CONDITIONAL VALUE AT RISK PORTFOLIO OPTIMIZATION WITH MACROECONOMIC FACTOR MODELS

by

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Abstract

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The objective of this paper is to study the possibility of applying factor models with macro variables to optimize the equity portfolio of the main selected capital markets (US, UK, Germany). In comparison with traditional optimization methods and basic factor models. To find the macro variables that affect the selected equity universe, I settled on the factors from Axioma Worldwide Macroeconomic Projection Equity Factor Risk Model (98 potential factors) using factor analysis to determine the leading 10 factors and solve the portfolio optimization problem by assessing the level of risk and return optimum Conditional Sharpe ratio portfolio with minimization risk strategy to avoid tail risk. The research shows that a strategy with a macroeconomic factor model compared with the classic equity portfolio optimization in the 2021-2022 interval showed a lower Expected Shortfall (CVaR) risk value in 87.4% of cases and a higher expected return in 56.3% of cases. Based on the factorial model, testing the logic of the model on more factors (>50) and a longer time interval (>2 years) will improve the results of the optimization model. The paper's findings can interest portfolio optimization/risk analysis software development as an additional portfolio risk analysis and decomposition model.

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LIST OF ABBREVIATIONS

US. United States of America.

UK. United Kingdom

FM. Factor Model

VaR. Value-at-Risk

CVaR. Conditional Value-at-Risk

MSV. Mean Standard Deviation

OWA. Ordered Weighted Averaging

FA. Factor Analysis

PCA. Principal Component Analysis

WCVaR. Worst-case Conditional Value-at-Risk and Multi-Factor Model

EFA. Exploratory Factor Analysis

CAPM. Capital Asset Pricing Model

APT. Arbitrage Pricing Theory

C h a p t e r 1

INTRODUCTION

The objective of this paper is to the possibility of applying factor models with macro variables to optimize the equity portfolio of the main selected capital markets (US, UK) in comparison with traditional optimization methods and basic factor models.

Countries In recent decades, the growth in the number of crises and market bubbles, especially after the 2008 crisis, caused a rapid increase in demand for services in risk management, not only in the banking environment but also in the field of investment. Since July 2022, the US normal yield curve has begun to invert, causing short-term Treasury rates to exceed long-term rates. Bonds lost significantly in price, which caused a decrease in the value of assets on the bank's balance sheet. Because of this, in March 2023, the US and European banking systems faced the risk of default of 6 large banks due to underestimation of the liquidity risk of securities, as in the case of Silicon Valley Bank, due to the sale at large losses of the treasury bond portfolio, which caused customer churn and default due to lack of deposit insurance.

Significant opportunities for applying risk avoidance techniques have also found application in the field of asset management, where, taking into account the client investor's attitude to risk, there is a need to determine the optimal portfolio from a certain number of assets, both one and many asset classes (stocks, bonds, gold, real estate, etc.). With the continuation of the coronavirus crisis in 2021-2022, the risk of changes in US monetary policy, changes in interest rates, banking crisis, etc., macroeconomic variables significantly impact the profitability of different asset classes. This causes at least the need to check the influence of macro variables on the basic practices of forming optimal portfolios of assets to achieve investment strategy.

A significant number of researchers have considered this problem of Conditional Value-at-Risk portfolio optimization approach theoretically: Rocjafellar and Uryasev (2000), Krokhmal et al. (2001), Yamai and Yoshiba (2002), Sarikalin et al. (2008), Lappalainen (2008), Ruan and Fukushima (2011) and others.

There are many approaches to portfolio optimization, from the modern Markowitz Portfolio Theory (MPT), published by Markowitz (1952) with significant simplification of estimation of expected return/loss and risk measure for each combination of assets in the portfolio. Currently, the following portfolio optimization models are mainly used:

1. Mean Risk Portfolio Optimization to find optimum portfolio asset weights that results from optimize one from four possible objective functions (Maximum Return Portfolio, Maximum Risk Portfolio, Maximum Risk Adjusted Return Ratio Portfolio, Maximum Utility Portfolio).

2. Risk Parity Portfolio Optimization, focuses on allocation of risk rather than allocation of capital. The risk parity approach asserts that when asset allocations are adjusted to the same risk level, the portfolio can achieve a higher Sharpe ratio and can be more resistant to market downturns, represented in Fast Design of Risk Parity Portfolios (2019).

3. Related Risk Parity Portfolio Optimization. Risk parity has been criticized as overly conservative. It is improved by re-introducing the asset's expected returns into the model and permitting the portfolio to violate the risk parity condition, presented by Gambeta and Kwon (2020).

4. Worst Case Mean Variance Portfolio Optimization with one from four objective functions (Worst Case Maximum Return Portfolio, Worst Case Minimum Risk Portfolio, Worst Case Maximum Utility Portfolio). It is a new approach for upper bounding the risk associated with a portfolio, for a given description of the uncertainty in the estimates of the first and second moments of the asset returns. It is a better approach is to explicitly account for such parameter uncertainty in the optimization, and to design a portfolio that performs reasonably for any set of parameters within the range of parameter uncertainty, presented by the research of Lobo and Boyd (2000).

5. Ordered Weighted Averaging (OWA) Portfolio, where calculated Higher L-Moment portfolio optimization model with possible objective functions (Minimum Risk Portfolio, Maximum Risk Adjusted Return Ratio Portfolio, Maximum Utility Portfolio). The main idea is to replace the classical mean and variance with the OWA operator. By doing so,

the new model can study different degrees of optimism and pessimism in the analysis being able to develop an approach that considers the decision-makers' attitude in the selection process. The main advantage of this method is the ability to adapt to many situations offering a more complete representation of the available data from the most pessimistic situation to the most optimistic one represented by Cajas (2021).

The choice of methodology depends on the target strategy of the investment fund or client: short-term or long-term investment, the presence of short sales, and the attitude to risk. This allows investors to maximize the expected return and minimize losses (risk).

As part of the research, we settled on the first method (Mean Risk Portfolio Optimization) because of the simplicity of its interpretation and the wide range of tested risk metrics. Mean Risk Portfolio Optimization is well suited for applying factorial models to estimate each asset's expected return. In classical factorial models, the risk is calculated mainly using the Standard Deviation metric (MV) or Semi Standard Deviation (MSV). However, given the non-normal distribution of stock prices, the CVaR (Expected Shortfall/ Conditional Value-at-risk) parameter was applied to avoid the tail risk that is not estimated by simpler risk metrics. Rickenberg (2020) showed measure the value of tail risk, the average loss of $\alpha\%$ worst-case scenarios. This will allow you to assess the impact of macroeconomic risk more accurately and, if necessary, minimize it in the portfolio of selected stocks, avoiding underestimated potential losses and reducing potential hedging costs (and a more accurate understanding of the direction of hedging securities or sectors, which will reduce costs). The selected portfolio is valued with the Conditional Sharpe ratio (C-Sharpe) defined as the ratio of expected excess return to the expected shortfall, presented by Chow and Lai (2015).

Portfolio optimization is usually considered in 2 stages: optimization of asset class weights and optimization of asset weights within the same asset class. In this study, only the second stage is considered, that is, the optimization of asset weights within the same asset class (because the asset class is constrained by the fund strategy).

The research question is whether factor models with macro variables are more suitable for portfolio optimization than factor models without macro variables and Markowitz optimization (traditional mean-variance portfolio method). To answer this question, I use the Axioma

Worldwide Macroeconomic Projection Equity Factor Risk Model to build an optimization model for a portfolio of 200 big-cap stocks from US / UK markets over 2 years (2021-2022). There are three main hypotheses that I test in the thesis.

The first hypothesis is the possibility of using macro variables derived from the Axioma Risk Model to optimize a stock portfolio. This means calculating and testing the statistical significance of 98 potential factors (26 major and 72 minor factors) and factor analysis to determine the maximum allowable number of factors and select the most important of them in a statistical factor model to avoid the multicollinearity problem and preserve the model's explanatory power, presented by Engelhardt (2013).

The second hypothesis is that the macroeconomic factor model is suitable for calculating the expected stock return, and CVaR shows an economically adequate result of the expected maximum loss in the simulated portfolio scenarios of the optimization model, compared to the Standard Deviation (MV) and Semi Standard Deviation (MSV) risk metrics. There are two possible explanations for this finding. Since this study does not focus on portfolio rebalancing research, one of the disadvantages of CVaR is avoided: "minimum CVaR portfolio is formulated with a single α and may output significantly different portfolios depending on the α. Kei Nakagava (2021) showed that the most portfolio allocation strategies do not account for transaction costs incurred by each rebalancing of the portfolio. In addition, Gotoh & Shinozaki & Takeda indicate that factor-model-based CVaR minimizing achieving better CVaR, turnover, standard deviation, and Sharpe ratio than the empirical CVaR minimization, presented in the research of Gotoh et al (2013).

The third hypothesis is that the macroeconomic factor model performs better than the basic and advanced factor model, the 5-factor Fama-French model. To do this, backtesting is carried out in optimal portfolio in each case, portfolio risk measure (CVaR) backtesting, Conditional Sharpe ratio (C-Sharpe) as measures of return to risk efficiency. This allows you to interpret the selected optimal portfolio's performance on historical data. If the percentage of historical scenarios in which the CVaR macroeconomic factor model in more than 50% of cases shows a lower risk value and in more than 50% of cases shows a bigger portfolio return compared to other selected models, then the hypothesis is accepted that the use of macro-factorial variables models is appropriate.

The paper is organized as follows. In Chapter 2, I review the literature on the factor dimension selection methods (factor analysis), which evaluate the number of needed factors for the model and factors the same and portfolio optimization methods with factor models. In Chapter 3, I discuss the factor analysis methodology, tail risk metrics calculation, CVaR factor model optimization problem formulation. In Chapter 4, I provide a data overview. Chapter 5 provides the estimation result of factor selection analysis, portfolio optimization model, asset allocation and risk analysis. Chapter 6 includes conclusions and practical implications of the model and opportunities for further improving steps in the next research papers.

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Chapter 2

LITERATURE REVIEW

The classical portfolio optimization approach for selecting the most effective asset allocation compared to the level of expected return and risk is well-discussed in literature for studying risk management and portfolio analysis. But most of the traditional approaches were limited by the assumption of normality distribution in asset price data, which is not always related to reality. The solution was to introduce a more conservative technique for assessing the risk of conditional value at risk (CVaR), as the expected return of the portfolio in the worst *b%* cases (equals the average of some percentage of the worst case loss scenarios), introduced by Rocjafellar and Uryasev (2000). The introduction of macroeconomic factor models for asset valuation is now actively developing due to their practical application in commercial risk models, such as Barra, Axioma, Barclays, Northfield. Usually, these methods were considered for different approaches to the formation of an optimal portfolio, factor models for assessing the expected return on assets, as well as calculating a covariance matrix for assessing less conservative risk metrics (portfolio dispersion); and the approach to CVaR optimization focused on researches Rocjafellar and Uryasev (2000), Sarikalin et al. (2008), Lappalainen (2008) and where factorial models are not applied. In contrast, several researchers, such as, Ruan and Fukushima (2011), combined these approaches using Japanese stock market data as an example. The amount of research in this field is still limited and is mostly concentrated on standard factor models (three and five-factor Fama-French models etc). This research will help to assess how appropriate it is to use factor models with macro variables for risk assessment compared to other factor models and the classical approach to portfolio optimization without using CVaR and factor models. Therefore, this chapter is divided into two parts. The first one explores general trends in research of CVaR portfolio optimization. The second part presents approaches for factor models and factors selection for these models.

