictures of motion (Schrödinger, Heisenberg, and interaction) are presented within a unified operator-theoretic framework. The course also introduces key approximation techniques, such as time-independent and time-dependent perturbation theory and the WKB method, offering essential tools for the analysis of systems beyond exact solutions. Throughout, the mathematical structure of quantum theory is treated as integral to its physical content.

The lectures cover:

- **Schrödinger wave mechanics:** experimental backgrounds, Schrödinger equation, continuity equation, Born's rule, observables in Schrödinger mechanics, stationary and non-stationary states, bound and unbound states, one-dimensional motion, tunnel effect, wave packets of a free particle, harmonic oscillator, central-force problem, Hydrogen atom, particle in a constant electric field;

- **Mathematical formalism of quantum theory:** Hilbert space of states, Dirac notations, operators and observables, pure and mixed states, density operator, unitary evolution, Dyson series, pictures of motion, representation theory;
- **Basic properties of quantum systems:** uncertainty relations, decoherence, quantum measurements, postulates of quantum theory, identical particles, bosons and fermions;
- **Algebraic methods:** harmonic oscillator in ladder-operator formalism, quantum electromagnetic field, coherent states, Bogolyubov transform, particle in a constant magnetic field, angular momentum operators, spin, Clebsch–Gordan coefficients;
- **Approximate methods:** time-independent perturbation theory, anharmonic oscillator, Stark effect, time-dependent perturbation theory, Fermi golden rule, WKB method.

**Recommended books:**

[1] A. Messiah, Quantum Mechanics. Dover Books on Physics, 2014.

[2] C. Cohen-Tannoudji, B. Diu, F. Laloë. Quantum Mechanics, Wiley-VCH, 2020.

[3] A.S. Davydov, Quantum Mechanics. Pergamon Press, 1976.

[4] Brian Hall, Quantum Theory for Mathematicians, Springer 2013.

[5] Keith Hannabuss, An Introduction to Quantum Theory, Oxford University Press, 1997.

[6] D. J. Griffiths, D. Schroeter. Introduction to Quantum Mechanics. Cambridge University Press, 2018.

[7] J. J. Sakurai, J. Napolitano. Modern Quantum Mechanics. Adison-Wesley, 2011.

[8] L.E. Ballentine. Quantum Mechanics: A Modern Development. World Scientific, 1998.

**ADVANCED QUANTUM PHYSICS** - back to the list of all courses

5 ECTS

Essential prerequisites: Differential Equations, Probability and Measure, Linear Algebra II, Multivariate Analysis, Complex Analysis with Applications, Analysis of Differential Equations, Classical Mechanics, Classical Electrodynamics, Quantum Physics

Desirable prerequisites: Groups, Topology and Metric Spaces, General Physics III: Optics, Atoms, and Particles, Analytical Mechanics, Analytical Electrodynamics

This optional lecture course is a natural continuation of the course Quantum Physics and focuses on advanced topics in relativistic quantum mechanics and relativistic quantum field theory. Beyond their fundamental physical significance, these subjects provide a rich introduction to various mathematical disciplines, including Clifford algebra, symmetry analysis, and gauge invariance. Many of the mathematical tools developed in quantum electrodynamics share deep connections with those used in its low-energy limit—quantum optics. Despite describing physically distinct regimes, these methods find common ground and wide applicability in modern contexts. They extend beyond traditional applications and form a foundation for emerging quantum technologies, including quantum computing, quantum communication, and quantum metrology.

The course also introduces advanced approximation techniques such as the Born–Oppenheimer method and mean-field theory. These methods offer powerful analytical tools for tackling quantum many-body problems and are widely used in atomic and molecular physics. In the final part of the course, students are introduced to selected topics in solid-state physics, providing a conceptual and mathematical basis for understanding key ideas in this major area of modern research.

The lecture cover:

- **Relativistic quantum mechanics:** Klein-Gordon equation, Feshbach-Villars formalism, interaction of zero-spin particle with electromagnetic field, Dirac equation, Foldy-Wouthuysen representation, position operator in relativistic quantum mechanics, Clifford algebra, charge conjugation, particles and antiparticles, the Hydrogen atom, spin-orbital interaction, hyperfine structure;
- **Introduction to quantum field theory:** second quantization, canonical quantization, spin-statistics theorem, perturbation theory, S-matrix formalism, Lamb shift, nonperturbative methods in quantum field theory, Schwinger effect, Jaynes–Cummings model, Rabi oscillations;
- **Advanced methods and systems:** Born–Oppenheimer approximation, molecules, self-consistent Hartree-Fock method, mean-field theory, periodic potential, introduction to solid-state physics.

