OPTIMAL PORTFOLIO HEDGING
IN A CRYPTOCURRENCY MARKET

by

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Kyiv School of Economics

Abstract

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The rapidly expanding cryptocurrency market has presented investors with unique opportunities for portfolio diversification and capital appreciation. This thesis investigates the dynamics of optimal portfolio hedging in the cryptocurrency market, addressing the research question: "How can investors optimally hedge their cryptocurrency portfolios with predictable cash flow to minimize risk exposure and maximize returns while taking into account market the unique characteristics of the digital asset ecosystem?" To answer this question I analyze various derivatives-based hedging instruments, specifically option-based and perps-based, and assess their effectiveness in mitigating risks associated with cryptocurrency investments. The findings of this thesis will provide investors with a comprehensive framework for managing risks in the cryptocurrency market, contributing to the digital asset management literature.

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List of Abbreviations

IV. Implied volatility. Market's forecast of a likely movement in an asset's price.

Perps. Perpetual futures.

ATM. At-the-money options.

OTM. Out-of-the-money options.

ITM. In-the-money options.

Chapter 1

INTRODUCTION

Over the past decade, the cryptocurrency market has rapidly expanded, evolving into a prominent player in the global financial landscape. Attracting both retail and institutional investors with its high returns and growing market capitalization, cryptocurrencies have garnered significant interest.

As the market has matured, a variety of cash-flow-generating crypto assets has emerged, with unique features and value propositions. This thesis focuses on a portfolio comprising 12 such yield-generating crypto assets, where the weights are determined based on their respective market capitalizations.

All of these assets exhibit a strong positive correlation with one another, implying that their price movements tend to follow the same direction, thereby increasing the need for effective risk management strategies.

Ethereum (ETH) plays a particularly important role in this context, as it is not only a key component of the portfolio but also the only asset for which hedging instruments are readily available in the market. The inherent volatility and unique characteristics of the digital asset ecosystem present investors with distinct challenges when it comes to managing risk and optimizing returns.

This thesis aims to explore the optimal strategies for hedging cash-flow-generating cryptocurrency portfolios, considering the various factors that influence risk exposure and return potential. The research question guiding this study is: "How can investors optimally hedge their cash-flow-generating cryptocurrency portfolios to minimize risk exposure and maximize returns while taking into account the unique characteristics of the digital asset ecosystem?"

By investigating this research question, this thesis will contribute to the growing body of literature on cryptocurrency investments and risk management. Through empirical analysis and the application of advanced financial theories, we will develop a comprehensive framework for constructing and managing cryptocurrency portfolios that efficiently balance risk and return.

In order to achieve this objective, this study will be organized into several sections. First, we will provide a literature review that examines the current state of knowledge regarding portfolio management and hedging techniques in the cryptocurrency market, highlighting the gaps in understanding that aim to address.

Following this, we will employ a multi-step methodology that encompasses a variety of analysis techniques to evaluate the effectiveness of different hedging strategies. This methodology will comprise the following steps:

1. Data Collection: The first step in our analysis will involve the collection of historical data for 12 different cryptocurrencies that make up our portfolio.

2. Calculation of Weights: In the weight calculation phase, the weight of each individual cryptocurrency is determined as the ratio of its market capitalization to the total market capitalization of all cryptocurrencies in the portfolio.

3. Portfolio Value Modeling: After it, to simulate the potential paths of the portfolio's value over the hedging period, we will use a multivariate Jump-Diffusion model. This stochastic process incorporates both continuous and discrete jump components to capture the dynamics of the portfolio assets. It allows for the Monte Carlo simulation of various scenarios for the price evolution of individual assets in the portfolio,

considering factors such as correlation among assets, volatility, drift, as well as the occurrence of jumps in asset prices. For each simulated path, we will calculate the expected portfolio value and construct a distribution of potential future portfolio values.

4. Hedging Exposure Calculation: Based on the expected portfolio value distribution obtained from the Monte Carlo simulation, the hedging exposure of the portfolio will be calculated. Hedging exposure refers to the sensitivity of the portfolio value to changes in the value of an underlying hedging instrument and to calculate it, we will employ a multivariate regression approach.

5. Hedging Techniques Simulation: With our portfolio value model, we will simulate the performance of various options-based and perpetual swaps (perps) based hedging techniques. This step will involve the application of both traditional and innovative financial instruments to manage the identified risks and achieve our desired hedging objectives.

6. Optimal Hedging Strategy Identification: Finally, we will compare the simulated performance of the different hedging techniques to identify the optimal strategy that provides the best trade-off between risk reduction and the cost of hedging. This step will enable us to make actionable recommendations for investors seeking to optimally hedge their cash-flow-generated cryptocurrency portfolios.

By employing this rigorous methodology, our study will not only contribute to the academic literature on cryptocurrency portfolio management but also provide practical insights and a quantitative framework that can be directly applied by investors in the real world. Through these findings, I hope to enhance the understanding of the unique challenges and opportunities presented by the digital asset market, ultimately promoting more informed decision-making and better risk management practice among cryptocurrency investors.

Chapter 2

LITERATURE REVIEW

In this section, we review the relevant literature on hedging strategies in financial markets, with a specific focus on the application of standard futures, options, and perpetual futures in the context of the cryptocurrency market. The literature on hedging strategies can be broadly divided into three main categories, corresponding to the three primary hedging instruments under consideration.

Standard Futures Contracts have long been recognized as a critical instrument in traditional financial markets, providing an effective mechanism for hedging against risks (Bodie, 1989). Such contracts provide investors the opportunity to secure the future price of an asset, thereby minimizing their exposure to potential price volatility.