2.1. Empirical studies on the Conditional Value at Risk

My paper is related to the literature studying the Conditional Value-at-risk portfolio optimization. Rocjafellar and Uryasev (2000) estimates a new approach to optimizing or hedging financial instruments to reduce risk is presented and tested on applications. It is focused on minimizing Conditional Value-at-risk (CVaR) rather than minimizing Value-at-risk (VaR). CVaR, mean shortfall (tail VaR), is considered a more consistent measure of risk than VaR because this approach calculates VaR and optimizes CVaR simultaneously. Authors used technique to find optimal portfolios from S&P500, government bonds, and small-cap portfolios, using Monte Carlo simulations set with different sizes (1,000-20,000). Calculations show that when the sample size is less than 10,000, the differences in CVaR and VaR obtained with the minimum CVaR and the minimum variance approaches are less than 1%. But with the growing sample size, portfolios are displayed that CVaR over VaR in capturing risk in the same modeling conditions; therefore, CVaR helps avoid uns realized loss.

Sarikalin et al. (2008) are shown research based on the same CVaR methodology, study the problem of choice between VaR and CVaR, especially in financial risk management, based on main differences in mathematical properties, stability of statistical estimation, simplicity of optimization procedures, acceptance by regulators. Authors show that CVaR has superior mathematical properties versus VaR because CVaR is a continuous and convex function, whereas VaR can even be discontinuous. Therefore, CVaR is easier to optimize with convex and linear programming methods, whereas VaR is difficult to optimize. VaR measure doesn't control scenarios exceeding VaR, but CVaR solves this problem and is a more conservative risk measure (because it can provide an adequate measure of risks reflected in extreme tail loss scenarios). Authors, based on the research of Yamai and Yoshiba (2002), show that VaR estimators are generally more stable than CVaR estimators with the same confidence level (selected part of the distribution) because it is not affected by very high tail losses, which are usually difficult to measure. For the cons of CVaR historical scenarios often don't provide enough information about tails. Therefore, the authors provide the assumption that it is better to assume a certain model for the tail to be calibrated on historical data, which provides additional assumptions to use factor models for evaluate potential tail loss.

The optimization problem from my research is based on Lappalainen (2008) optimization problem, which summarizes Rocjafellar and Uryasev (2000) and Krokhmal et al. (2001) next authors' assumptions to simplify the calculation. The modified methodology helps to avoid calculating VaR, starting directly from calculating CVaR risk measures. Compared with Rocjafellar and Uryasev (2000), where minimizing CVaR while requiring a minimum expected return, author focus on minimizing the negative expected return with a CVaR constraint and add modified method of approaching the efficient frontier with Monte Carlo simulations (100-50,000 scenarios). Differences in my approach with the author based on that in this research used factor model for calculate expected returns, but author use Black-Sholes model for pricing the share prices and calibrate the optimization using risk tolerance level. The greatest strength of the author's method is, without a doubt, the freedom of choosing parameters and assets. The optimization result also follows the commonly accepted rules of a good investment, moving from the risk-free asset through a diversified portfolio and to a portfolio with less diversification as the risk tolerance increases.

Ruan and Fukushima (2011) authors used pervious mentioned researches and implement them to the portfolio selection model with a Worst-case Conditional Value-at-risk and Multi-Factor Model (WCVaR) to real market data in Japan. The comparison reveals that the WCVaR minimization model is more robust than traditional one in a market recession period. Authors used modified Rocjafellar and Uryasev (2000) methodology for CVaR portfolio optimization, but mentioned that in practice researcher does not have enough information about probability distribution, but form definition of CVaR, portfolio return vector assumed to follow a probability distribution represented by a density function. New way of view applies an uncertainty about future probability distribution and also reflects our reliance on the universe of possible scenarios of distribution (mixtures of some predetermined distributions). Authors estimate expected returns by factor model, based on Asset Pricing Model (APT), approved by Ross (1976) and use Fama-French three factor model, published by Fama-Frech (1993), with assumption of multivariate normal distribution with a fixed variance-covariance matrix ($\sum$). Portfolio, formed from 23 stocks (selected from 50), and proposed model is shown to be more robust than the traditional model for real market data (with assumption of semi-strong efficient market hypothesis, where it is problematic to use past information to predict future prices).

The key discussion between my paper and the abovementioned studies is that I focus on Conditional Value-at-risk (CVaR) optimization with factor model with macroeconomic variables (but not just in the recession period) and compare it with classic models. In my paper, I attempt to do it for a selected stock universe for the first time on this topic.

2.2. Pricing factor models and factor selection

Classical models, which are used for financial asset price evaluation are included Capital Asset Pricing Model and Fama-French models (three and five-factor models). Started developing CAPM by Sharpe (1964), with a new step in factor modeling derived by Ross (1996) using the Arbitrage Pricing Theory (APT), one period model in which investor believes that the stochastic properties of returns of capital assets are consistent with a factor structure. Ross pointed out that linear pricing relation is necessary for equilibrium in a market where agents maximize certain types of utility. After that, increasing practical usage of researcher multifactor models for estimate asset prices, for example, three Fama-Frech (1993) and five Fama-Frech (2014) models. Fama-French models significantly increase exploratory power to 70-90% by using 5 factors. Fama-Frech (2014) includes to the factors list the value-weight return on the market portfolio of all sample stocks minus one month Treasury bill rate, SMB (small minus big) as the size factor, HML (high minus low B/M) is the value factor; RMW (robust minus weak OP) is the profitability factor; and CMA (conservative minus aggressive Inv) is the investment factor. However, this model is still unable to describe a significant part of the anomalies in the pricing of different groups of shares.

My paper is based on the comparison of optimal portfolios from basic models (three and five factors Fama-French models), advanced model (with trading indicators), and macroeconomic model. The macroeconomic model was built by the methodology of one of the most widely used commercial Axioma risk model – Axioma Worldwide Macroeconomic Projection Equity Factor Risk Model (2021). This model is the transformation of the main equity factor risk WW4 fundamental model combined with an additional set of market-traded macroeconomic factors. This model consists of 98 potential factors, related to macro factors (interest rate factors, inflation factors, corporate credit factors), currency factors, style factors (market-based factors, fundamental factors), industry factors, global market factors, country factors, and local factors.

Factor analysis is used to preserve the quality of the model and avoid the problem of multicollinearity when dependent variables can influence one another, which causes a biased result. Daoud (2017) mentioned that when multicollinearity occurs, one of the main assumptions in regression analysis (which is used for factor models) is violated. Multicollinearity appears when two or more independent variables in the model are correlated; if this happens, the standard errors of the coefficients will increase. Increased standard errors mean that the coefficients for some or all independent variables may be found significantly from 0, or overestimating the errors, multicollinearity makes some variables statistically insignificant. Brown (2006), in his research, introduced the exploratory factor analysis (EFA) methodology for selecting the appropriate number of potential factors, which should be guided by substantive considerations in addition to the statistical guidelines discussed upper. A more detailed description of the methodology is given in Section 3.11. I use this methodology to estimate the number of potential factors which need to select from the Axioma risk model factors list.

C h a p t e r 3

METHODOLOGY

3.1. Financial assets universe

Consider a portfolio consisting of *d* assets. The evolution of the market in time is represented by a sequence of financial instruments price vectors $y_1, y_2, \ldots \in R_+^d$, where

$$y_1 = (y_n^{(1)}, \ldots, y_n^{(d)}), \tag{3.1}$$

such that the *j*-th component $y_n^{(j)}$ denotes the price of *j*-th asset on the *t*-th trading period.

In order to apply the usual prediction techniques for time series analysis one has to transform the sequence price vectors $\{y_n\}$ into a more or less stationary sequence of return vectors $\{r_n\}$ as follows:

$$y = (y_1^{(1)}, \ldots, y_n^{(d)}), \tag{3.2}$$

such that

$$r_n^{(j)} = \frac{y_n^{(j)}}{y_{n-1}^{(j)}}, \tag{3.3}$$

Thus, the *j*-th component $r_n^{(j)}$ of the return vector r_n denotes the amount obtained after investing a unit capital in the *j*-th asset on the *n*-th trading period.

3.2. Defining the risk measure

For this part I select method, representing by the Krokhmal et al. (2001) and Lappalainen (2008). Let $f(r, w)$ – loss function, where *r* – portfolio return vector, chosen from subset of possible portfolio scenarios $r \in R^n$ and vector *x* from $X \in R^m$ – optimal assets weights. Loss value $f(r, w)$ – random variable, affected by *w* (uncertain market variables affecting the loss function value - scenario), and $p(w)$ – probability density function of w. The probability of the potential loss $f(r, w)$ not exceeding given threshold value ζ, where $\zeta \in R$ is given by

$$\psi(r, \zeta) = \int_{f(r,w) \leq \zeta} p(w)dw, \tag{3.4}$$

For portfolio, which fixed at point of time, *r* the function $\psi(r, \zeta)$ becomes a cumulative distribution function for the loss of the portfolio and shows the behavioral of the random variable. For simplicity Rocjafellar and Uryasev (2000) give assumption that by choosing *p(w)* to be continuous we get $\psi(r, \zeta)$ to be continuous with respect to α.

Therefore, we need convex function for find optimal solution from optimization problem, where we have α-VaR and α-$CVaR$ for $\alpha \in [0,1]$ is given by

$$\alpha\text{-}VaR: \quad \zeta_\alpha(r) = min\{\zeta \in R: \ \psi(r, \zeta) \geq \alpha\}, \tag{3.5}$$

$$\alpha\text{-}CVaR = \varphi_\alpha(r) = \frac{1}{1-\alpha} \int_{f(x,w) \geq \zeta_\alpha(r)} f(r, w)p(w)dw, \tag{3.6}$$

From formula (3.5) can be calculated threshold value $\zeta_\alpha(r)$, which shows, when the probability of loss exceeds α level ($\psi(x, \zeta) \geq \alpha$). From formula (3.6), using $\zeta_\alpha(r)$. Therefore, equation has limit value for the interval in which α-$CVaR$ is calculated, which can be represented by

$$\zeta_\alpha(r) \leq \varphi_\alpha(r), \text{ where } \forall x \in X \tag{3.7}$$

Rocjafellar and Uryasev (2000) provide the more complicated formula for calculating CVaR, without the external calculation of VaR, which can be represented by

$$F_{\alpha}(r,\zeta) = \zeta_{\alpha}(r) + \frac{1}{1-\alpha}\int_{w\,\epsilon\,R^m}[f(r,w)-\zeta]^{+}p(w)\partial w, \tag{3.8}$$

where $[f(r,w)-\zeta]^{+}$ = max{a,0}.

Theorem 3.1. As a function of $F_{\alpha}(r,\zeta)$ is convex and continuously differentiable. The α-$CVaR$ of the loss associated with any with any $r \in R^n$ can be determined from the formula, presented by Rocjafellar and Uryasev (2000):

$$\zeta_{\alpha}(r) \leq \arg\min F_{\alpha}(r,\zeta), \tag{3.9}$$

$$\varphi_{\alpha}(r) = min_{\zeta \in R} F_{\alpha}\big(r,\zeta_{\alpha}(r)\big), \tag{3.10}$$

Therefore, with properties from Theorem 3.1 of convex, continuously differentiable function can be minimizing: a local minimum can be directly seen as a global one. Therefore, formula (3.8), provided by Rocjafellar and Uryasev (2000) provide calculation α-$CVaR$ without calculation α-VaR.