**Recommended books:**

[1] W. Greiner. Relativistic Quantum Mechanics: Wave Equations. Springer, 2000.

[2] A.S. Davydov, Quantum Mechanics. Pergamon Press, 1976.

[3] J. J. Sakurai. Advanced Quantum Mechanics. Pearson Education, 1967.

[4] M. Peskin, D. Schroeder. An Introduction to Quantum Field Theory. Addison-Wesley, 1995.

[5] S. Weinberg. The Quantum Theory of Fields, Vol. I. Cambridge University Press, 1995.

[6] M. Kaku. Quantum Field Theory: A Modern Introduction. Oxford University Press, 1993.

[7] S.S. Schweber. An introduction to relativistic quantum field theory. Dover Publications, 2005.

[8] D.F. Walls, G.J. Milburn. Quantum Optics. Springer 2008.

[9] A. Messiah, Quantum Mechanics. Dover Books on Physics, 2014.

[10] C. Cohen-Tannoudji, B. Diu, F. Laloë. Quantum Mechanics, Wiley-VCH, 2020.

**QUANTUM MEASUREMENTS AND CORRELATIONS** - back to the list of all courses

4 ECTS

Essential prerequisites: Differential Equations, Probability and Measure, Linear Algebra II, Multivariate Analysis, Complex Analysis with Applications, Classical Mechanics, Classical Electrodynamics, Quantum Physics

Desirable prerequisites: Groups, Topology and Metric Spaces, General Physics III: Optics, Atoms, and Particles, Advanced Quantum Physics

This lecture course continues the study of quantum physics, with an emphasis on fundamental concepts and modern applications that underpin contemporary quantum technologies. Students will explore the foundations of quantum measurement theory and the theory of quantum correlations, including entanglement, Bell nonlocality, and related phenomena. These ideas are further developed within the framework of quantum resource theories, which provide a modern mathematical language for describing and quantifying nonclassical features of quantum systems. The course concludes with an introduction to the mathematical principles underlying key quantum technologies, including quantum metrology, quantum communication, and quantum computing.

The lectures cover:

- **Quantum measurement theory:** Quantum observables, Born's rule, projective measurements, generalized measurements, positive operator-valued measure;
- **Quantum information:** Shannon entropy, von Neumann entropy, mutual information, Holevo limit;
- **Quantum correlations:** Quantum entanglement, Bell nonlocality, quantum steering, quantum discord, introduction to quantum resource theory;
- **Modern quantum technologies:** introduction to quantum metrology, classical and quantum Fisher information, mathematical backgrounds of security analysis for quantum key distribution, basics of quantum computing.

**Recommended books:**

[1] M.A. Nielsen, I.L. Chuang. Quantum Computation and Quantum Information. Cambridge University Press, 2010.

[2] P. Busch, P. Lahti, P. Mittelstaedt . The Quantum Theory of Measurement. Springer, 1996.

**FEYNMAN PATH INTEGRAL** - back to the list of all courses

5 ECTS

Essential prerequisites: Differential Equations, Probability and Measure, Linear Algebra II, Multivariate Analysis, Complex Analysis with Applications, Analysis of Differential Equations, Classical Mechanics, Classical Electrodynamics, Quantum Physics, Advanced Quantum Physics

Desirable Prerequisites: Groups, Topology and Metric Spaces, General Physics III: Optics, Atoms, and Particles, Advanced Quantum Physics

The path integral formulation, introduced by Richard Feynman, offers a powerful and intuitive alternative to the standard operator approach in quantum mechanics and quantum field theory. By expressing quantum amplitudes as sums over all possible histories, it provides deep connections between quantum physics, classical action principles, statistical mechanics, and modern field theory, and serves as a foundational tool in areas ranging from condensed matter to high-energy physics.

