BLACK MARKET EXCHANGE RATE AND EXCHANGE RATE PATH STABILITY: A DYNAMIC-STOCHASTIC MODEL ANALYSIS FOR UKRAINE

by

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Abstract

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In this work, a dynamic-stochastic currency substitution model will be developed to study the emergence of a black market exchange rate and the stability of the exchange rate path in the presence of exchange and interest rate regulations and external shocks. Then such policy will be evaluated for its effectiveness in the case of the Ukrainian economy and improved by creating an exchange rate rule, similar to the interest rate one, which acts simultaneously as the determinant of Central Bank policy.

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Only a crisis - actual or perceived - produces real change. When that crisis occurs, the actions that are taken depend on the ideas that are lying around. That, I believe, is our basic function: to develop alternatives to existing policies, to keep them alive and available until the politically impossible becomes the politically inevitable.

Capitalism and Freedom
Milton Friedman

Chapter 1.

INTRODUCTION

As the Russo-Ukrainian conflict persists in an intense phase, the National Bank of Ukraine (NBU) has implemented a pegged exchange rate regime to impede an imminent "balance of payments"[1] crisis. This crisis arises due to a sudden drop in foreign investment and a surge in outbound capital flow, as exchange rate expectations deteriorate and the confidence in the nation's financial stability decreases.

In conjunction with stringent capital controls, this has resulted in the flourishing of a black market for currency with free market exchange rates. Such a market operates outside regulatory oversight in unlicensed facilities, such as "exchange kiosks," and has become more accessible and appealing to currency holders. In the face of these challenging economic conditions, it is understandable that the NBU has sought to employ a pegged exchange rate regime to address both inflation and the issue of a widening black market premium.

To slow down the rise in the black market premium over the official rate, the NBU has implemented an "adjustment-pegged" regime, where the official exchange rate adjusts to a black market rate once the premium reaches 20%. Thus, the NBU has two primary objectives: maintaining low inflation (its main goal) and keeping the exchange rate low while simultaneously controlling the black market exchange rate premium.

The efficacy and stability of this policy must be evaluated to ensure that the stated goals are achieved.

This work will outline a model that explains the existence of the black market

1. "Balance of payments crisis" or a "currency crisis", occurs when a nation is unable to pay for essential imports or service its debt, as nation's currency devaluates

exchange rate under rational expectations. Additionally, it will examine the criteria for stability in the black market exchange rate path under a monetary policy designed to limit the devaluation of the local currency, impede a balance of payments crisis, and simultaneously control the black market premium to prevent a disastrous feedback loop.

Subsequently, a case for Ukraine's dual monetary policy will be presented, the optimal policy derived, and the black market exchange rate forecasted.

Chapter 2.

LITERATURE REVIEW

2.1. Fixed exchange

Countries that have sought to safeguard their foreign reserves amid payment difficulties have regularly used exchange restrictions. Governments have historically considered exchange controls as a replacement for less desired adjustment measures, despite the fact that controls do little to address the fundamental causes of external imbalance. In reality, regulations may worsen existing distortions in relative pricing and resource allocation, making it more difficult to deal with the issues they were intended to address.

Under a pegged exchange rate regime, it is controlled by a monetary authority and usually fixed against the value of a reference, like another currency, a basket of currencies, or even gold.

There are two main mechanisms for managing the fixed exchange rate:

Flat: The government simply makes it illegal to trade currency at any other rate, so the government enforces a monopoly over currency conversion. The main drawback of this kind of policy is that it is extremely hard to enforce for prolonged periods. This method is employed by the Chinese government to maintain a currency peg of the yuan against the U.S. dollar.

Currency Interventions: The government, pursuing a fixed exchange rate, buys and sells its currency on an open market, like the interbank market, effectively controlling supply and demand for currency, which allows the trade of currency for agents without much control but puts a strain on the nation's international reserves. This method is currently used in Ukraine.

2.2. Currency black markets

The IMF (IMF 1985) identifies black markets as a market comprised of currency vendors offering floating rates that do not comply with the monetary authority's requirements.

Exchange controls, like other sorts of limitations, by their very nature, promote evasion. When this happens in a deliberate and organized way, currency exchange black markets form. In the majority of instances, governments frown upon the presence of such markets because they divert limited foreign exchange from official channels to uses that the authorities are aiming to prevent. Even more crucially, black markets have the capacity to affect the economy as a whole and have a number of critical policy consequences.