Theorem 3.2. Minimizing the α-$CVaR$ of the loss associated with *r*: $\forall r \in R$, it is equivalent to minimizing $F_{\alpha}(r,\zeta)$ over all (r,ζ) from Rocjafellar and Uryasev (2000) can be represented by

$$min_{x\,\epsilon\,X}\varphi_{\alpha}(r) = min_{x\,\epsilon\,X}F_{\alpha}(r,\xi), \tag{3.11}$$

therefore, optimum result r^{*} - minimum α-$CVaR$ and corresponding to this value α-VaR.

3.3. Efficient frontier constraint

In this paper I oriented to the technic based on minimizing the negative return, potential risk, with a CVaR constraint from Krokhmal et al. (2001) and Lappalainen (2008) provides in his research 3 equivalent formulations technic of the optimization problem, each of them created the same efficient frontier (different portfolios generate a different portfolio return, and the efficient frontier shows portfolio that produce the best return for a given level of risk).

In this research was selected minimization problem, represented by

$$min_{(r,\zeta \in R^n \times R^m)} = -R(r),\ F_\alpha(r,\zeta) \le \omega,\ x \epsilon X \tag{3.12}$$

where ω – percentage of initial portfolio value / risk tolerance level; and minimization of $-R(r)$ over (r,ζ) produces a pair (r^*,ζ^*) such that r^* maximizes the return (minimizing loss) and ζ^* gives the corresponding α-VaR.

3.4. Formulating the optimization problem

For optimization linear objective function (selected in 3.3). The minimization problem based on Lappalainen (2008) can be represented by

$$\min r^T x \tag{3.13}$$

where in this case r – prices vector, x – weights vector.
subject to the list of the linear constraints

$$Ax \le b \tag{3.14}$$

where x - vector of variables, r – coefficients in the objective function, vector b and the matrix A are made up of the coefficients in the constraint equations.

Together all linear equations divide possible space of solutions to the feasible space (space intersection between all constraint lines).

3.5. Discretization and Linearization Constraints

From a programming point of view we transform integral from CVaR formula (3.8) to linear format, which can be representing by

$$F_\alpha(r,\zeta) = \zeta_\alpha(r) + \frac{1}{1-\alpha}\int_{w \, \epsilon \, R^m}[f(r,w)-\zeta]^+ p(w)\partial w \; \text{->}$$

$$F_\alpha\widetilde{(r,\zeta)} = \zeta + \frac{1}{1-\alpha}\sum_{j=1}^{J}\pi_j[f(r,w) - \zeta]^+ \tag{3.15}$$

where π_j are probabilities of scenarios w_j.

To express $F_\alpha(r,\zeta)$ as linear equation, we need to linearize initial function $F_\alpha(r,\zeta)$ by adding new dummy variable – $z \, \epsilon \, [1, \ldots \; , J]$, and can be representing by

$$F_\alpha\widetilde{(r,\zeta)} = \zeta + \frac{1}{1-\alpha}\sum_{j=1}^{J}\pi_j z_j \tag{3.16}$$

Regarding to constraint ω, and $\varphi_\alpha(r) \leq \omega$, and therefore $F_\alpha\widetilde{(r,\zeta)} \leq \omega$, we can rewrite (3.16), which can be representing by

$$\zeta + \frac{1}{1-\alpha}\sum_{j=1}^{J}\pi_j z_j \leq \omega \tag{3.17}$$

3.6. Factor model for estimation expected return

For estimation expected financial instrument return I use risk factors model. Using research derived by Ross (1977) about Arbitrage Pricing Theory and Chamberlain and Rothschild (1983) in a large economy multi-factor model assume that the return vector follows a linear factor model as represented by Fan et al (2008):

$$r_j = b_{j1}f_1 + \cdots + b_{jm}f_m + \varepsilon_j, \tag{3.18}$$

where $j = 1, \dots , d$; and $f_1, \dots f_m$ are the excessive returns (excess returns) of *m* factors; b_{jd} from $j = 1, \dots , d$; and $i = 1, \dots , m$, are unknown factor loadings; and $\varepsilon_1, \dots \varepsilon_n$ are *d* idiosyncratic errors uncorrelated given $f_1, \dots f_m$. For ease of presentation, we can rewrite the factor model (3.18) in matrix form

$$R(x) = A + B_d f + \varepsilon \text{ or } R(x) = A + BF + \varepsilon, \tag{3.19}$$

where *R(x)* is the return series, *A* is the intercept, *B* is the loadings matrix, *F* is the expected returns vector of the risk factors.

Next estimate the expected returns vector based on the risk factors models, we can rewrite factor model *R(x)* it the same in matrix form:

$$\mu_f = A + BE(F) \tag{3.20}$$

For loss function we use prices calculation to find portfolio value difference and after potential return from initial investment, which can be represented by formula (3.3).

3.7. Loss function

Remember that portfolio consists from d assets in the financial instrument universe, without risk-free instruments. Let $x^0 = (x_1^0, \dots, x_d^0)^T$ – initial position vector, $x = (x_1, \dots, x_d)^T$ – optimized portfolio vector weights / position. $y = (y_1, \dots, y_d)^T$ – vector with uncertain expected prices of the given instruments; $q = (q_1, \dots, q_d)^T$- vector of current prices. Future asset value y is depended from factor model. Krokhmal et al. (2001) assumed in that the loss return function equals difference between future and present value of the portfolio, which can be represented by

$$f'(x, y) = \frac{-y^T x + q^T x^0}{q^T x^0}, \tag{3.21}$$

The return on a portfolio is the sum of the returns on the individual instruments *j* in the portfolio with weights x_j. The portfolio expected return / loss function $R(x)$ at the end of the period for each scenario *w* can be represented by

$$R(x) = E[r^T x] = -E[f'(x, y(w)] = \sum_{i=1}^{d} E[r_j] x_j \tag{3.22}$$

3.8. CVaR and positioning constraints

Krokhmal et al. (2001) and Lappalainen (2008) formulate risk exposure constraint for each scenario *w* can be represented by

$$\varphi_\alpha(r) \leq \omega q^T x^0 \tag{3.23}$$

$$\zeta + \frac{1}{1-\alpha} \sum_{j=1}^{J} \pi_j z_j \leq \omega q^T x^0 \tag{3.24}$$

$$\zeta + \frac{1}{1-\alpha}\sum_{j=1}^{J}\pi_j z_j \ \leq \ \omega\sum_{j=1}^{d} q_j x_j^0, \qquad j = 1, \ \ldots, J \tag{3.25}$$

$$z_j^{\ +} \geq \sum_{i=1}^{d}(-y_{ij}x_i + \ q_i x_i^0) - \ \zeta \geq \ 0 \tag{3.26}$$

where $\varphi_\alpha(r)$ - α-$CVaR$, ω – percentage of the initial portfolio value (selected target risk).

To avoid a portfolio consisting of one single asset, we add the bonds on positions. This research used a proportional approach, which does not handle short positions in the portfolio ($x_i \ \geq 0$) and maximum proportional part in the portfolio ($x_i \ \leq \ \overline{x_i}$), which can be formulated by

$$\underline{x_i} \leq \ x_i \ \leq \ \overline{x_i} \tag{3.27}$$

$$x_i \ \geq 0 \tag{3.28}$$

$$\sum_{i=1}^{d} x_i = 1 \tag{3.29}$$

3.9. The optimization problem

The first selected optimization problem, minimization CVaR risk, which optimizes objective function subject to CVaR constraints and bounds described in this chapter (3), can be formulated by

$$\min \sum_{i=1}^{d} -E[r_i]x_i \ , \tag{3.30}$$

subject to

$$\zeta + \frac{1}{1-\alpha}\sum_{j=1}^{J}\pi_j z_j \ \leq \ \omega\sum_{j=1}^{n} q_j x_j^0, \qquad j = 1, \ldots, J$$

$$z_j{}^{+} \geq \sum_{i=1}^{d}(-\, y_{ij}x_i + \, q_i x_i^0) - \zeta \geq \, 0$$

$$\underline{x_i} \leq \, x_i \, \leq \, \overline{x_i} \,, \quad i = 1, \ldots, d$$

$$x_i \, \geq 0$$

$$\sum_{i=1}^{d} x_i \, = 1$$

The optimization problem returns $E[r]x^*$ for optimal portfolio x^* with selected risk tolerance level (which helps to obtain the CVaR efficient frontier).

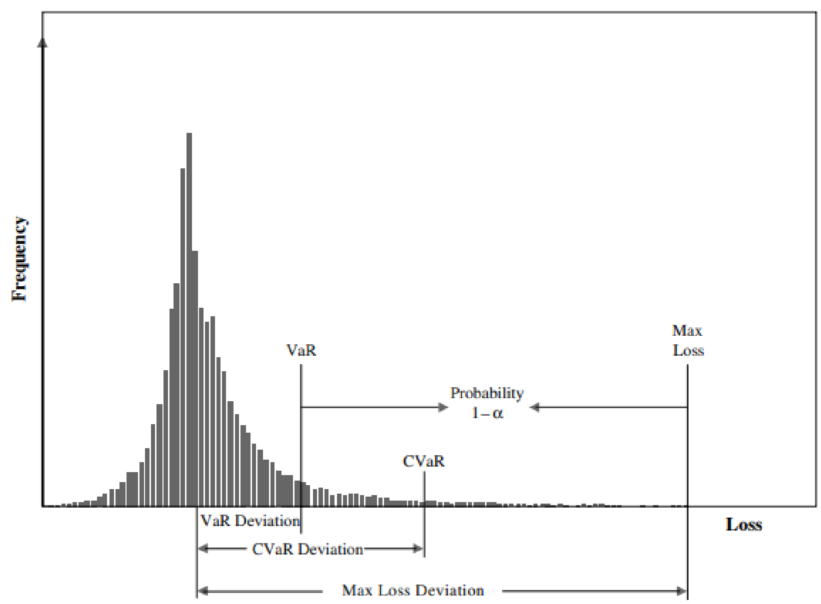

Figure 1. Risk metrics distribution

Source: Sarykalin et al. (2008)

The second optimization problem is oriented not just to optimize risk measure value but also to maximize expected portfolio return. One of the most popular measures of the return-to-risk efficiency ratio is the Sharpe ratio. For the goal of this optimization problem using the Conditional Sharpe ratio (CSR) or Modified Sharpe ratio (MSR) from Pär Lorentz, 2012 , Blaise Labriola which can be represented by

$$CSR = \frac{R(x)-r_f}{CVaR}, \tag{3.31}$$

where r_f – risk free rate, $(R(x) - r_f)$ – excess return, $CVaR$ – α-$CVaR$ value.

Deviation and risk are quite different risk management concepts. A risk measure evaluates outcomes versus zero, whereas a deviation measure estimates wideness of a distribution. For instance, CVaR risk may be positive or negative, whereas CVaR deviation is always positive. Therefore, the Sharpe-like ratio (expected reward divided by risk measure) should involve CVaR deviation in the denominator rather than CVaR risk, represented by Sarykalin et al. (2008). Therefore, Maximum Risk Adjusted Return Ratio optimization problem can be represented by

$$\max \sum_{i=1}^{d} \frac{E[r_i]x_i}{F_\alpha(r,\zeta)}, \tag{3.32}$$

subject to

$$\zeta + \frac{1}{1-\alpha}\sum_{j=1}^{J}\pi_j z_j \ \le\ \omega \sum_{j=1}^{n} q_j x_j^0, \qquad j = 1,\dots,J$$

$$z_j{}^+ \ge \sum_{i=1}^{d}(-\, y_{ij}x_i +\, q_i x_i^0) - \zeta \ \ge\ 0$$

$$\underline{x_i} \ \le\ x_i \ \le\ \overline{x_i}\,, \quad i = 1,\dots,n$$

$$x_i \geq 0$$

$$\sum_{i=1}^{n} x_i = 1$$

The optimization problem returns $E[r]x^*$ for optimal portfolio x^* with selected risk tolerance level (which helps to obtain the CVaR efficient frontier).