In recent times, futures contracts have grown in popularity (Harris, 2018). This trend was primarily spurred by the Chicago Mercantile Exchange (CME) with the introduction of Bitcoin futures in December 2017, followed by Ethereum futures in February 2021. These advancements have significantly broadened the scope of financial derivatives, providing investors with novel ways to navigate the unique risks tied to cryptocurrency investments.

Even with the rising popularity of cryptocurrency futures contracts, empirical research investigating their efficacy as a risk management tool is still developing. Initial research, such as that by Billio et al. (2021), offers early empirical evidence pointing to the potential effectiveness of futures contracts in reducing risk exposure related to cryptocurrencies.

Perpetual Futures - perpetual futures, or “perps’ are a relatively new financial instrument that has risen within the cryptocurrency market (Cont & Kotani, 2020), offering a unique financial tool specifically designed to handle the market's inherent volatility. With no fixed expiration date, perpetual futures differentiate themselves from standard futures and are increasingly recognized as a potentially more effective risk management tool within the fast-paced crypto market dynamics. This innovative financial instrument has become popular across various cryptocurrency exchanges, including platforms like Binance and BitMEX.

The effectiveness of risk mitigation strategies in the cryptocurrency market, especially those utilizing perpetual futures, has recently garnered academic interest. Major research efforts have been dedicated to solving the standard minimum-variance hedging portfolio problem.

In this context, Alexander et al. (2020) emphasize the notable hedge effectiveness of BitMEX's inverse perpetual futures against cryptocurrency spot prices. Similarly, Deng et al. (2021) highlight the effectiveness of OKEx's inverse futures contracts as strong risk management tools for handling spot price risk. Their research suggests that these contracts surpass CME's standard futures in terms of hedge effectiveness.

Carol et al. (2021) conducted a comparative study of these financial derivatives across various trading platforms with the goal of identifying the one that provides both minimum hedged portfolio variance and the lowest chance of liquidations due to insufficient collateral. Using the extreme value theorem, they found an optimal strategy, highlighting that margin constraints and loss aversion are crucial determinants of its features.

The study by Nimmagadda and Ammanamanchi (2019) examines the relationship between Inverse Perpetual Swap contracts, a Bitcoin derivative akin to futures, and the margin funding interest rates levied on BitMEX. The research establishes the Heteroskedastic nature of funding rates, meaning that the variability of the funding rate is not constant but changes over time. This characteristic of the funding rate is crucial for the development of predictive models, as it implies that the volatility of the funding rate itself is a random variable that needs to be modeled.

Options are another common financial instrument used for hedging risk in traditional markets (Black and Scholes, 1973). Providing the right, but not the obligation, to buy or sell an asset at a predetermined price on or before a specified date, allow investors to manage their risk exposure more flexibly. Recently, cryptocurrency options have gained traction in risk management (Bakshi et al., 2021). Platforms like Deribit and LedgerX offer options on various cryptocurrencies, including Bitcoin and Ethereum.

In an examination of hedge performance associated with options, Branger et al. (2012) employed a range of methodologies including delta, delta-vega, and minimum variance hedging. They utilized three distinct models for their investigation: the Black and Scholes (1973) model, the Merton (1976) model, and the Heston (1993) model. The performance of these models was then juxtaposed with the Stochastic Volatility with Jumps (SVJ) model, which was proposed by Bakshi et al. (1997). Their analysis showed that the Black-Scholes model, in spite of its traditional status, exhibited superior performance in the context of portfolio delta-hedging when compared to the other models. However, the model demonstrated suboptimal performance during periods characterized by extreme market fluctuations, while the Heston model showed proficiency during 'normal times' but fell short during 'extreme events' marked by large market movements.

Sun et al. (2015) conducted a simulation study where the data-generating process adhered to the double Heston jump-diffusion model. They aimed to discern the more effective approach between accurately predicting the market model and utilizing an options hedging model that fits well. Their study revealed that the hedging performance was insignificantly impacted by a misestimation of the hedge model, corroborating the findings of Green et al. (1999). The latter investigated the risk exposure from model risk in the event of mispricing and inaccurate volatility estimation in forecasting. They found a material risk exposure stemming from model risk under the assumption that an option is priced based on the Black-Scholes model. Consequently, they proposed a 'volatility markup' for call/put options pricing.

El Karouni et al. (1998) also explored hedging under the assumption of volatility misspecification. They demonstrated that, under certain conditions, the Black-Scholes option pricing model offers a robust and efficient hedge. The hedge performance remained robust under claim convexity and accurate volatility fitting.

In light of these insights, Matic (2019) recommended the application of the Jump-Diffusion model as a synthetic data-generating process within a stochastic process framework.

The Jump-Diffusion model, as recommended by Matic (2019) offers a compelling choice for a synthetic data-generating process in the context of hedging cryptocurrency portfolios with options. The continuous aspect of the Jump-Diffusion model, mirroring the traditional GBM model, captures the continuous, day-to-day fluctuations in the prices of cryptocurrencies. In conjunction with this, the jump aspect of the model, which introduces an additional source of randomness, accurately represents the sudden and dramatic price changes, or “jumps”, often seen in the cryptocurrency market. These jumps might be driven by various factors, such as regulatory announcements, technological changes, or shifts in market sentiment, which often result in drastic changes in cryptocurrency prices.