Insofar as black market activities circumvent foreign exchange controls, these activities serve a useful purpose in that they mitigate the negative effects of quantitative restrictions on resource allocation. This is only true to the extent that these activities circumvent these controls. They increase prices for exporters who do not sell foreign currency on the official market and allow importers to purchase goods they could not otherwise obtain at prices they are willing to pay. Consequently, exporters who do not sell foreign currency on the official market obtain higher prices. In this way, the black market serves as a mechanism that decreases the value of the domestic currency relative to other currencies, even though the official exchange rate remains unchanged.

This serves to benefit those exporters and importers who use the black market; however, it can create an overall detriment to the economy.

2.3. Current exchange regime in Ukraine

If combined with a fixed interest rate policy, capital controls are needed to prevent capital outflows from agents' pursuit of uncovered interest arbitrage profits.

Table 1: History of capital controls under fixed exchange rate regime in Ukraine during full-scale invasion

February.22	March.22	April.22
All currency interbank operations are banned	Interbank exchange is unfrozen	
Withdrawal from deposits is banned	Currency withdrawal limit is set to the equivalent of 30 th. UAH	
All abroad currency payments are banned	Abroad payments limit is set to the equivalent of 100 th. UAH	Abroad payment delay is lowered from 365 to 90 days

This created a black market for free-floating currency, which is used to bypass capital controls. To compete with the black market and prevent cash speculation, the NBU lifted some regulations and adopted a crawling peg regime, where the exchange rate is allowed to fluctuate in a narrow band, which adjusts to the black market exchange rate when the black market premium reaches 20%. Thus, UAH is pegged both to USD and itself on the black market. This kind of recursion may lead to a dangerous kind of positive feedback loop, where a controlled exchange rate combined with a controlled interest rate and unfastening capital controls may result in an uncontrolled devaluation of the currency, followed by a balance of payment crisis, which this policy was thought to avert.

IMF identifies in their Annual Report on Exchange Arrangements and Exchange Restrictions (IMF 2022), that a nation that is having difficulty sustaining a balance of payments has three options: enacting adjustment measures through the use of fiscal, credit, income, and exchange rate policy tools; borrowing money from other nations, or placing limitations on

international trade and payments. The first alternative may have undesirable short-term effects on income distribution, employment, or inflation; the second option, borrowing, is often restricted in scope, costly, and only capable of providing temporary relief. All of these are problems that governments wish to avoid. Quantitative restrictions, on the other hand, may appear to offer a solution with quick and direct consequences, but they are less problematic in terms of the social and economic costs they impose. As a result, it should not come as a surprise that restrictions have been widely employed as a response to external imbalances, particularly in emerging economies.

When quantitative restrictions are enforced in the form of constraints on the supply of foreign exchange through official channels, the emergence of foreign exchange black markets is inevitable. If a central bank is unable or unwilling to fulfil all of the demand for foreign exchange at its official exchange rate, those whose demand is not met will be willing to pay a higher price. As long as the risks and costs of violating exchange control restrictions are not exorbitant, this is the case. If such offers are made, those who earn or borrow foreign money will have the incentive to sell it on the black market rather than to the central bank. In these situations, the existence of foreign exchange black markets is a consequence of a mismatch between supply and demand for foreign exchange at the official rate

Thus, this review will be divided into two parts: the first part, where the development of models for the balance of payments crisis will be covered, and the second one where model of a dual exchange rate regime will be described.

2.4. Balance of payments crisis model

The term "crisis" in the balance of payments refers to the situation that arises when the government is no longer able to defend a set parity because of the limits that are placed on its operations. But all of these policy instruments are subject to limits. A government attempting to keep its currency from

depreciating may find its foreign reserves exhausted and its borrowing approaching a limit. A government attempting to prevent its currency from losing its value may find the cost of domestic inflation unacceptable. When the government is no longer able to defend a fixed parity because of the constraints on its actions, there is a "crisis" in the balance of payments.

A model for describing such crises was developed by Krugman (Krugman 1979), and it describes in detail why efforts to defend fixed exchange rates so often lead to crises. These possible instances of currency crises are self-fulfilling: if everyone believes that the peg will fail, then everyone will perform a speculative attack, causing the peg to fail, thereby validating the initial belief. The model, however, has its limitations, as it uses a simplified macroeconomic foundation that disallows the government to trade in anything but its reserves, meaning that the list of factors triggering a crisis is incomplete as the government plays a passive role and has little realistic possibility for optimizing behaviour. However, it clearly describes a mechanism by which utility-maximizing agents perform a series of speculative attacks on the nation's reserves, as there exist equilibria, where it is rational for government to depreciate the nation's currency.