The course develops the path integral method from its conceptual foundations in quantum mechanics to its powerful applications in quantum field theory. We begin by exploring how quantum evolution can be reformulated as a sum over classical paths, using this idea to compute propagators and understand familiar systems like the free particle and harmonic oscillator from a new perspective. Building on this, we examine how the path integral naturally leads to perturbation theory and Feynman diagrams, uncovering its deep connection to statistical mechanics through the Euclidean formulation and the partition function. Semi-classical methods, such as the WKB approximation and instanton techniques, reveal the path integral's strength in describing quantum tunneling and nonperturbative phenomena. The course culminates in the application of these ideas to quantum field theory, where we construct path integrals for scalar and fermionic fields, introduce Grassmann variables, and demonstrate how generating functionals and diagrammatic expansions emerge naturally in this framework.

The lectures cover:

- **Introduction to path integral in quantum mechanics:** Gaussian integrals and Wick theorem, Feynman propagator in quantum mechanics, a free particle and harmonic oscillator in path integral formalism, relation to Schrodinger equation;

- **Advanced topics of path integral in quantum mechanics:** perturbation theory, Feynmann diagrams for anharmonic oscillator, relation to statistical mechanics, partition function and energy spectrum, quasi-classical (WKB) approximation, quantum tunneling and instantons, quantization of constrained systems via path integrals;

- **Path integral in quantum field theory (QFT):** generating functionals and correlation functions for scalar fields, Wick theorem and formalism of Feynmann diagrams in perturbation theory, equivalence of operator and path integral approaches in QFT, Grassmann algebra, Grassmann path integral for fermions.

**Recommended books:**

[1] R.P. Feynman, A.R. Hibbs, Quantum Mechanics and Path Integrals, New York: McGraw-Hill, 1965.

[2] J. Zinn Justin, Path Integrals in Quantum Mechanics, Oxford University Press, 2004.

[3] M. Peskin, D.V. Schroeder, An Introduction to Quantum Field Theory, Addison-Wesley, 1995.

[4] S. Weinberg, The Quantum Theory of Fields, vol. I, Cambridge University Press, 1995.

[5] H. Kleinert, Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets, Singapore: World Scientific, 2009.

[6] P. Etingof, Geometry and Quantum Field Theory, MIT OpenCourseWare, 2002.

**QUANTUM COMPUTATION** - back to the list of all courses

4 ECTS

Essential prerequisites: Differential Equations, Probability and Measure, Linear Algebra II, Multivariate Analysis, Complex Analysis with Applications, Classical Mechanics, Quantum Physics, Quantum Measurements and Correlations

Desirable prerequisites: Topology and Metric Spaces, General Physics III: Optics, Atoms, and Particles, Advanced Quantum Physics, Mathematical Topics for Machine Learning.

This course provides an accessible yet rigorous introduction to the principles and techniques of quantum computation. Building on a basic understanding of quantum systems, it explores the manipulation of quantum information through single- and multi-qubit operations, quantum circuits, and the implementation of key quantum algorithms. Students will learn about the fundamentals of quantum error correction, noise models, and fault-tolerant architectures, as well as current approaches in quantum machine learning. The course balances mathematical formalism with algorithmic thinking, equipping students with tools to understand and analyze quantum protocols and their potential applications.

The lectures cover:

- **Introduction and Mathematical Framework:** Qubits, quantum states, tensor products, basic measurements, Bloch sphere representation, quantum gates and unitary transformations, circuit model;

- **Quantum Gates and Circuits:** Single-qubit gates, multi-qubit gates, universal quantum gate sets, quantum circuit design and compilation;
- **Quantum Algorithms:** Quantum parallelism and interference, Deutsch and Deutsch-Jozsa algorithms, Bernstein-Vazirani algorithm, Simon's algorithm, Grover's search algorithm and amplitude amplification, Quantum Fourier transform and phase estimation, Shor's algorithm for integer factorization;
- **Quantum Error Correction and Fault Tolerance:** Sources of noise and quantum decoherence, bit-flip and phase-flip codes, Shor code and Steane code, stabilizer formalism, threshold theorem and basic ideas of fault tolerance;
- **Intro to Quantum Machine Learning (QML):** Parameterized quantum circuits and variational algorithms, quantum data encoding, quantum classifiers and kernels, overview of hybrid quantum-classical models, current limitations and future directions in QML.

**Recommended books:**

[1] M.A. Nielsen, I.L. Chuang. Quantum Computation and Quantum Information. Cambridge University Press, 2010.