3.10. Scenario generation

For this research we use Monte Carlo approach for scenario generation simulations vectors of 10 000 scenarios with different probability densities for each input of each financial instrument. Scenarios generation. In the optimizations the used Monte Carlo scenarios have been created from a normal distribution to simulate Brownian motion. Expected price calculated from factor model with different range of the factor variables selected from Axioma Worldwide Macroeconomic Projection Equity Factor Risk Model and Five-Factor Fama-French model. Framework, where factor model pricing formula can be found in Fan (2008), Axioma Risk Model Factsheet (2021), Fama-French (2014).

3.11. Factor selection approach

For factor model selection problem to avoid problem multicollinearity, we select part of potential factors from the list of the factors with better explanatory power.

Brown (2006) mentioned that factor analysis partitions the variance of each indicator (derived from the sample correlation / covariance matrix that is used as input for the analysis) into two parts: 1) common variance, or variance accounted for by the latent factor, which is estimated on the basis of variance shared with other indicators in the analysis; and 2) unique variance, which is combinations of reliable variance that is specific to the indicator (systematic latent factors that influence only one indicator) and random error variance (measurement error or unreability in the indicator).

In this research selected exploratory factor analysis (EFA), because it is a data-driven approach such that no specifications are made in regard to the number of latent factors (initially) or to the pattern of relationships between the common factors and the indicators (factor loadings). Exploratory technique to determine the appropriate number of common factors to uncover which measured variables are reasonable indicators of the various latent dimensions (by the size and differential magnitude of factor loadings).

In the EFA factor loadings are completely standardized estimated of the regression slopes for predicting the indicators from the latent factor, and thus are interpreted along the lines of standardized regression (b) or correlation (r) coefficients as in multiple regression / correlational analysis (Cohen, 2003).

A fundamental equation of the factor model from Brown (2006) research can be represented by

$$y_j = \lambda_{j1}\eta_1 + \lambda_{j2}\eta_2 + \cdots + \lambda_{jk}\eta_k + \varepsilon_j \ , \tag{3.33}$$

where y_j – represents the *j*-th of p indicators, obtained from a sample of *n* independent subjects, where λ_{jk} – factor loading relating variable j to the *m*-th factor η, ε – the variance that us unique to indicator y_j and independent of all η and all ε.

System of indicators (p) an be represented at matrix form

$$\sum = \Lambda_y \Phi \Lambda_y' + \Theta_\varepsilon \ , \tag{3.34}$$

where $\sum$ - $p \times p$ symmetric matrix of p indicators, Λ_y - $p \times m$ matrix of factor loadings λ, Φ - $m \times m$ symmetric correlation matrix of factor correlations and Θ_ε is the $p \times p$ diagonal matrix of unique variances ε.

Therefore Brown (2006) explained variance can be calculated by

$$VAR(P1) = \sigma_{11} = \lambda_{11}^2 \Phi_{11} + \varepsilon_1 \,, \tag{3.35}$$

where Φ_{11} – the variance of the factor η_1 (equal 1, because the EFA model is standardized), and ε_1 – unique variance of p1.

Model-implied correlation of the indicators is the product of their completely standardized factor loadings

$$COV(P1, P2) = \sigma_{21} = \lambda_{11} \Phi_{11} \lambda_{21} \tag{3.36}$$

After determining that EFA, we should decide which indicators to include in the analysis and determine if the size of nature of the sample are suitable for research (selection of the appropriate number of factors).

Brown (2006) indicates that the results of the initial analysis are used to determine the appropriate number of factors to be extracted in subsequent analyses. Factor based on differential relationships among indicators that stem from extraneous or methodological artifacts. Number of factors m is limited by the number of observed measures (p) that are submitted to the analysis.

The maximum number of factors is mathematically limited by $a \geq b$, but can create potential problems with small set of indicators, therefore data may not support extraction of the small set of indicators. Mathematical explanations can be presented by

$$a = (p * m) + \frac{m*(m+1)}{2} + p - m^2 \tag{3.37}$$

$$b = [p * (p + 1)]/2 \tag{3.38}$$

where a – number of parameters in the factor solution, b – number of elements in the input correlation or covariance matrix, m – number of factors, p – number of observed variables (indicators)

Visually number of required factors can be represented by eigenvalues. Eigenvalue (p) - the proportion of variance in the indicators that is accounted for by the factor model (Pct of Var) / percentage of the explained variance. Thus, the eigenvalues guide the factor selection process by conveying whether a given factor explains a considerable portion of the total variance of the observed measures.

In this research, based on Brown (2006), we use two factor selection procedures are based on the eigenvalues: Kaiser-Guttman rule and scree test.

Kaiser-Guttman rule obtain the eigenvalues derived from the input correlation matrix by the methodology of the Fabrigar et al, (1999), after determine how many eigenvalues are greater than 1 (the corresponding factor accounts for less variance than the indicator with variance equals 1) and use that number to determine the number of nontrivial latent dimensions that exist in the input data.

Second technique that we used – scree test, presented by Cattel (1966) the same uses eigenvalues from correlation matrix to provide visual realistic illustration of eigenvalues from the vertical axis and the factors from horizontal axis. The graph is inspected to determine the last substantial decline magnitude of the eigenvalues or the point where lines drawn through the plotted eigenvalue slope.

Chapter 4

DATA OVERVIEW

The data used in this study can be divided into two groups: data on the prices of financial assets considered for inclusion in the portfolio, and data used to calculate the factors for the factor model (to estimate the price of each selected asset).Financial asset type for simplicity was limited by equity stock market from major markets (US, UK) for the largest capital stocks from each of them. Stocks distributed between different sectors downloaded from public data sources - Yahoo Finance.

Factors for the factor model are calculated according to the methodology Axioma Worldwide Macroeconomic Projection Equity Factor Risk Model. For this, macro variable was downloaded from FRED, government bond data from Investing.com, inflation-linked bonds data from Refinitiv Eikon, as well as public financial reporting data of companies and adjusted close price of the selected companies with Yahoo Finance are used; Fama-French factors data is loaded from the Kenneth R.French Data Library.

The factor model requires the same dimension of data of all factors and prices of financial assets. Automatic loading and processing of data arrays in Python has a 3-year array limit due to Yahoo Finance reporting only for the last 3-4 years (accordingly, the entire array is automatically reduced to 3 years or less to exclude missing data points). According to the Axioma Worldwide Macroeconomic Projection Equity Factor Risk Model methodology, the daily data frequency is used, for a number of factors - the monthly frequency (which is duplicated for each trading day in the month).

4.1.Financial assets universe

Stock list for each country was selected from CompaniesMarketCap according to the market cap criterion, since the model's target audience is asset management funds, whose asset portfolios are often subject to macro risk since they are designed for long-term investment.

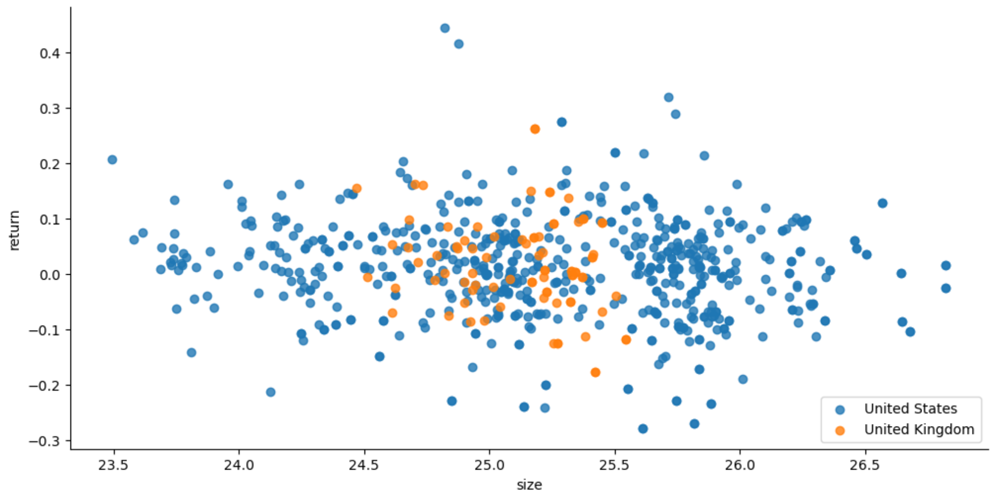

Figure 2. Stock universe country distribution (2017–2022).

Source: author's calculations on the data from Yahoo Finance.

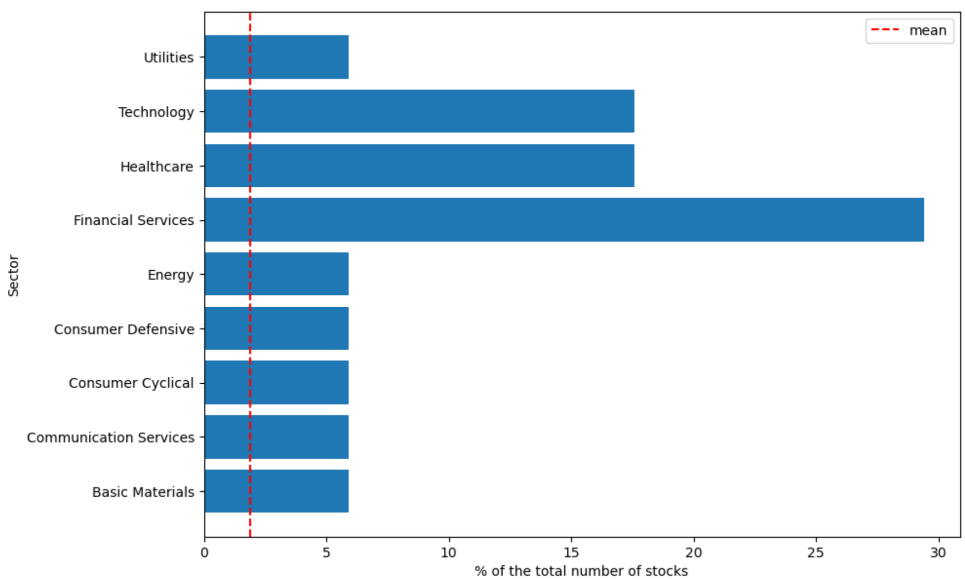

Figure 2. Stock universe sector distribution (2017–2022).

Source: author's calculations on the data from Yahoo Finance.

The long-term investment strategy assumes that assets should have stable dividend payments and a low risk of bankruptcy, which, according to research, is more relevant to stable companies with a large market capitalization (with a market capitalization of 10 billion dollars or more), calculations of return and risk components are presented in the Warren (2020) research.

As a result, after eliminating the missing data points due to the lack of financial statements and stock prices on stock days, 136 stock tickers with a time period from 2017 to 2022 remained in the array, the results of which are shown in Figures 2-3. Share price data is not limited, however, due to the selected time frame.

Most of the stocks selected in the array are traded on stock exchanges in the United States (88.2%), United Kingdom (11.8%) showing that the impact of macroeconomic factors from these regions will have a more significant effect than the macroeconomic factors of the Eurozone. Figure 3 shows that the potential portfolio is dominated by the financial services, technology and healthcare sectors.

Figure 4. Interest rate factors (2017–2022).

Source: author's calculations on the data from Investing.com.