Implementing the Jump-Diffusion model for hedging cryptocurrency portfolios with options can offer several advantages. Firstly, it may provide a more accurate representation of the actual price dynamics of cryptocurrencies, which can contribute to improved hedging performance. Secondly, the model's flexibility in capturing extreme price movements could potentially lead to more efficient risk management strategies, as it can help to identify potential hedging opportunities that might be missed by more traditional models.

Despite these potential advantages, we should also consider the challenges associated with the use of the Jump-Diffusion model in the context of crypto assets. These include the complexity of the model, which requires the estimation of additional parameters compared to more traditional models, and the lack of historical data on cryptocurrencies, which complicates the model calibration process. Moreover, the very nature of the cryptocurrency market, with its extreme volatility and frequent price jumps, might make any model, even the Jump-Diffusion model, less reliable over longer time horizons.

Chapter 3

METHODOLOGY

This chapter presents the comprehensive methodology employed in this thesis to identify the optimal hedging strategies for cash-flow-generating cryptocurrency portfolios. The research aims to address the question: "How can investors optimally hedge their cash-flow-generating cryptocurrency portfolios to minimize risk exposure and maximize returns, while taking into account market volatility, liquidity, and the unique characteristics of the digital asset ecosystem, given the positive correlation among the assets and the availability of hedging instruments for Ethereum?" In order to accomplish this, a multi-step process is designed.

3.1 Data collection

We collect historical data for 12 different cryptocurrencies that constitute the portfolio of interest. The data was collected in CSV format and included the daily closing prices of the cryptocurrencies over a defined historical period.

3.2 Calculation of Weights

The weight of each individual cryptocurrency is calculated as the ratio of its market capitalization to the total market capitalization of all cryptocurrencies in the portfolio. The max weight is set to 50% and assigned to the ETH.

$$w_i = \frac{M_c}{M_t} \quad (1)$$

where w_i - the weight of the individual asset, M_c - market capitalization of the individual asset, M_t - total market capitalization of all cryptocurrencies in the portfolio.

3.3 Portfolio Value Modeling

In this step, we will model the portfolio value using a multivariate Jump-Diffusion model, which is a stochastic process that incorporates both continuous and discrete jump components to capture portfolio asset dynamics. This model allows for the simulation of various scenarios for the price evolution of individual assets in the portfolio, considering factors such as volatility, drift, and correlation, as well as the occurrence of jumps in asset prices.

The multivariate Jump-Diffusion model can be represented by the following equations:

$$dS_i(t) = \mu_i S_i(t)dt + \sigma_i S_i(t)dW_t(t) + J_i(t)dN_i(t) \qquad (2)$$

$$dM(t) = rM(t)dt \qquad (3)$$

Here, $dS_i(t)$ represents the change in the price of the asset i at the time t, μ_i is the drift (expected return) of asset i, σ_i is the volatility (standard deviation of returns) of asset i, $dW_i(t)$ denotes the differential of a standard Wiener process for the asset i, J is the jump size as a multiple of stock price, while $N_i(t)$ is the the number of jump events that have occurred up to time t. $N(t)$ is assumed to follow the Poisson process $P(N(t) = k) = \frac{(\lambda t)^k}{k!}e^{-\lambda t}$, where λ is the average number of jumps per unit of time. Where the jump size follows log-normal distribution

$$J \sim m \cdot exp(-\frac{v^2}{2} + vN(0, 1)) \qquad (4)$$

where $N(0, 1)$ is the standard normal distribution, m is the average jump size, and v is the volatility of jump size. The three parameters $\lambda,\ m,\ v$ characterize the jump-diffusion model.

3.3.1 Parameters Estimation

Then we will estimate the parameters of the multivariate Jump-Diffusion model using the historical returns of the cryptocurrencies. The returns will be calculated as the natural logarithm of the ratio of successive daily closing prices.

$$r_t = ln(\frac{P_t}{P_{t-1}}) \tag{5}$$

where r_t is the return at time t, P_t is the price at time t, and P_{t-1} is the price at time $t - 1$.

The drift μ and volatility σ will be calculated from these historical returns. The drift will be the mean of the returns, mathematically represented as:

$$\mu = \frac{\sum_{i=1}^{N} R_t}{N} \tag{6}$$

where r_t is the return at time t, and n is the number of observations.

The volatility will be the covariance matrix of the returns, calculated as follows:

$$\sigma = Cov(r) \tag{7}$$

where r is the vector of returns, and Cov denotes the covariance operation.

The jump amplitude (J) will be estimated as the mean of the returns that exceeds two standard deviations from the mean return.

$$J = \sum \frac{r_t}{m},\ for\, r_t > \mu + 2\sigma \tag{8}$$

where r_t is the return at time t, μ is the mean return, σ is the standard deviation of returns, and m is the number of returns that exceed two standard deviations.

The jump frequency λ will be set as a fixed parameter, representing the average number of jumps per day.

3.3.2 Estimation Method

We will estimate the parameters using Maximum Likelihood Estimation (MLE). The MLE method involves finding the parameter values that maximize the likelihood function, which measures the probability of observing the data given the parameters.

$$\ln[L(\mu,\ \sigma^2)] = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum(x_i - \mu)^2 \qquad (9)$$

We will maximize the log-likelihood function with respect to μ and σ^2 to obtain the MLEs of these parameters.

The estimation of the parameters will be done separately for each cryptocurrency, assuming independence between cryptocurrencies. The correlation between cryptocurrencies will be accounted for in the simulation step by using the covariance matrix of the returns to generate correlated random time series.