[2] P. Kaye, R. Laflamme, M. Mosca. An Introduction to Quantum Computing. Oxford University Press, 2007.

[3] E. Rieffel, W. Polak. Quantum Computing: A Gentle Introduction. The MIT Press, 2011.

[4] J. Preskill. Lecture Notes on Quantum Computation. https://www.preskill.caltech.edu/ph229/.

[5] M. Schuld, F. Petruccione. Supervised Learning with Quantum Computers. Springer, 2018.

**STATISTICAL PHYSICS AND THERMODYNAMICS** - back to the list of all courses

5 ECTS

Essential prerequisites: Differential Equations, Probability and Measure, Multivariate Analysis, Complex Analysis with Applications, Analysis of Differential Equations, Classical Mechanics, Classical Electrodynamics, Quantum Physics

Desirable prerequisites: General Physics II: Electromagnetism and Thermal Physics.

Statistical physics and thermodynamics describe the behavior of systems on a macroscopic scale. Their methods are broadly applicable, encompassing phenomena ranging from refrigerators and chemical reactions to stellar interiors, electrical conductivity and superconductivity. Statistical mechanics and thermodynamics are deeply interconnected disciplines, despite having distinct historical origins. The pioneers of thermodynamics formulated their theories without knowledge of the atomic structure of matter. In contrast, statistical mechanics, grounded in this atomic perspective, derives the behavior of systems in a way that gives rise to thermodynamic principles. In essence, while statistical mechanics emerged later in history, it serves as a conceptual foundation for thermodynamics. Deep understanding of the thermodynamic states functions requires knowledge of such mathematical concepts as an exact differential and Legendre transforms.

The lectures cover:

- **Entropy.** Mathematical background: the classical ideal gas (configurational entropy), temperature, pressure, chemical potential;
- **Thermodynamics:** Postulates and laws of thermodynamics, thermodynamic processes, thermodynamic potentials, thermodynamic identities Extremum principles, phase transitions;
- **Classical statistical mechanics:** Ensembles in classical statistical mechanics, refining the definition of entropy, irreversibility, entropy and gravity;
- **Quantum statistical mechanics:** Quantum ensembles, black-body radiation, the harmonic solid, ideal quantum gases, Bose-Einstein statistics, Fermi-Dirac statistics;
- **Basics of nonequilibrium statistical physics:** The Boltzmann transport equation, H theorem.

**Recommended books:**

[1] K. Huang. Statistical Mechanics.. Willey, 2nd edition 1991.

[2] R.H. Swendsen. An Introduction to Statistical Mechanics and Thermodynamics. Oxford, 2nd edition 2020.

[3] V. Nair. Thermodynamics and Statistical Mechanics. Set of lectures. The City College of New York., 2015.

[4] R. Kubo. Thermodynamics. North-Holland Publishing Co., 1968.

[5] L.D. Landau, E.M. Lifshitz. Statistical Physics, Part 1, Volume 5. Elsevier, 3rd edition 1980.

[6] E.M. Lifshitz, L.P. Pitaevskii. Statistical Physics, Part 2, Volume 9. Elsevier, 3rd edition 1980.

[7] H. Callen. Thermodynamics and an Introduction to Thermostatistics. John Wiley & Sons, 2nd edition, 1985.

[8] C. Kittel, H. Kroemer. Thermal physics. New York: W.H. Freeman, 2nd edition, 1980.

[9] P. Landsberg (editor), Problems in Thermodynamics and Statistical Physics, 1971.

**ADVANCED STATISTICAL PHYSICS** - back to the list of all courses

5 ECTS

Essential prerequisites: Differential Equations, Probability and Measure, Multivariate Analysis, Complex Analysis with Applications, Analysis of Differential Equations, Classical Mechanics, Classical Electrodynamics, Quantum Physics, Statistical Physics and Thermodynamics

Desirable prerequisites: General Physics II: Electromagnetism and Thermal Physics.

This optional lecture course is a natural continuation of the course Statistical Physics and Thermodynamics and focuses on advanced topics in statistical physics. While the basic course is more focused on equilibrium phenomena, here a stronger accent is put on nonequilibrium phenomena. The course includes more advanced topics such as the Carathéodory statement of the second law of thermodynamics developed by Constantin Carathéodory who formulated thermodynamics on a purely mathematical axiomatic foundation.