4.2. Factors calculation (Axioma risk model, additional models' factors)

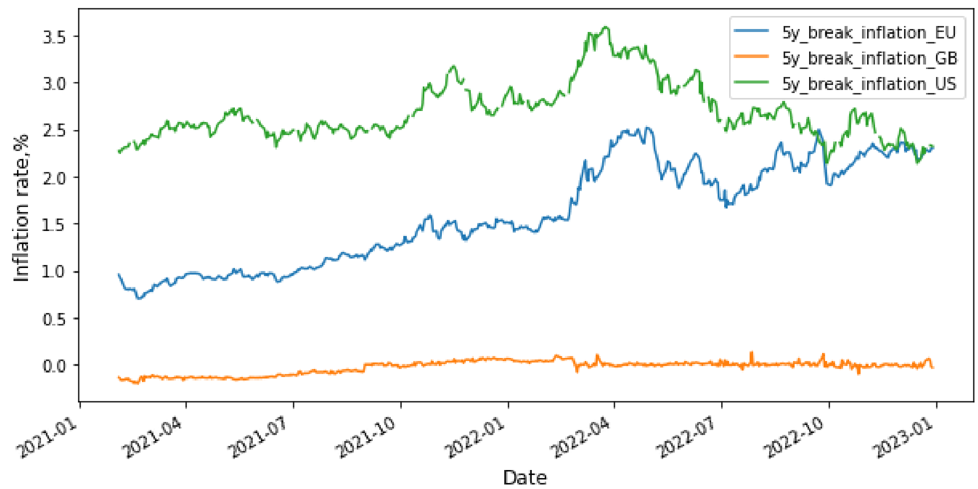

Figure 5. Inflation rate factors (2021–2022).

Source: author's calculations on the data from Investing.com, Refinitiv Eikon.

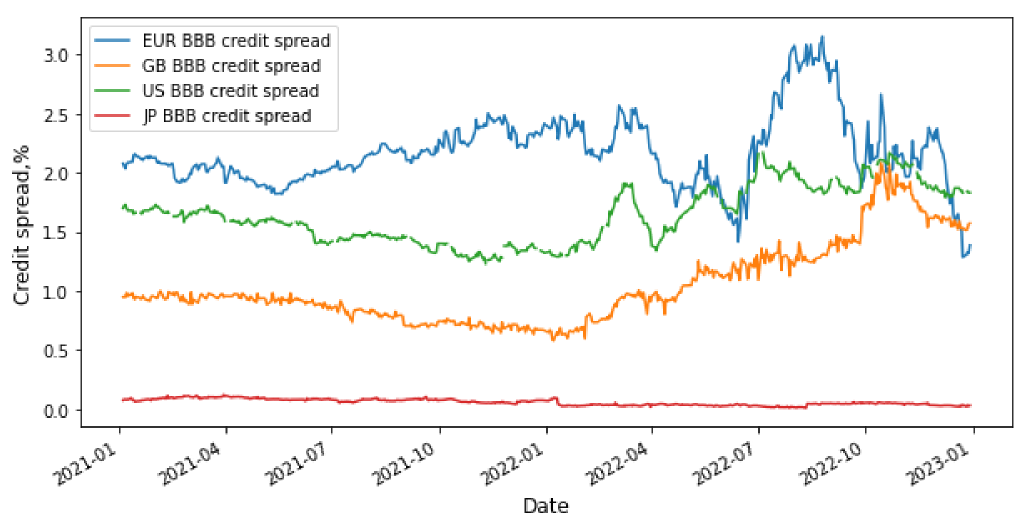

Figure 6. Corporate Credit Spread (2021–2022).

Source: author's calculations on the data from Investing.com, Refinitiv Eikon.

Corporate Credit Factors group consists of USD BBB Corp Spread, GBP BBB Corp Spread, EUR BBB Corp Spread, and JPY BBB Corp Spread. They are calculated as daily log returns of the 5-year node of the USD/ GBP/ EUR/ JPY BBB credit spread curve (corporate bond yield - government bond yield). The historical dynamics of the effects are shown in Figure 6. Daily yields have the same 2 years limitation. These spreads may indicate how investors are viewing economic conditions. The narrowing of spreads in 2022 will lead to a negative yield curve, indicating nonstable economic conditions in the future.

Last part of the macroeconomic factors based on Commodity Factors: commodity (daily returns of GSCI non-energy commodity spot index), gold (daily returns of the GSCI gold spot index), oil (daily returns of NYMEX:CL 1 month oil futures). The historical dynamics of the effects are shown in Figure 7. Oil is the most volatile, potentially explaining some of the volatility in stock prices.

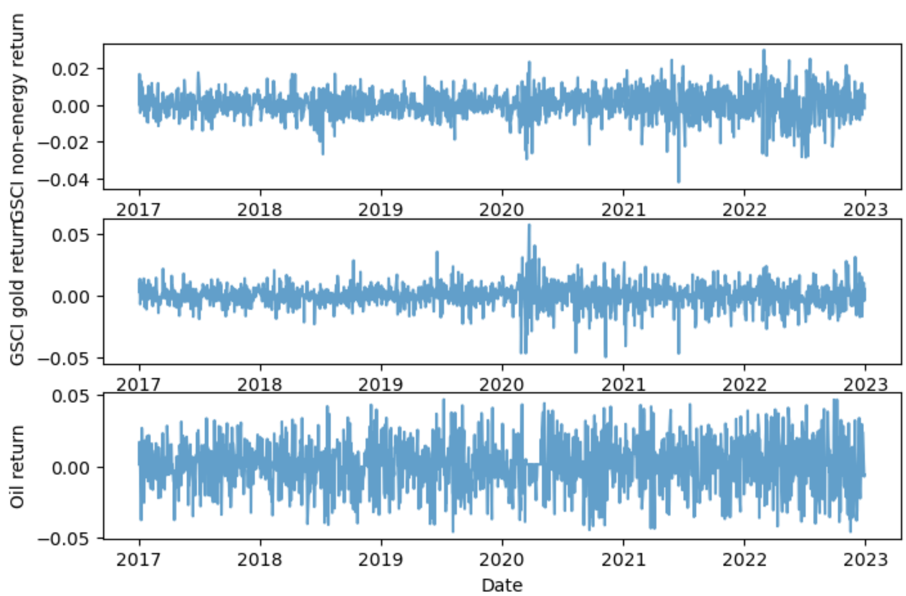

Figure 7. Commodities Factors (2017–2022).

Source: author's calculations on the data from Investing.com, Refinitiv Eikon.

Currency Factors include an important factor; however, given that 94.4% of shares are traded in dollars, the impact on the selected portfolio will be insignificant (the effect of other currencies on the portfolio is low).

Style Factors (12) consist of two main groups of factors by characteristics: Market-Based Factors (market sensitivity, volatility, liquidity, exchange rate sensitivity, medium-term momentum, size); and Fundamental Factors (value, earnings yield, leverage, growth, profitability, dividend yield). Analysis of market sensitivity (beta) shows that, on average, the selected stocks are less volatile than the market. The same conclusions can be drawn from the analysis of historical exchange rate sensitivity data relative to the currency market basket (basket consisting of equal shares of USD, EUR, GBP, JPY, and CNY currencies) because most of the portfolio is associated with the dollar. The historical dynamics of the effects are shown in Figure 8.

The volatility factor is calculated as the 6-month average of absolute returns over cross-sectional standard deviation, fully orthogonalized to market sensitivity. It is a good measure of the risk of stock price, which is directly related to the Size (logarithm of market capitalization), which is shown in Figure 9.

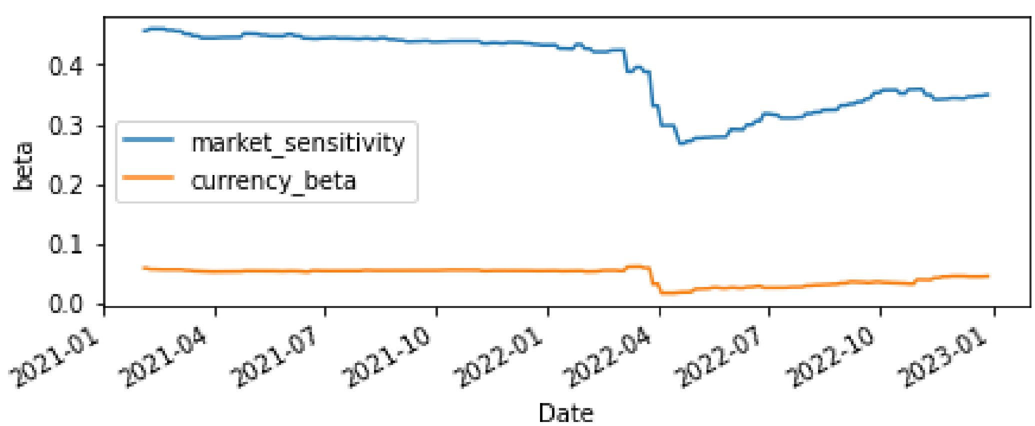

Figure 8. Market sensitivity and currency market beta (2021–2022).

Source: author's calculations on the data from Yahoo Finance.

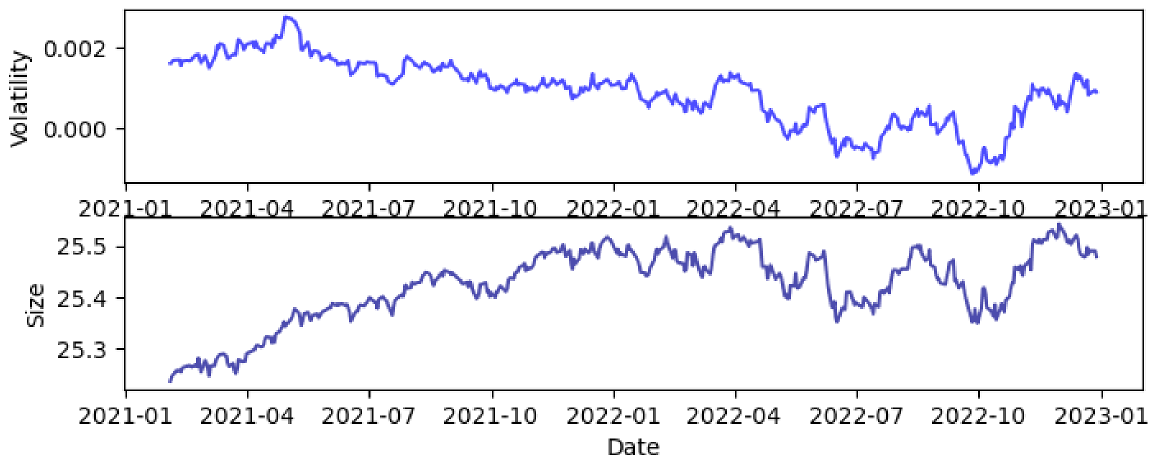

Figure 9. Volatility and size factors (2021–2022).