3.3.3 Simulation

To generate possible paths for the cryptocurrency prices over the pre-defined hedging period (30 days), we will perform a Monte Carlo simulation. The simulation will be repeated 10000 times to obtain a distribution of possible portfolio values. Each simulation incorporated random jumps and normal variations in prices, as dictated by the Jump-Diffusion model.

For each simulated price path, we will calculate the portfolio value and construct a distribution of potential future portfolio values. This distribution will provide insights into the range of possible outcomes including the likelihood of achieving certain portfolio values and the associated risks. It allows us to assess the portfolio's exposure to extreme events and evaluate the effectiveness of risk management strategies.

$$V_{t+1} = \sum_{p=1}^{12} w_p S_{p,\,t+1} \tag{10}$$

where V_{t+1} is the value of the portfolio at the end of the hedging period $t + 1$, w_p represents the weight of each individual portfolio asset and $S_{p,(t+1)}$ is the simulated price of the respective asset at that time. The distribution of portfolio values further will be used for risk management and decision-making purposes, allowing for an assessment of potential gains or losses within the given time frame.

3.4 Hedging Exposure Calculation

Based on the expected portfolio value distribution obtained from the Monte Carlo simulation, the hedging exposure of the portfolio will be calculated. Hedging exposure refers to the sensitivity of the portfolio value to changes in the value of an underlying hedging instrument or portfolio.

To calculate the hedging exposure, we will employ a multivariate regression approach to estimate the portfolio elasticity with respect to the ETH price, a specific cryptocurrency, and the only asset for which hedging instruments are presented.

$$\ln(\widehat{r}_p) = \beta_0 + \beta_1 \ln(\widehat{r}_1) + \ldots + \beta_i \ln(\widehat{r}_i) + \varepsilon \tag{11}$$

where $\hat{r}_p$ - expected daily returns of the portfolio, $\hat{r}_1$ - expected daily returns of cryptocurrencies, β_i - are the regression coefficients, ε - represents the error term.

The estimated coefficient β_1 in the multivariate regression model represents the portfolio price elasticity - sensitivity of the portfolio returns to changes in the returns of ETH.

The portfolio price elasticity (PE) provides valuable insights into the responsiveness of the portfolio value to changes in the price of ETH. A higher elasticity indicates a greeted sensitivity of the portfolio to ETH price movements, while a lower elasticity implies a lower sensitivity.

Now, based on the estimated portfolio price elasticity with respect to the ETH, we can calculate the hedging exposure (HE) of the portfolio. HE represents the change in the portfolio value for a 1% change in the ETH price and can be calculated using the following formula:

$$HE = \sum V_i(1 + y_i) \cdot PE \cdot 1\% \qquad (12)$$

where HE - is the hedging exposure of the portfolio, V_i - is the current value of the individual assets, y_i is the expected yield, and PE is the estimated portfolio price elasticity with respect to the ETH price.

The hedging exposure quantifies the potential impact on the portfolio value due to changes in the ETH price and provides valuable insights for designing effective hedging strategies to mitigate risks associated with ETH price fluctuations.

3.5 Hedging Techniques Simulation

In this step, we simulate the performance of options-based and perps-based hedging techniques, to evaluate their effectiveness in protecting the portfolio against adverse price movements. Due to the unavailability of historical options prices, we will use the theoretical Jump-Diffusion Model for Option Pricing. This step allows us to identify the optimal hedging strategy that provides the best trade-off between risk reduction and cost of hedging.

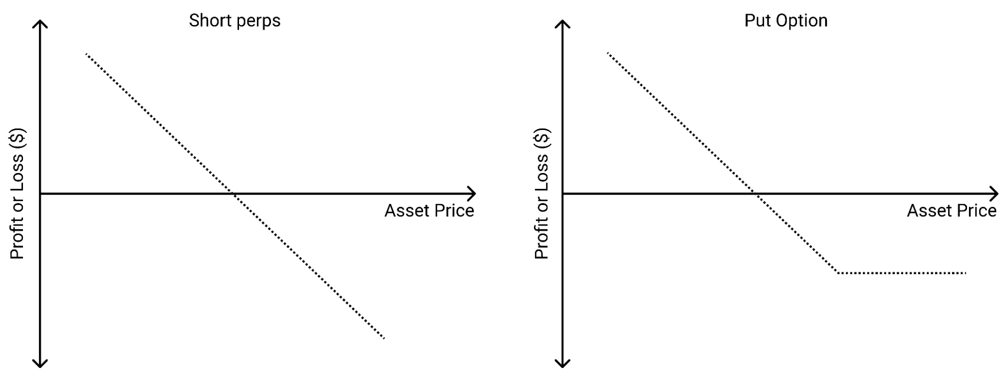

Figure 1. Short perps and Put option payoff diagrams

For this, we will design hedging strategies for each of the considered techniques, emphasizing short perps and put options. These strategies aim to protect the portfolio from adverse price movements and limit potential downside risk.

Short perps are a type of derivative that allows the investor to profit from a decrease in the price of the underlying asset, in our case, Ethereum (ETH). In such a way, we offset any changes in the total portfolio value that might be a result of price changes of underlying assets.

The number of shorts needed to fully hedge the portfolio can be calculated by dividing the hedging exposure by the contract size.

We will simulate the performance of a hedging strategy that involves opening a short perps position at the beginning of the hedging period and closing it at the end of the period. The cost of hedging using short perps will be determined by the funding rate paid to maintain the position over the hedging period.