The lectures cover:

- **The Carathéodory principle:** Carathéodory statement of the second law;
- **Nonequilibrium statistical physics:** Theory of fluctuations, Brownian motion, the Fokker–Planck equation;
- **Thermodynamics of irreversible processes:** The Onsager reciprocal relations, thermoelectricity;
- **Fermi systems:** Insulators and semiconductors, negative temperatures, the theory of white dwarf stars, Pauli paramagnetism, Bohr - van Leeuwen theorem, Landau diamagnetism, the de Haas - van Alphen effect, the quantized Hall effect;
- **Bose systems:** Bose-Einstein condensation, an imperfect Bose gas, superfluidity.

**Recommended books:**

[1] K. Huang. Statistical Mechanics. Willey, 2nd edition 1991.

[2] R.H. Swendsen. An Introduction to Statistical Mechanics and Thermodynamics. Oxford, 2nd edition 2020.

[3] V. Nair. Thermodynamics and Statistical Mechanics. Set of lectures. The City College of New York., 2015.

[4] L.D. Landau, E.M. Lifshitz. Statistical Physics, Part 1, Volume 5. Elsevier, 3rd edition 1980.

[5] E.M. Lifshitz, L.P. Pitaevskii. Statistical Physics, Part 2, Volume 9. Elsevier, 3rd edition 1980.

[6] H. Callen. Thermodynamics and an Introduction to Thermostatistics. John Wiley & Sons, 2nd edition, 1985.

[7] P. Landsberg (editor). Problems in Thermodynamics and Statistical Physics, 1971.

**SUPERCONDUCTIVITY** - back to the list of all courses

2 ECTS

Essential prerequisites: Differential Equations, Probability and measure, Multivariate Analysis, Complex Analysis with Applications, Analysis of Differential Equations, Classical Mechanics, Classical Electrodynamics, Quantum Physics, Statistical Physics and Thermodynamics

Desirable prerequisites: General Physics II: Electromagnetism and Thermal Physics.

Whether undergoing an magnetic resonance imaging scan in a hospital, reading about the discovery of the Higgs boson at the Large Hadron Collider, or hearing about a new speed record set by a train levitating on a superconducting magnetic cushion, all these remarkable technological achievements are made possible by the practical application of superconductivity. Superconductivity was discovered in 1911, but its first microscopic explanation, now known as the Bardeen-Cooper-Schrieffer (BCS) theory, was developed only in 1957, revealing its quantum nature. Research in this field continues, driven by the goal of achieving superconductivity at room temperature. This course presents the fundamental experimental facts and theoretical approaches related to superconductivity.

*This is a short course consisting of 15 lectures and no tutorials.*

The lectures cover:

- **History of discovery and the main goal of studying superconductivity:** Basic experimental facts, ideal conductivity and superconductivity, ideal diamagnetism, Josephson effects;
- **Earlier phenomenological theories of superconductivity:** Thermodynamic consideration of superconductivity, the electrodynamics of a superconductor by London brothers;
- **Microscopical theory of superconductivity:** Direct attraction between electrons as the result of phonon exchange, Cooper pairing, the Bardeen-Cooper-Schrieffer (BCS) theory, the Bogolyubov method;
- **The Ginzburg-Landau theory:** Landau's theory of the second order phase transitions, free energy and Ginzburg-Landau equations, coherence length and penetration depth;
- **Superconductivity and particle physics:** Goldstone mode, Higgs phenomenon and so on.

**Recommended books:**

[1] M. Tinkham. Introduction to Superconductivity: Second Edition, McGraw-Hill, Inc 1996.

[2] V.V. Schmidt. The Physics of Superconductors Introduction to Fundamentals and Applications. Springer, 1997.

[3] A.C. Rose-Innes, E.H. Rhoderick. Introduction to Superconductivity. Pergamon

Press, Oxford 1978.

[4] J.R. Schrieffer. Theory Of Superconductivity. Advanced Book Classics, 1999.

[5] E.M. Lifshitz, L.P. Pitaevskii. Statistical Physics, Part 2, Volume 9. Elsevier, 3rd edition 1980.