As the European Exchange Rate Mechanism (ERM) crisis unfolded in Europe in the early 1990s, a number of analysts asserted that the Krugman model was inadequate for explaining such complex financial crises. Rather than a unique equilibrium, they determined that these crises were best explained by self-fulfilling speculative attacks with added investor herding behaviour. It was proposed, that the ERM crisis is the result of a "game" between investors on one side and governments on the other, with each side having clearly defined goals and objectives. This viewpoint led to the conclusion that discrepancies between the government's fiscal and monetary policies were not the only fundamental cause of crises.

This class of models is in the context of the logic of currency crises, as described by Obstfeld (Obstfeld 1994), and then a type of "exchange bias," as in "inflatory bias" in the Barro-Gordon model (Barro and Gordon 1983), is

introduced into the model as a direct result of the government's discretionary exchange policy. This results in a new understanding of the balance of payments crisis: both a devaluation and a fixed exchange rate are rationalizable as equilibria (Sachs, Tornell, and Velasco 1996), so multiple equilibria exist and due to added herding behaviour the crisis becomes self-fulfilling.

In further studies (Morris and Shin 1998), it was discovered that the public's understanding of the fundamentals is the source of the multiple equilibria that are included in the models. If one were to introduce a little amount of uncertainty regarding the actual fundamentals of the economy in such a way that each investor would only perceive a noisy signal of the real fundamentals, then the equilibrium would become unique and no crisis will occur. Since the earnings from holding long or short positions on the currency are contingent on the actions of other investors, investors need to intuitively set expectations regarding the expectations of other investors when there is some uncertainty about the underlying fundamentals. Even while an individual's own signal could suggest that the fundamentals are in very poor shape, it is impossible to know for certain whether or not other investors have the same knowledge and would, as a result, sell the currency. Consequently, investors build expectations regarding the expectations of other investors, which typically leads to a unique equilibrium based on a threshold value for the noisy signal that is received by investors.

In this context, the introduction of the black market into the improved, a la Sachs-Tornell-Velasco, balance of payments crisis model would serve multiple purposes: as an additional factor triggering a speculative attack, as the rapid growth of the black market premium over the official rate clearly signals that the official rate is far below equilibrium and therefore the government acting on discretion will surely devalue the currency, and as an additional mechanism of relief of reserve pressure as part of the cash flow diverges from official markets where rates are set using market interventions by spending reserve.

As the exchange rate in these models is purely exogenous and there is an endogenous relation between official and black market exchanges, as a

self-standing addition to the balance of payments crisis models, endoginating both exchange rates also has to be introduced.

2.5. Dual exchange rate regime model

First, it is crucial to understand, why agents tend to prefer to switch to another currency and perform a "currency substitution".

Currency substitution is the practice of agents having a strong incentive to diversify the currency composition of their cash balances to facilitate their operations in different countries. Even individuals and businesses domiciled in a particular nation frequently have transactional, precautionary, or even speculative reasons for diversifying their currency holdings. These motivations may involve hedging against potential risks, volatility, or even inflation (Miles 1978). Individuals and businesses can reduce the costs associated with international transactions by utilizing diversification, which also provides certain risk-reduction benefits typically associated with asset diversification. Nevertheless, the mere possession of a diversified portfolio of currencies is insufficient for significant currency substitution to occur. This is because it is possible for every nation to have amassed a certain amount of foreign currency reserves. These holdings must shift in response to changes in the relative opportunity costs of holding foreign currencies for a country to engage in currency substitution (a currency that seems likely to depreciate rapidly is substituted for another currency that looks set to appreciate). In addition, currency substitution requires the presence of a group of individuals and organizations that hold both domestic and foreign currencies and are indifferent between holding more domestic or more foreign currencies. These individuals and institutions must hold both domestic and foreign currencies.

When the black market is incorporated into this framework (Agénor and Khan 1992), it establishes an intrinsic connection between black market premium, currency portfolio composition and economic fundamentals, like interest rates, domestic and foreign, money supply and bond volumes.

This model, however, unlike the balance of payments crises models presented earlier, is governmentless and focuses purely on agents' optimizing behaviour.

2.6. Summary

In conclusion. There is no black market in the balance of payments crisis models present in the literature. There is no government part in currency substitution models with the black market present in the literature. As a result, there is research potential in the creation of a new model for the balance of payments crisis with the black market by merging the two models above.

Chapter 3.

DATA

For the purpose of calibrating this model, following data was collected or produced:

- Official/Black Market ER — NBU
- Official Interest Rate — NBU
- Monetary aggregates — NBU
- Exchange rates expectations (12 months ahead, monthly frequency, for every data point) — NBU & derivation
- Interest rate expectations (12 months ahead, monthly frequency, for every data point) — NBU
- Sentiment Index (from Search Volume Index) — Google & derivation

Chapter 4.