Where the funding rate in a perpetual futures contract is related to the difference between the futures price and the spot price. When the market is bullish and traders are willing to pay a premium for leverage, the futures price tends to be higher than the spot price, leading to a positive funding rate. Conversely, in a bearish market, the futures price might fall below the spot price, resulting in a negative funding rate. This dynamic suggests that there might be a relationship between a daily price change and the funding rate.

We will use a linear regression model to quantify this relationship and use it to forecast future funding rates.

$$\ln(fr) = \beta_0 + \beta_1 \ln(r) + \varepsilon \qquad (13)$$

where fr- ETH perpetual futures funding rate, r- daily returns of ETH, β_1 - is the regression coefficients, ε - represents the error term.

Put options - a strategy that involves acquiring put options that give the holder the right to sell assets at a predetermined price (the strike price) within a specified timeframe. By purchasing put options on our portfolio, we create a form of insurance against potential price declines. The put options will gain value as the underlying asset prices decrease, offsetting any losses in the portfolio. The cost of hedging using put options will be calculated as the total premium paid for the options contracts.

To hedge the portfolio using put options, we will calculate the number of options contracts needed to fully hedge the portfolio by dividing the hedging exposure by the contract size.

Strike selection process: For the sake of simplicity of this thesis, the strike price is selected equal to the current price of the underlying asset at the time of the option purchase. This selection strategy is known as at-the-money (ATM) options trading. By choosing a strike price that equals the current market price, we're attempting to balance the cost and potential benefits of the put options.

Put options with strikes set at the current price generally have higher premiums than out-of-the-money (OTM) options, but lower than in-the-money (ITM) options. The reason for this is due to the higher probability of the options expiring in the money, which would trigger a payout. This balance between cost and potential payout makes ATM options a popular choice for many traders and investors, especially in volatile markets, like the cryptocurrency market.

Moreover, by choosing an ATM strike, we ensure that our put options start gaining value as soon as the underlying asset price decreases, without needing the price to move below a lower strike price first This can provide quicker and more immediate protection for our portfolio against price declines, making it an essential part of our risk management strategy.

Options Pricing: to estimate the theoretical put options prices we will use the Jump-Diffusion model.

In the Jump-Diffusion model, the asset price S_t follows the random process as described in (2).

For European call-and-put options, closed-form solutions for the price can be found within the jump-diffusion model in terms of Black-Scholes prices. If we write P_{BS} as the Black-Scholes price of a call or put option with spot S, strike K, volatility σ, interest rate r (assumed constant for simplicity), and the time to expiry T, then the corresponding price within the jump-diffusion model can be be written as

$$P_{JD} = \sum_{k=0}^{\infty} \frac{exp(-m\lambda T)(m\lambda T)^k}{k!} P_{BS} \tag{14}$$

where $\sigma_k = \sqrt{\sigma^2 + kv^2/T}$ and $r_k = r - \lambda(m - 1) + k\frac{\log m}{T}$. The k^{th} term in this series corresponds to the scenario where k jumps occur during the life of the option.

The Black-Scholes Formula for Options Pricing:

$$C = S_t N(d_1) - Ke^{-r(T-t)} N(d_2) \tag{15}$$

where:

$$d_1 = \frac{ln\frac{S_t}{K} + (r + \frac{\sigma_v^2}{2}(T-t))}{\sigma_s \sqrt{T-t}} \tag{16}$$

$$d_2 = d_1 - \sigma_s \sqrt{T - t} \tag{17}$$

here:

C - call option price;

S - current ETH/USDC price;

K - strike price;

r - risk-free interest rate;

t - time to maturity;

N - a normal distribution;

The price of a corresponding put option based on put-call parity with a discount factor $e^{-r(T-t)}$:

$$P = N(-d_2)Ke^{-r(T-t)} - N(-d_1)S_t \tag{18}$$

3.6 Optimal Hedging Strategy Identification

Finally, we implement the designed strategies in the simulated paths obtained from the portfolio modeling step and rank them based on their performance in terms of the highest expected value minus hedging cost and then choose the optimal strategy that will provide the most effective protection for the portfolio against adverse price movements while minimizing the costs associated with implementing and maintaining the hedge.

For the short perps strategy, the hedging cost is the funding rate paid to maintain the position over the hedging period. Hedging cost for short perps:

$$HC_{sp} = \sum fSn \tag{19}$$

where f - funding rate, S - position size, n - number of days.

For the put option strategy, the hedging cost is the total premium P_{JD} paid for the options contracts.

By following this systematic approach to identifying the optimal hedging strategy, we ensure a comprehensive evaluation of the available hedging techniques, taking into account both qualitative and quantitative aspects. The resulting optimal strategy will provide a robust and effective means of hedging the portfolio's risk exposure while maximizing returns, given the unique characteristics and challenges of the digital asset ecosystem.

Chapter 4

DATA

To conduct the analysis and implement the methodology outlined in this thesis, we rely on a variety of data sources to collect the necessary information.