**COMPUTATIONAL BIOPHYSICS** - back to the list of all courses

3 ECTS

Essential prerequisites: Differential Equations, Probability and Measure, Multivariate Analysis, Complex Analysis with Applications, Analysis of Differential Equations, Classical Mechanics, Classical Electrodynamics, Quantum Physics, Statistical Physics and Thermodynamics

Desirable prerequisites: General Physics II: Electromagnetism and Thermal Physics

Computational biophysics is an interdisciplinary field at the intersection of physics, biology, chemistry, mathematics, and computer science. Within the framework of this lecture course students will learn the basics in various methods used in the mathematical modeling of biological molecules across different levels of organization, from the electronic and atomic scales to the mesoscopic and macroscopic levels. To master the material, students will be offered various practical tasks that involve solving problems and analyzing the properties of biological macromolecules using modern visualization and molecular structure analysis programs. The practical sessions will be conducted using the computational cluster of the Bogolyubov Institute for Theoretical Physics of the NAS of Ukraine. Through these exercises, students will have the opportunity to apply their mathematical knowledge to solve practical problems in the field of biophysics.

The lectures cover:

- **Basics of atomistic modeling:** The idea of molecular dynamics and Monte Carlo methods, algorithms for integrating equations of motion, potentials for describing the interaction between atoms, limitations and problems of the methods;
- **Classical molecular dynamics simulations:** Force fields, construction of molecular topology in the classical method, software packages for implementing VMD and NAMD, simulation protocols and selection of optimal simulation parameters, simulation of molecular systems on the computing cluster, methods for processing and analyzing the obtained simulation trajectories;
- **Coarse grained molecular dynamics simulations:** From top to bottom and bottom-up philosophies of constructing a force field, Inverse Boltzmann method, Inverse Monte Carlo method, software packages for implementing coarse-grained MD, MagiC program;
- **Basics of quantum mechanics of molecules:** Hartry-Fock approach, post-HF methods, density functional theory, software packages;
- **Quantum molecular dynamics simulations:** Quantum molecular dynamics in the Born-Oppenheimer approximation, quantum molecular dynamics in the Car-Parinello approximation, quantum mechanics / molecular mechanics methods, software packages.

**Recommended books:**

[1] T. Schlick. Molecular Modeling and Simulation. An Interdisciplinary Guide. New York: Springer-Verlag, 2002..

[2] P. Atkins., R. Friedman. Molecular quantum mechanics, fourth edition. Oxford University Press (2005).

[3] H. Goldstein, Ch.P. Poole, and J.L. Safko. Classical Mechanics. Addison-Wesley, 2002.

[4] A.S. Davydov, Quantum Mechanics. Pergamon Press, 1976.

[5] K. Huang, Statistical Mechanics. Willey, 2nd edition 1991.

[6] R. Kubo, Thermodynamics. North-Holland Publishing Co., 1968.

[7] Tutorial and user guide VMD. http://www.ks.uiuc.edu/Training/Tutorials/

[8] Tutorial and user guide NAMD. http://www.ks.uiuc.edu/Training/Tutorials/

**GEOMETRY OF GENERAL RELATIVITY** - back to the list of all courses

5 ECTS

Essential prerequisites: Differential Equations, Multivariate Analysis, Analysis of Differential Equations, Geometry, Classical Mechanics, Classical Electrodynamics, Topology and Metric Spaces

Desirable prerequisites: General Physics I: Mechanics, Oscillations, and Waves, General Physics II: Electromagnetism and Thermal Physics, General Physics III: Optics, Atoms, and Particles

This course assumes familiarity with the language of differentiable manifolds, but develops the theory of affine connections and enough pseudo-riemannian geometry (metric tensor, curvature) in order to describe the theory of General Relativity. This is done via the postulates of General Relativity and the Einstein field equations. The course then explores solutions of the Einstein field equations,including the famous Schwarzschild black hole and the cosmological solutions, which introduces the geometric notions of homogeneity anisotropy.

The lectures cover:

- **Affine connections:** covariant derivative, torsion, curvature, parallel transport, geodesics, geodesic deviation.
- **Riemannian geometry:** metric tensors, Lorentzian metrics, Levi-Civita connection, curvature tensors, moving frames, Cartan structure equations, isometries, Killing vector fields.
- **General Relativity:** special relativity and Minkowski spacetime, Maxwell's equations, postulates of General Relativity, spacetime, general covariance, energy-momentum tensor, Einstein equations.
- **Causal structure and Penrose diagram for Minkowski spacetime.**
- **Schwarzschild solution:** static and spherically symmetric spacetimes, black hole, Kruskal extension, causal structure and Penrose diagram.
- **Cosmological models:** homogeneity and isotropy, the Friedmann-Lemaître-Robertson-Walker metric.