Source: author's calculations on the data from Yahoo Finance

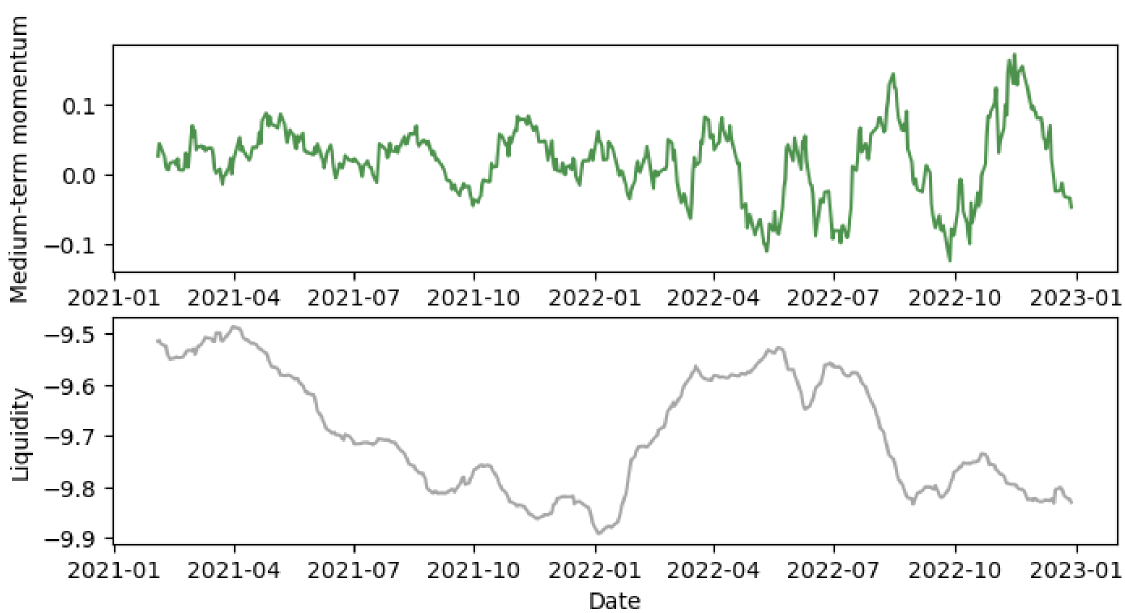

Figure 10. Liquidity and medium-term momentum factors (2021–2022).

Source: author's calculations on the data from Yahoo Finance

Liquidity is a natural logarithm of the ratio of 3-month average daily volume and 1-month average market capitalization. Medium-Term Momentum - cumulative return over the past year, excluding the most recent month. Both of these parameters showed decreasing liquidity and stabilization momentum in 2022, which can result: in reducing liquidity (demand) for stocks – the risk of price decreasing, stabilization momentum does not give any results (because it can be increasing or decreasing).

The Fundamental Factors group consists of value, earning yield, leverage, and profitability. In this part, we oriented for value, which is calculated as a book-to-price ratio, and earning yield – earning-to-price or estimated earning-to-price ratio. From the graphs, we can see a high underestimation of companies' prices due to their real values because of the recession and energy crisis and the reducing companies' margins. Earning yield (P/E) ratio showed a decrease in stock returns (caused by a decrease in company margins due to the crisis). The historical dynamics of the effects are shown in Figure 11.

Figure 11. Value (B/P) and Earning yield (E/P) factors (2021–2022).

Source: author’s calculations on the data from Yahoo Finance

Additional factors like Industry factors, Global Market Factors, Country Factors, and Local Factors are not used because they are not specified in the logic of the research – there factors (72 factors) will be an overcomplicated macroeconomic factor model.

To compare macroeconomic model with traditional factor models, used in industry, I use Fama-French factors for three and five factors model.

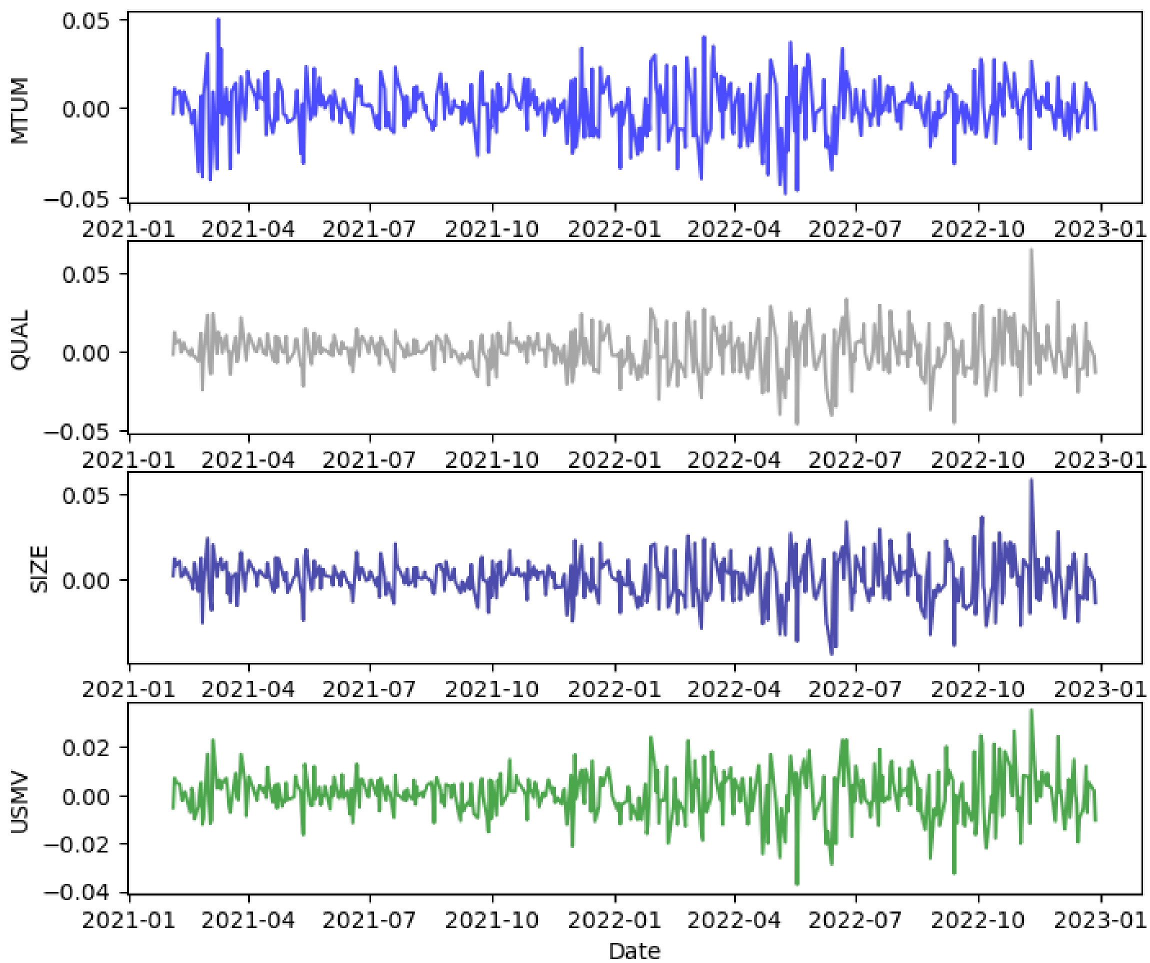

Figure 12. QUAL, SIZE, USMV and VLUE factors (2021–2022).

Source: author's calculations on the data from Yahoo Finance

Five-factor Fama and French model factors: small minus big (SMB), which represents the return spread between small- and large-cap stocks; high minus low (HML), which measures the return spread between high book-to-market and low book-to-market stocks; robust minus weak (RMW), which compares the returns of firms with high, or robust, operating profitability, and those with weak, or low, operating profitability; and conservative minus aggressive (CMA), which gauges the difference between companies that invest aggressively and those that do so more conservatively.

Additional factors, which are used for advanced factor model, ETF factors: MTUM (IShares MSCI Momentum Factor ETF); QUAL (iShares MSCI USA Quality Factor ETF); SIZE (iShares MSCI USA Size Factor ETF), USMV (iShares MSCI USA Min Vol Factor ETF); VLUE (iShares MSCI USA Value Factor ETF), as a variant of the weighted average index on the US market feature of interest (which occupies a large part of the portfolio).

Chapter 5

ESTIMATION RESULTS

This chapter provides the optimal factor selection process, portfolio optimization model results for each of the selected models, and comparison risk and performance results of each optimal portfolio with the optimal portfolio from the macroeconomic model (model with macro variables). Section 5.1 provides an analysis of the factor selection between the models and factor analysis. In Section 5.2, I construct the CVaR optimization model and show results for the macroeconomic factor model, advanced factor model, and three- and five-factor Fama-French models. After I compare the efficiency of each model for the final best model selection.

5.1. Results of factor selection

As part of factor analysis and selection of the required number of factors, we encounter a number of difficulties: a large number of factors and a significant number of financial assets that correlate differently with different factors.

To form a macroeconomic model, we first check the required mathematical number of factors, after increasing the number of which the explanatory power of the model will not increase significantly. This will allow us to avoid the problem of multicollinearity, since many factors are being tested.

To determine the suitability of these factors for factor analysis, we conduct Kaiser-Meyer-Olkin (KMO). It determines the adequacy of each observed variable and the complete model. KMO estimates the proportion of variance among all the observed variables. Lower proportion id more suitable for factor analysis. KMO values range between 0 and 1. The value of KMO less than 0.6 is considered inadequate. In our case, the potential factors for the macroeconomic model showed a test value of 0.74. That is, these factors can be used in factor analysis.

The next stage of factor analysis according to the methodology described in Section 3.11 is screeplot. The scree plot is used to determine the number of factors to retain in an exploratory factor analysis (FA). Based on the scree plot, using eigenvalues of factors greater than 1, we determine that the optimal number of factors for the macroeconomic model is no more than 10.

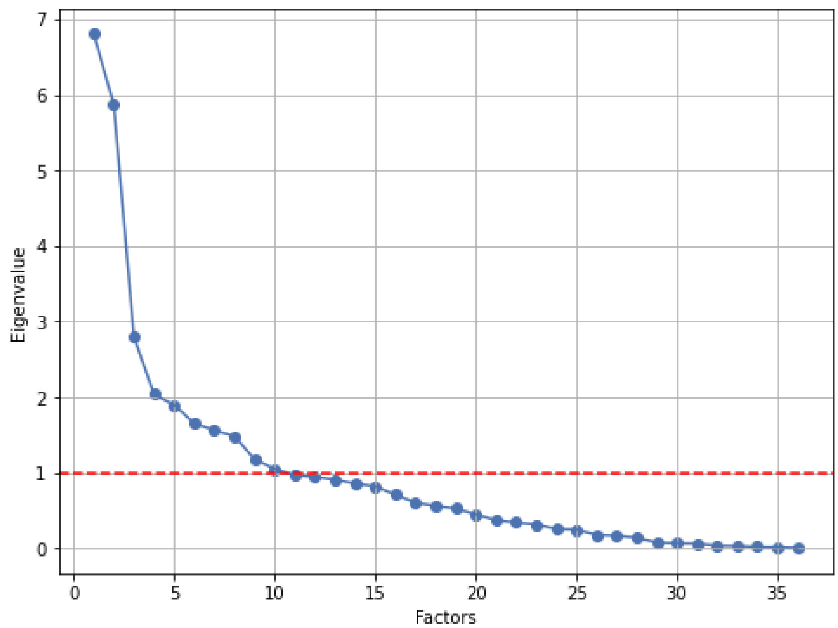

Figure 13. Scree plot.

Source: author's calculations.

The next part of the analysis is to determine which of the factors are best included in the macroeconomic model. Considering that different factors correlate differently with other stocks, we determine the percentage of the sample subject to non-zero loadings. This shows how much of the sample is correlated with each of the factors. Having sorted them in ascending order, we see in Figure 14 (first above) that a significant part of macroeconomic factors does not affect the portfolio level, but, for example, at the sector level (correlation matrices detailed at the portfolio level show a blurry picture, however, if we sort the array for the sector, shows a strong correlation with certain factors). Therefore, for example, factors common to the US market, such as VLUE and SIZE, affect a large part of the sample. Other factors that can be included in the macroeconomic model are non-energy return, oil return, GSCI gold return, GB BBB credit spread, EU interest rate, medium-term momentum, EUR BBB credit spread. An interesting observation is that the US macro variables were found to be less significant within the analysis.