Table 1. Daily price data for 12 crypto assets

Asset Name	Min	Standard deviation	Max
ETH/USDT	518.68	1004.94	4807.98
ADA/USDT	0.14	0.659	2.97
MATIC/USDT	0.02	0.592	2.88
SOL/USDT	1.20	60.39	258.44
DOT/USDT	4.29	12.5	53.82
AVA/USDT	0.48	1.297	6.27
ATOM/USDT	4.40	9.425	44.27
NEAR/USDT	0.87	4.183	20.18
EOS/USDT	0.82	1.915	14.52
EGLD/USDT	8.81	85.12	490.99
XTZ/USDT	0.72	1.77	8.71

The Asset Name column enumerates the 12 cryptocurrencies studied, which are all paired with Tether (USDT), a stablecoin tied to the value of the US dollar. The cryptocurrencies include Ethereum (ETH), Cardano (ADA), Polygon

(MATIC), Solana (SOL), Polkadot (DOT), Avalanche (AVA), Cosmos (ATOM), Near (NEAR), EOS (EOS), Elrond (EGLD), and Tezos (XTZ). The Minimum (Min) and Maximum (Max) columns present respectively the lowest and highest recorded daily price for each cryptocurrency during the observed period. The Standard Deviation column indicates the level of price volatility for each cryptocurrency during the period under consideration. This is a measure of how spread out the prices are from the average price. For example, ETH/USDT has a standard deviation of X%, indicating the lowest price volatility among all of the presented portfolio assets.

Table 2. Market Capitalization and Annual Yield.

Asset Name[1]	Market capitalization, $B	Annual yield, %
ETH	230	6
ADA	14.09	4
MATIC	10.18	4.8
SOL	9.28	7.15
DOT	7.82	15
AVA	5.91	9
ATOM	3.29	21.39
NEAR	1.93	10.07
EOS	1.33	1.26
MVRSX	1.03	9
XTZ	1.01	5.6

[1] Data source: Coingecko https://www.coingecko.com/en

The Asset Name column lists the ticker symbols of the 12 cryptocurrencies. The market Capitalization column displays the total market value of each of the listed cryptocurrencies as of 11.04.2023, in billions of dollars. For instance, ETH has the highest market capitalization at $230 billion, while Tezos (XTZ) has the lowest at $1.01 billion. The Annual Yield column is an annual return for each cryptocurrency as a percentage that is continuously distributed among token holders who have staked (locked) their tokens over 30 day period.

Here, ATOM offers the highest expected yield at 21.39%, while EOS has the lowest expected yield at 1.26%.

The final column, Portfolio Weight shows the percentage weight of each cryptocurrency in our investment portfolio. The portfolio is most heavily weighted towards ETH at 50%, and least weighted towards Tezos (XTZ) at 0.7%.

Perpetual Futures Funding Rate Data: The funding rate for perpetual futures contracts is essential to understand the cost of holding these positions as a hedging instrument.

Table 3. Annualized perpetual futures funding rate, %

Name[2]	Min	Mean	Max
Perps annualized funding rate, %	-333.91	-6.11	10.95

Table 3 provides data on the costs associated with the upkeep of perpetual futures contracts as a hedging instrument.

[2] Data source: Laevitas https://app.laevitas.ch/dashboard

By collecting and organizing the data from these various sources, we ensure that the analysis conducted in this thesis is based on accurate and comprehensive information. This data-driven approach allows us to build a robust methodology and derive meaningful insights into the optimal hedging strategy for cryptocurrency portfolios in the context of the digital asset ecosystem.

Chapter 5

ESTIMATION RESULTS

The estimation results are organized according to the different steps of the methodology, providing insight into the key aspects of the optimal hedging strategy for a cryptocurrency portfolio. As an example, we will showcase the results for one specific hedging period 0.1.03.2023-01.04.2023, demonstrating the application of the methodology and resulting implications for portfolio risk management.

After data collection, the first step is to calculate portfolio weights.

Table 4. Portfolio Weights

Asset Name	Portfolio weight, %
ETH	50
ADA	9.7
MATIC	7.01
SOL	6.39
DOT	5.38
AVA	4.07
ATOM	2.27
NEAR	1.33
EOS	0.92
MVRSX	0.71
XTZ	0.7

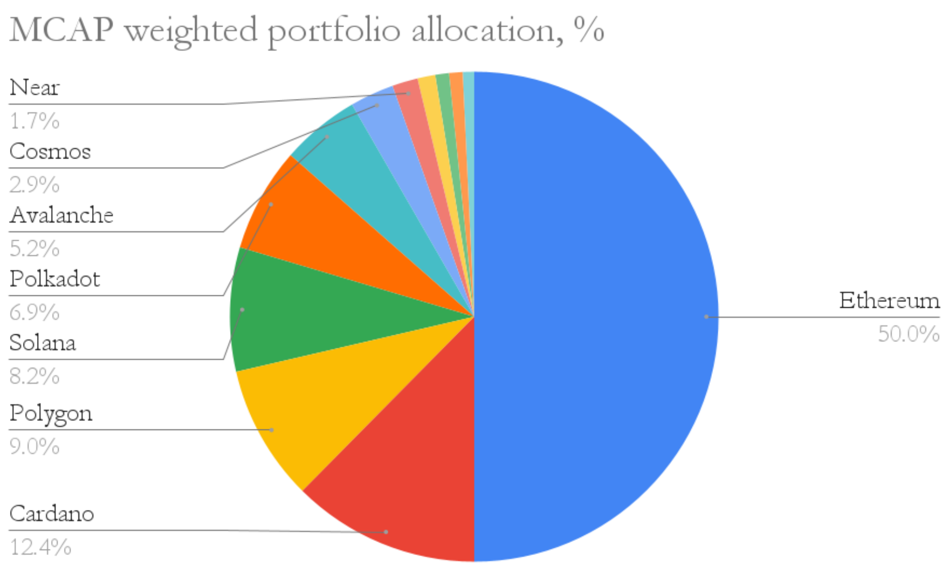

Figure 2. Portfolio composition

The process begins with the calculation of portfolio weights based on individual cryptocurrency market capitalizations. This approach ensures that cryptocurrencies with larger market sizes carry more significant weight in the portfolio.