**Recommended books**:

[1] An Introduction to General Relativity, L.P Hughston and K.P. Tod (LMS, CUP, 1990)

General Relativity, R. M. Wald, University of Chicago Press (1984)

**ASTROPHYSICS AND COSMOLOGY** - back to the list of all courses

4 ECTS

Essential prerequisites: Differential Equations, Probability and Measure, Multivariate Analysis, Complex Analysis with Applications, Analysis of Differential Equations, Classical Mechanics, Classical Electrodynamics, Quantum Physics, Statistical Physics and Electrodynamics, Geometry of General Relativity.

Desirable prerequisites: General Physics III: Optics, Atoms, and Particles.

This course offers a mathematically oriented introduction to key topics in astrophysics and cosmology. It emphasizes the formulation and analysis of physical models that describe stars, galaxies, and the universe as a whole. Topics include radiative processes, stellar structure and evolution, galactic dynamics, and the standard model of cosmology. Applications are drawn from both classical and modern observational data. Students will explore differential equations, dynamical systems, and geometric aspects of spacetime as tools for understanding the cosmos.

The lectures cover:

- **Mathematical Foundations of Astrophysics:** Coordinate systems, proper motion, parallax, units and scaling laws in astrophysics, flux, luminosity, magnitude scales, methods of measuring distances;

- **Radiation and Transport:** Blackbody radiation, Planck's law, Stefan–Boltzmann law, radiative transfer equation, optical depth and scattering

- **Stellar Structure and Evolution:** Hydrostatic equilibrium: Lane–Emden equation, polytropic models, energy generation and transport equations, nuclear fusion as a source of energy in stars, timescales of stellar evolution, Hertzsprung–Russell diagram, white dwarfs: Chandrasekhar limit, neutron stars and black holes;

- **Galactic Dynamics:** Mass profiles and rotation curves, spherical vs. disk potentials, evidence for dark matter, Jeans instability and gravitational collapse;

- **Relativistic Cosmology:** Review of general relativity (geodesics, metric tensor, Einstein field equations), Friedmann–Lemaître–Robertson–Walker (FLRW) metric, Friedmann equations and cosmic expansion, Hubble law and redshift;

- **Thermal History of the Universe:** Cosmic Microwave Background (CMB), recombination and decoupling, Big Bang nucleosynthesis;

- **Contemporary Topics:** Dark matter and gravitational lensing, dark energy and accelerating universe, basics of cosmic inflation.

**Recommended books:**

[1] B. Ryden. Introduction to Cosmology. Cambridge University Press, 2016.

[2] A. Liddle. An Introduction to Modern Cosmology. Wiley, 2013.

[3] D. Prialnik. An Introduction to the Theory of Stellar Structure and Evolution. Cambridge University Press, 2009.

[4] J.V. Narlikar. An Introduction to Cosmology. Cambridge University Press, 2002.

[5] S. Weinberg. Cosmology. Oxford University Press, 2008.

[6] T. Padmanabhan. Theoretical Astrophysics. Cambridge University Press, 2000.

# MISCELANIA

**INTRODUCTORY AND RESEARCH PRACTICUM** - back to the list of all courses

Starting from the second year, students participate in research activities within the KSE and its partner research institutes. Each trimester, students may join one research group, allowing them to explore different fields and develop hands-on experience in real scientific work. The practicum is designed to build core research competencies along with essential soft skills, including scientific writing and oral presentation. At the end of each academic year, students are required to submit a written report and deliver an oral presentation summarizing their work in one of the research groups.

**CAPSTONE PROJECT** - back to the list of all courses

In their fourth year, students undertake an individual Capstone Project under the supervision of a faculty member or research scientist. Each project is based on a real, yet unsolved research problem, giving students the opportunity to contribute—at an appropriate level—to ongoing scientific inquiry. The work involves independent study and research, typically aligned with the student's specialization or interests. It culminates in a written thesis and an oral defense, demonstrating the student's ability to formulate a research question, apply appropriate methods, and communicate results effectively in both written and oral form.