METHODOLOGY

As it was earlier identified in the Literature Review chapter, there is no single model, that exists to describe a balance of payments crisis with the black market present. However, separate aspects of these problems are described in separate models. Government behavior and unique crisis behavior of agents are described in the balance of payments model, while the existence of the black market and currency substitution through it is described in Currency Substitution Model.

Hence a merging attempt will be made.

The first step in merging two economic models is to identify whether the models are compatible, meaning that they should share some common features, assumptions, theoretical frameworks, and data inputs. Understanding these features will help identify the areas of overlap between the two models and where the models differ. This information will be used to determine how the two models can be merged.

The second step is to determine the integration method. There are several methods for merging two economic models, such as model calibration, model combination, or model extension. Model calibration involves adjusting one model to fit the data from the other model. The model combination involves merging the two models into a single model that incorporates the features of both models. The model extension involves adding features from one model to the other.

The final step is to test the merged model. This involves comparing the predictions of the merged model to real-world data to determine how accurate it is. If the merged model is accurate, it can be used to make predictions and analyze economic behavior.

4.1. Key features and assumptions

The Balance-of-Payments Crises model seeks to explain why some countries experience balance-of-payments crises, which occur when a country cannot meet its international financial obligations and may be forced to default on its debt. The model assumes that a country's central bank is responsible for managing the exchange rate and maintaining a fixed exchange rate regime. The key features and assumptions of the model include:

Fixed exchange rate regime: The model assumes that the central bank is committed to maintaining a fixed exchange rate between the domestic currency and a foreign currency. The fixed exchange rate is assumed to be a credible commitment, which means that the central bank will take action to defend the exchange rate if it comes under pressure.

Sterilization: The model assumes that the central bank can use monetary policy to sterilize the impact of its exchange rate interventions on the domestic money supply. Sterilization involves selling or buying domestic currency in the foreign exchange market in order to maintain the fixed exchange rate, while also using open market operations to offset the impact of these interventions on the domestic money supply.

Capital mobility: The model assumes that capital can move freely across borders, which means that investors can easily buy or sell domestic assets in response to changes in the exchange rate.

Speculative attacks: The model assumes that investors may engage in speculative attacks on the fixed exchange rate, which involves selling domestic currency in the foreign exchange market in order to pressure the central bank to abandon the fixed exchange rate.

Balance-of-payments crisis: The model predicts that a balance-of-payments crisis will occur when the central bank's foreign exchange reserves are depleted due to interventions to defend the fixed exchange rate, and the

central bank can no longer maintain the fixed exchange rate. At this point, investors will engage in speculative attacks and sell off the domestic currency, causing a devaluation of the domestic currency and a balance-of-payments crisis.

The currency Substitution model is a theoretical framework that explains the relationship between foreign currency deposits and the demand for money. There are only agents present in the model and no monetary authority that exhibits some form of oversight or regulation exists. Some key assumptions featured are:

Currency basket: Individuals have two options for holding their wealth: domestic currency deposits and foreign currency deposits. This assumption is a key feature of the model, as it assumes that individuals have access to both domestic and foreign currency deposits in the banking system. This assumption is realistic for many countries, as foreign currency deposits are often available in the banking system alongside domestic currency deposits.

Demand for money: It is assumed that demand for money is positively related to income and negatively related to interest rates and exchange rates.

Stability: Foreign currency deposits are relatively more stable than domestic currency deposits, and individuals may prefer to hold their wealth in foreign currency to avoid the risks associated with domestic currency. This assumption reflects the fact that foreign currency deposits are often seen as a relatively safe store of value, as they are not subject to the same risks as domestic currency deposits, such as inflation or exchange rate fluctuations. Preference for foreign currency deposits can lead to a reduction in the demand for domestic currency deposits.

Money Velocity: The presence of foreign currency deposits in the banking system can reduce the demand for domestic currency deposits, which

> can lead to an increase in the velocity of money and cause inflationary pressures in the economy. This assumption reflects the key hypothesis of the model, which is that the presence of foreign currency deposits can reduce the demand for domestic currency deposits and lead to inflationary pressures in the economy. The authors argue that this reduction in demand for domestic currency deposits can lead to an increase in the velocity of money, as individuals and firms may spend more quickly when they hold their wealth in foreign currency. This increase in the velocity of money can lead to inflationary pressures in the economy.

Now similarities and differences in assumptions have to be addressed to aid in the further model development.