In addition, we tested the basic factor model (US market indexes factors such as MTUM, QUAL, SIZE, USMV, VLUE model), the advanced factor model (five-factor Fama-Frech factors with US market indexes factors from the basic factor model), and five-factor Fama-French model. The results shown in Figure 16 reflect the significance of each of the selected factors for the model (since they correlate with a significant portion of the sample of financial instruments).

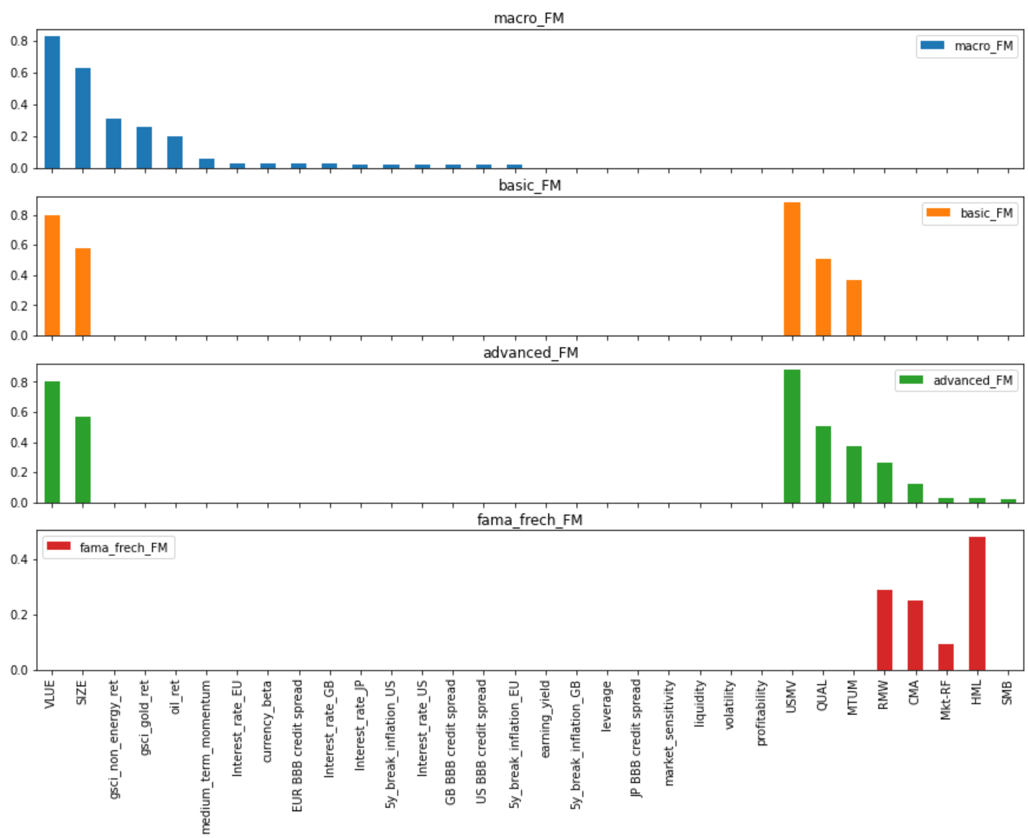

Figure 14. Factor selection for macro, basic, advanced and five-factor Fama-French model.

Source: author's calculations.

5.2. Conditional Value-at-Risk portfolio optimization results

The main optimization task is to select the optimal number of assets from the previously selected 200 stocks. By removing stocks with gaps in stock price data or no financial statements, 68 stocks meet the criteria.

The first step for portfolio analysis is the analysis of expected return (historical mean and factor model) and risk metrics (Mean-Variance, Value-at-risk, Conditional Value-at-risk) for the study period of 2021-2022 (due to the limited available data of several factors only for this range). Since this period was characterized by high volatility, more than in previous historical periods, for example, 2015-2020 (before the start of high volatility in financial markets caused by COVID-19 and the recession). Analysis of historical data for the period, as well as expected asset returns calculated based on factor models (Fig.14), explain the bias in the analysis of the risk-return ratio of investing in these assets (C-Sharpe ratio).

To find the optimal portfolio, it is necessary to determine the risk measure (risk metric) and the factors included in the asset valuation model (factor model). Based on the results of Fig.14, the factor model consists of the following parameters: value, size, GSCI non-energy return, oil return, GSCI gold return, EU interest rate, UK interest rate (GB interest rate), medium-term momentum, EUR BBB credit spread and currency beta.

To select a risk model, it is necessary to check whether the stock returns data are normally distributed. If some of the stocks in the portfolio have a non-normal distribution, then the use of traditional metrics such as Standard Deviation and Value-at-risk carries the risk of underestimating tail risk. In such a case, the best alternative is Conditional Value-at-risk. The Jargue-Bera test evaluates the normal distribution of data with the normal hypothesis about normality distribution (H0: sample 1 and sample 2 is not significantly different from normal distribution; H1: sample 1 and sample 2 is significantly different from a normal distribution). Therefore, values less selected significance level (a=0.05) allows us to reject the null hypothesis. Fig 15 shows that some of the stocks don’t follow the normal distribution, which means that Conditional Value-at-risk (CVaR) should be applied to solve the portfolio optimization problem.

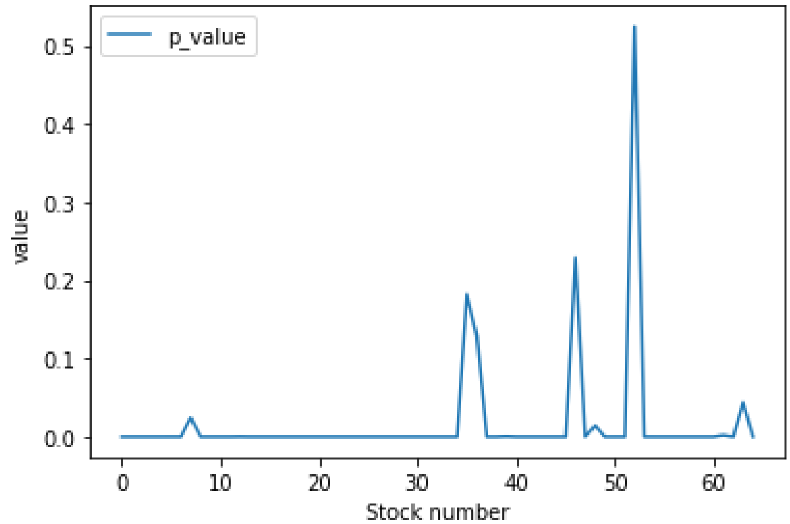

Fig.15. Jargue-Beta test p-value results

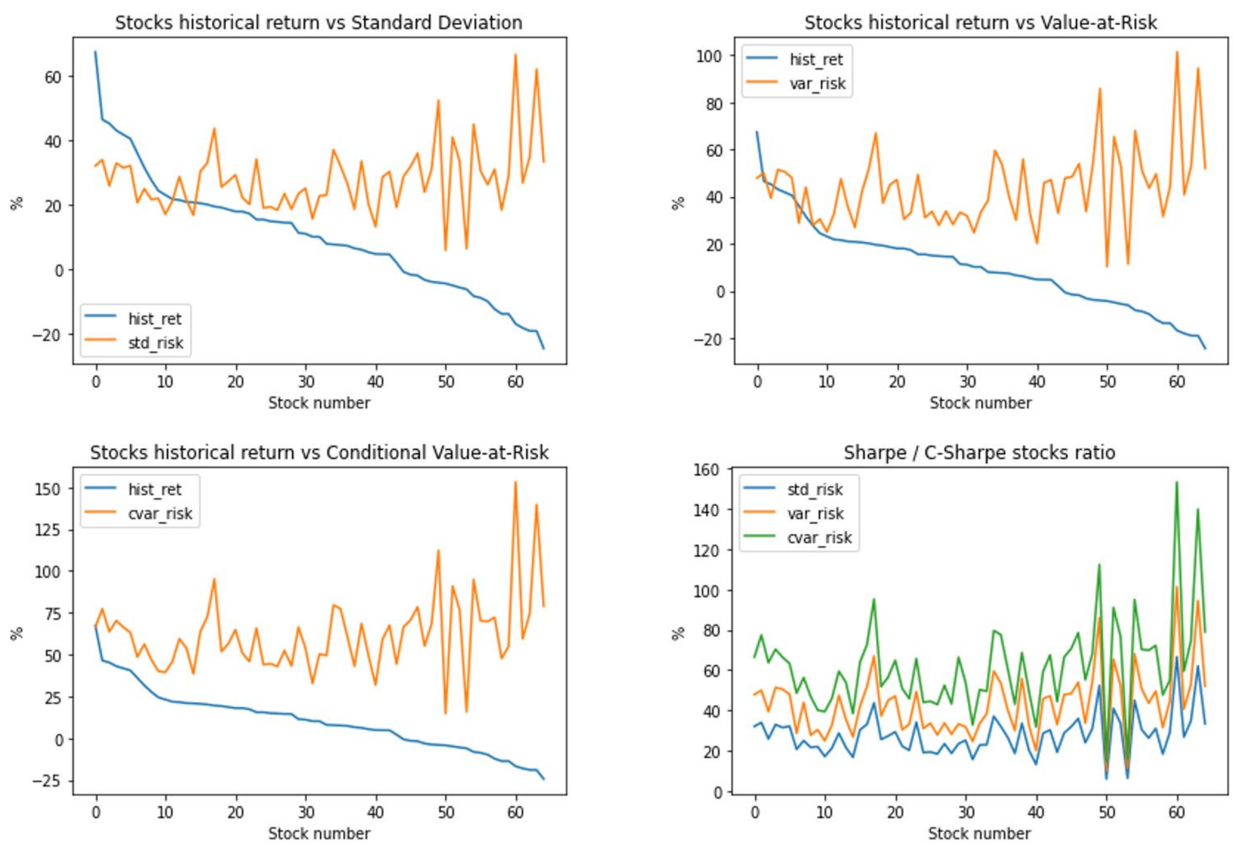

Figure 16. Comparison of risk metrics from selected assets universe

The need to use CVaR is confirmed in Fig 16 by comparing mean stock historical return with the risk metrics Standard Deviation, Value-at-risk, and Conditional Value-at-risk. Annualized risk and return values show the effectiveness of an investment in a particular financial instrument. The upper left subplot shows that the first 10 stocks sorted by return can be chosen by the optimization model because the return exceeds the risk (Standard Deviation), but the comparison with other more accurate metrics (top right subplot and bottom left subplot). The volatility values are biased because the entire study period falls into a period of high volatility in financial markets (COVID-19, recession). An underestimation of tail risk can lead to unforeseen losses if, when choosing an optimal portfolio, the strategy focuses on maximizing the ratio of profitability and risk (Sharpe ratio / C-Sharpe ratio).

Solving the optimization problem for the selected target strategies (minimum risk, maximum return, maximization Sharpe / C-Sharpe ratio) will help evaluate the purpose of the work - whether the assessment of macro variables (macro model) affects the quality of portfolio formation compared to other factor models (model with trading factors, 5 Factor Fama-French model, and advanced model (consisting of factors of two other models) with optimization based on CVaR, non factor model (Markovitz approach), and also compared with the same models based on Standard Deviation.