For this, we sum up the market caps of all cryptocurrencies and then divide the individual market cap of each cryptocurrency by this total sum. This results in a set of weights, each lying between 0 and 1 and summing to 1, representing the fraction of the total portfolio investment in each cryptocurrency. The largest weight 50% belongs to ETH - the only portfolio asset for which hedging instruments are available, while the lowest weight 0.7% belongs to XTZ.

After it, we simulate the portfolio value over the next 30 days. This step involves two major components: the parameters estimation and the simulation of portfolio value evolution.

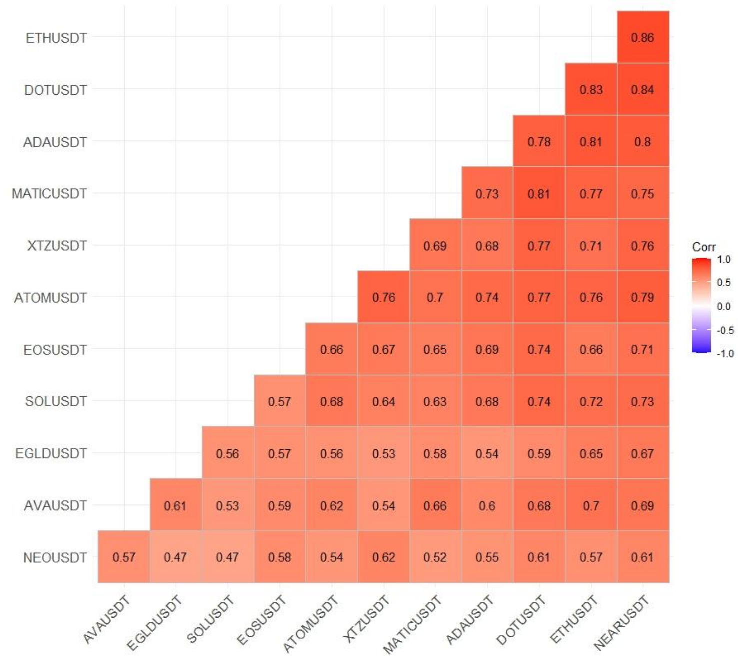

Figure 3. Correlation Matrix

One of the simulation parameters is the correlation matrix that represents how different cryptocurrencies in the portfolio move with respect to each other. This matrix helps us capture the interactions among different assets in the portfolio.

By using this matrix, we then proceed to simulate the portfolio value over the next 30 days using a Monte Carlo approach based on a multivariate Jump-Diffusion model. This model captures both gradual and sudden changes in cryptocurrency prices. By taking into account asset correlation, it randomly generates potential future price paths based on the diffusion and jump parameters derived from historical data.

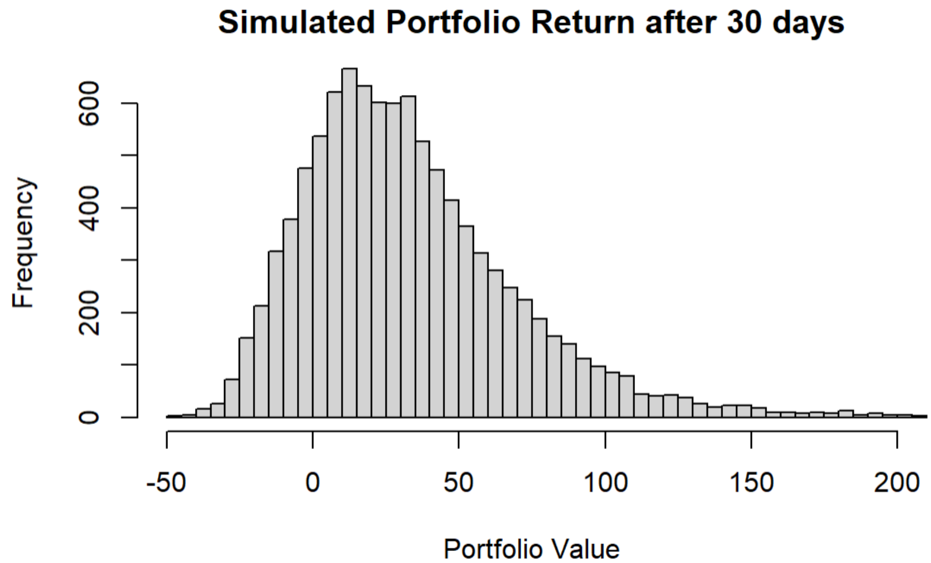

Figure 4. Portfolio Value distribution after 30 days.

The result is a distribution of possible portfolio returns after 30 days, allowing us to understand the potential future performance of the portfolio.

Where the most likely portfolio return is +33%.

The next step is to estimate the hedging exposure. This value provides an indication of how sensitive the portfolio value is to changes in the price of ETH, which is used as the underlying hedging instrument.

To estimate this sensitivity, we perform a regression analysis with the most likely portfolio returns picked from the distribution as the dependent variable and the individual ETH returns as an independent variable.

The coefficient corresponding to ETH in this regression gives us the elasticity of the portfolio with respect to ETH price changes. This elasticity is then used to calculate the hedging exposure, which quantifies the change in portfolio value for a 1% change in the ETH price.

Table 5. Portfolio price elasticity estimation results

Coefficients	Estimate	Std. Error	t value	
Intercept	-0.00103	0.005054	-0.205	
log_eth_returns	1.352481	0.215054	6.289	***

The log_eth_returns coefficient β_1 is the sensitivity of the portfolio returns to change in the returns of ETH. This is also referred to as the portfolio's exposure to ETH. This coefficient is statistically significant. The value of the coefficient suggests that for every 1% increase in the returns of ETH, we expect a 1.35% increase in the portfolio value.

The next step involves pricing put options on ETH as one possible method of hedging the portfolio. We use the Jump-Diffusion option pricing model. The input for this model is the most likely price path over the next 30 days, derived previously from the Monte Carlo simulation.

$$P_{JD} = 106.21\$$$

The output is the estimated price for the put option, which is the cost of hedging the portfolio's exposure to ETH using these derivatives.

After it, we focus on the second hedging strategy, which involves the use of perpetual futures contracts. The cost of maintaining a position in perpetual futures is determined by the funding rate. We build a linear regression model to predict this funding rate based on the historical relationship between daily price changes of ETH and the funding rate.

Table 6. Funding rate regression estimation results

Coefficients	Estimate	Std. Error	Pr(>|t|)	
Intercept	-5.505e-05	3.246e-05	0.0929	.
dailyEthReturns	6.115e-04	5.802e-04	0.2944	*

The Intercept represents the funding rate when the daily return is zero. The p-value associated with the intercept (0.0929) is slightly less than 0.1, indicating a borderline significance. We'd typically look for a p-value of less than 0.05 to consider a coefficient statistically significant.

The dailyEthReturns coefficient 6.115e-04 represents the change in the funding rate for a 1-unit change in the daily return. Its p-value (0.0944) is again borderline, so we should interpret this result with caution. It suggests that the funding rate might increase as daily returns increase, but the evidence is not strong enough to definitely say there is a relationship that requires further theoretical exploration.

The forecasted funding rate is then used to calculate the cost of hedging with perpetual futures.

Table 7. Cost of hedging for each strategy for a given hedging period

Strategy	Cost of hedging
Put option	7.4%
Perpetual futures	1.33%

Here, the cost of hedging with put options is 7.4% of the portfolio value, while the cost of hedging with perpetual futures is considerably lower, at 1.33%.

Put options provide insurance against a potential decline in the value of ETH. They require an upfront cost, known as a premium, which is 7.4% of the initial portfolio value in this case. However, it's important to note that the capital required for put options is considerably lower compared to that for perpetual futures. This is because you only need to pay the premium to buy the options and no additional collateral is necessary.

On the other hand, hedging with perpetual futures requires full collateralization, which means you need to maintain an amount of capital equal to the portfolio value. While the cost of hedging seems significantly lower at 1.33%, the full capital requirement can be a major drawback, as it reduces the capital available for other investment opportunities, such as reinvesting in the portfolio.

Moreover, in the context of our portfolio where all assets are staked to receive yield, the opportunity cost of hedging with perpetual futures becomes even more significant. Staking offers attractive returns in the form of staking rewards. The capital required for perpetual futures, if not locked up in futures, could be invested back into the portfolio for staking, potentially earning higher returns.

Therefore, while perpetual futures seem cheaper from a hedging cost perspective, the high capital requirement and the associated opportunity cost in terms of forgone staking rewards can make put options a more attractive hedging strategy in certain scenarios. The choice between the two methods should consider not only the direct hedging cost but also the capital requirements, opportunity cost, risk tolerance, market outlook, and potential rewards from staking.

Chapter 6

CONCLUSIONS

This thesis aimed to explore the dynamics of optimal portfolio hedging in the cryptocurrency market, focusing on how investors can optimally hedge their cryptocurrency portfolios with predictable cash flow to minimize risk exposure and maximize returns. The study was guided by the research question: "How can investors optimally hedge their cryptocurrency portfolios while taking into account the unique characteristics of the digital asset ecosystem?"

The study has demonstrated that derivatives-based hedging instruments, specifically options-based and perps-based, can be effective in mitigating risks associated with cryptocurrency investments. The application of these instruments in the context of a portfolio comprising 12 yield-generating crypto assets has been investigated, with Ethereum (ETH) playing a particularly important role due to the availability of hedging instruments for this asset.

The research has employed a multi-step methodology that includes data collection, calculation of weights, portfolio value modeling, hedging exposure calculation, and hedging techniques simulation.

The study's findings contribute to the growing body of literature on cryptocurrency investments and risk management. They provide a comprehensive framework for constructing and managing cryptocurrency portfolios that efficiently balance risk and return. This framework can be directly applied by investors in the real world, enhancing the understanding of the unique challenges and opportunities presented by the digital asset market.

In terms of policy recommendations, the study suggests that investors should consider the use of derivatives-based hedging instruments to manage their risk exposure in the cryptocurrency market. The choice of hedging instrument should take into account the unique characteristics of the digital asset ecosystem, including market volatility, and the positive correlation among assets. Furthermore, given the crucial role of Ethereum in the portfolio, investors should pay particular attention to the availability of hedging instruments for this asset.

For future research, it would be beneficial to explore different data-generating processes to further validate the robustness of the proposed hedging framework. Additionally, investigating the prediction of the funding rate and its impact on the hedging strategy could provide valuable insights into the cost-effectiveness of the hedging techniques. Finally, the exploration of various option-based techniques could offer a broader range of hedging strategies for investors to consider. This could potentially lead to the development of more sophisticated and effective risk management strategies in the cryptocurrency market.

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