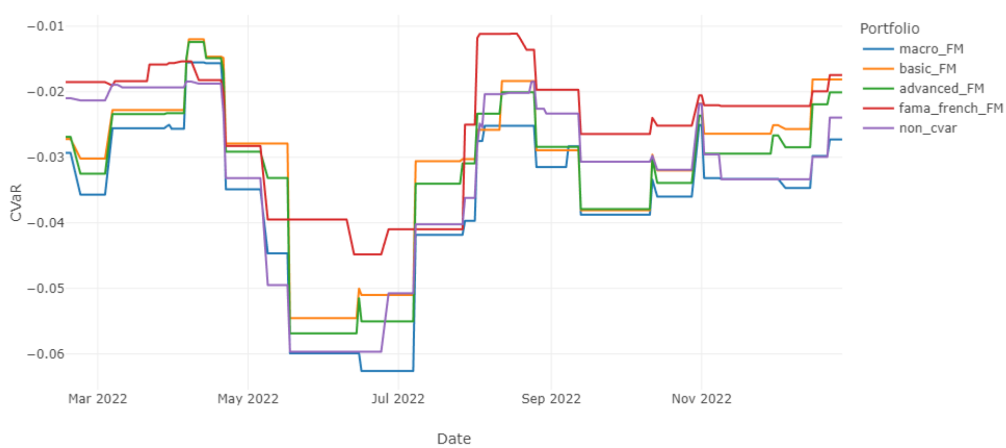

Figure 17. Comparative structure of models' CVaR

Thus, the main finding in this research can be that macroeconomic factor models can be used for portfolios designed to minimize risk (for example, macro risk) during volatile periods, since this model historically more accurately measures the measure of risk, allowing you to more accurately determine the necessary reserve funds for the fund, which can be proven due to the scenarios analysis in cases in Table 1.

Table 1. Comparisons between minimum risk, maximum return, and maximum Sharpe strategies

Model	Risk measure	Return – Min Risk	Return – Max Ret	Return – Max Sharpe	Return – Min Risk	Return – Max Ret	Return – Max Sharpe	Return – Min Risk	Return – Max Ret	Return – Max Sharpe
Macro	CVaR	50.2%	51.6%	51.2%	23.9%	99.6%	67.3%	50.4%	41.3%	48.0%
Basic	CVaR	50.2%	51.6%	50.7%	23.9%	99.6%	76.%	50.4%	41.3%	48.8%
Advanced	CVaR	54.7%	53.8%	49.1%	96.7%	99.6%	66.1%	51.8%	40.3%	52.0%
Fama-French	CVaR	56.3%	48.4%	49.1%	87.4%	0.0%	30.1%	54.9%	50.8%	48.5%
Non Factor	CVaR	48.5%	46.0%	53.12%	17.4%	0.5%	44.4%	53..9%	59.4%	48.0%
Macro	SD	48.7%	46.0%	52.9%	17.4%	0.5%	44.4%	53.7%	59.4%	48.5%
Basic	SD	50.2%	47.5%	49.1%	71.1%	3.8%	29.7%	51.1%	56.3%	50.4%
Advanced	SD	56.3%	48.4%	49.1%	87.4%	0.0%	30.1%	54.9%	50.8%	48.5%
Fama-French	SD	50.2%	51.6%	51.2%	23.9%	99.6%	67.3%	50.4%	41.3%	48.0%
Non Factor	SD	50.2%	51.6%	50.7%	23.9%	99.6%	76.%	50.4%	41.3%	48.8%

A comparison of the main assets that make up the main models (with CVaR) is shown in Fig.17.

The ‘Return’ column in Table 1 shows the percentage of historical scenarios in the optimal portfolio chosen by the macro model that showed a higher return than other models. The 'Risk' column in Table 1 shows how many percentages of historical scenarios the optimal portfolio chosen by the CVaR macro model showed less portfolio risk than other strategies. Regarding the risk measure, it should be considered that due to non-normality data, this parameter more accurately considers the risk and shows that other factor models (basic, advanced) show a greater risk. Since these models do not assess the impact of macroeconomic variables, this confirms the assumption that adding macro variables allows for minimizing risk (taking into account these variables allows you to bypass periods of shocks in the financial market caused by changes in macroeconomic parameters). Thus, if we compare the backtesting macro model strategy with the CVaR risk measure, in more than 50% of cases, this strategy is suitable for finding the optimal portfolio, considering the maximum return and risk ratio. Since the model includes macro variables, this portfolio will perform better than other models, especially during periods of high volatility.

Risk analysis (CVaR) part shows that macroeconomic model strategies based on the Minimum Risk strategy (Min Risk) and Maximum Sharpe / C-Sharpe strategy, which create much less risk compare to the benchmark (^GSPC), therefore using macroeconomic variables helps to closely predict the potential risk (optimal portfolio will have less risk than benchmark): in case of Min Risk strategy risk was less in 99.66% of the cases or the Max Sharpe strategy, where in 67% cases macro model measure less risk due to the impact from macroeconomic variables.

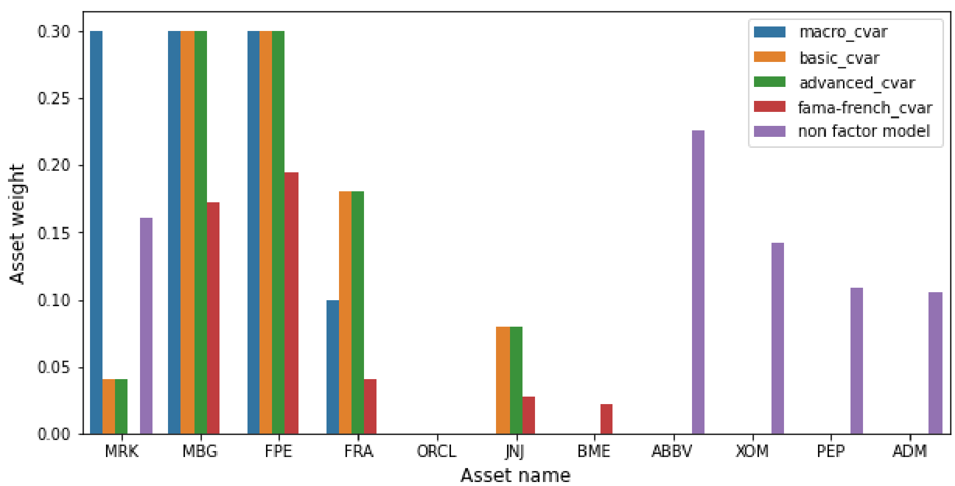

Figure 18. Comparison of the main models' optimal portfolio structure

The optimal macro model portfolio is selected from the assets of ABBE(30%), MRK(30%), XDM (29%), XOM (11%) and others. The portfolio structure is shown in Fig.19, also the optimal portfolio is shown on the efficiency frontier in Fig.20.

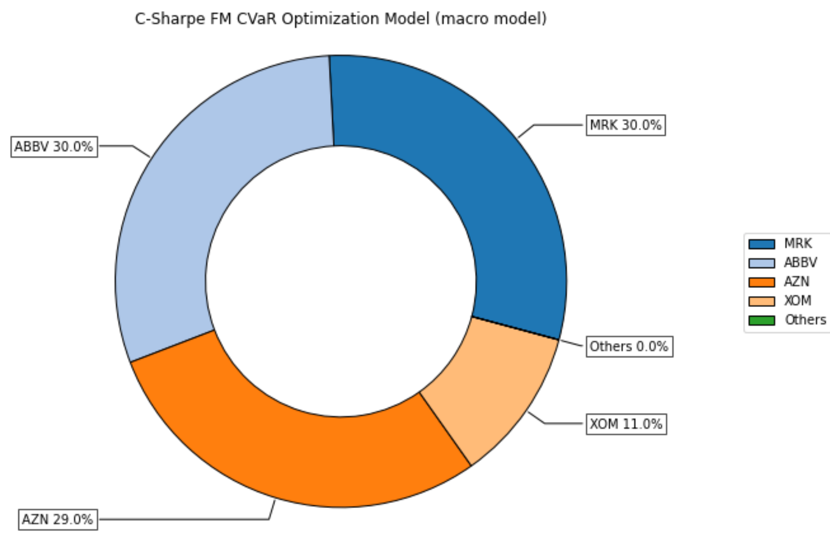

Figure 19. Macro model optimal portfolio structure

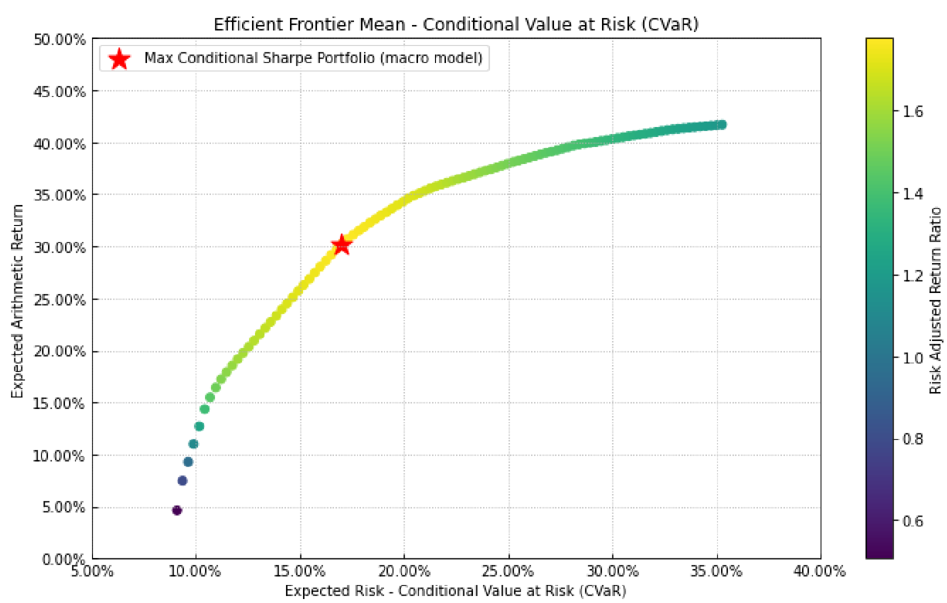

Figure 20. Macro model efficient frontier

Chapter 6

CONCLUSIONS AND POLICY RECOMENDATIONS

In this thesis, the assumptions were confirmed that using macro variables when using the Conditional Value at Risk optimization method works better for Minimum Risk strategy scenarios (Table 1). Using macro variables improves the quality of models and investment efficiency for risk-averse clients: asset management companies, pension funds, and insurance companies. In all the studied scenarios, the Minimum Risk strategy showed greater profitability, lower risk, and higher investment efficiency, confirming the need to use macro variables for strategies focused on minimizing the client's risk.

The issues in the advantages of this model for calculating the expected price are related to the blurring of the influence of macro variables; that is, fundamental variables that are widely used in trading are more suitable for factor models because they affect all financial instruments when the influence of macro variables is concentrated on individual sectors (which can be seen if the correlation matrices are detailed by sectors, where the correlation significantly increases). Thus, the study showed that factor models are susceptible to the selection of factors to select from a large number of factors that will influence most of the array of selected financial instruments. Using more conservative risk measurement metrics when building a portfolio helps optimize the required reserves of asset management funds. Moreover, the use of risk metrics that avoid tail risk is overestimated due to non-optimal values of the Skewness and Kurtosis parameters. Considering the long-term strategy of investing in the fund and revising the structure of the fund every six months or a year, the most optimal is the portfolio with account Conditional Value-at-Risk metric.

Further research on this topic with the selection of more factors will improve the quality of the models. This model can be used as a basis for selecting instruments and index funds, as there are possible better results in sectoral detail (taking into account macro factors, for example). Examples of factor models with macro variables have already been implemented in several analytical tools for asset management. Expanding the arrays of available data and the number of factors will improve the model.

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