There are several common features, that these models share.
Balance-of-payments crisis model assumes that capital can move freely across borders, which means that investors can easily buy or sell domestic assets in response to changes in the exchange rate. This assumption is similar to the assumption made in Currency Substitution model, which also assumes that individuals have access to both domestic and foreign currency deposits in the banking system. Balance-of-payments crisis model also predicts that a crisis will occur when the central bank's foreign exchange reserves are depleted due to interventions to defend the fixed exchange rate, and the central bank can no longer maintain the fixed exchange rate. This is similar to the prediction made in Currency Substitution model, which argues that the presence of foreign currency deposits in the banking system can reduce the demand for domestic currency deposits and lead to inflationary and reserve pressures, which may provoke a balance-of-payments crisis.

However, there are several key differences that need to be accounted for when merging these models.

Balance-of-payments crisis model assumes that the central bank can use monetary policy to sterilize the impact of its exchange rate interventions on

the domestic money supply, it also assumes that investors may engage in speculative attacks on the fixed exchange rate, which involves selling domestic currency in the foreign exchange market in order to pressure the central bank to abandon the fixed exchange rate. This is different from Currency Substitution model, which does not explicitly address theese issues.

In summary, while models mostly share common assumptions, they also differ in important ways, including mathematical formulation, whilst the BoP crisis model uses continious time framework, the CS model is discrete.

4.2. Integration method

It is possible to incorporate aspects of Balance-of-Payments crisis model into Currency Substitution model, so the goal is to extend a Currency Substitution model, but it would require significant modifications to both models.

To merge the models one would need to introduce a fixed exchange rate regime into Currency Substitution model and to explicitly model the behavior of agents in the foreign exchange market. This would allow the model to capture the possibility of a balance-of-payments crisis, as predicted by Balance-of-Payments model, when the central bank's foreign exchange reserves are depleted due to interventions to defend the fixed exchange rate.

Another modification would be to introduce a mechanism for sterilization into Currency Substitution model, which would allow the central bank to offset the impact of its foreign exchange interventions on the domestic money supply. This would bring the model closer to Balance-of-Payments model, which explicitly models the behavior of the central bank in response to external shocks.

It is also possible to incorporate a black market for foreign currency into a merged model. One way to do this would be to introduce a premium on the black market exchange rate, which reflects the scarcity of foreign currency in the economy. This would lead to a divergence between the official exchange

rate and the black market exchange rate.

There are several possible trade-offs that would need to be considered.

One potential trade-off is between exchange rate stability and monetary autonomy. Under a fixed exchange rate regime, the central bank must be prepared to use its foreign exchange reserves to defend the exchange rate, which can limit its ability to pursue an independent monetary policy. This can lead to inflationary pressures in the economy if the central bank is forced to monetize its foreign exchange interventions, which is exactly what happened in Ukraine during russian invasion.

Another trade-off is between short-term stabilization and long-term growth. While a fixed exchange rate regime may help to stabilize the exchange rate in the short run, it may also discourage investment and reduce long-term growth if it leads to a misallocation of resources or if it hinders the development of export-oriented industries. While macroeconomic stability is important for maintaining low inflation and promoting economic growth, it may also have distributional consequences that affect social welfare. For example, policies that prioritize price stability over employment may exacerbate inequality and social exclusion.

A third trade-off is between financial stability and capital mobility. While capital mobility can facilitate the flow of investment and boost economic growth, it can also increase the risk of speculative attacks and destabilize the financial system. This risk may be heightened in the presence of foreign currency deposits, which can make the domestic currency more vulnerable to sudden shifts in investor sentiment, which is one of the key behavioral mechanisms considered in the more recent generation of Balance-of-Payments crisis models.

Finally, there may be trade-offs between different policy objectives, such as inflation targeting, exchange rate stability, and financial sector development. These trade-offs may require a careful balancing of competing priorities and the use of multiple policy tools to achieve desired outcomes.

4.3. New and improved Currency Substitution Model

The total output of an economy is fixed at a level and consists of a continuum of exportable goods. The price of these goods is set on the world market. Agents consume a continuum of goods, where they could be only imported using official and illegal exchange markets, respectively. Agents also hold domestic and foreign currency and bonds.

The notion of "legal" and "illegal" goods is introduced to then introduce the black market into the economy. Whilst legal goods can only be imported by exchanging using the official exchange rate $\bar{e}$, illegal goods can only be purchased for a black market exchange rate e.

There are also some additional minor assumptions:

- No commercial banks
- Commercial transactions are settled partly on the legal market using the official exchange rate and partly on the black market using the parallel exchange
- Capital transactions are settled in the parallel market
- Total economy output s fixed at a level
- There are no domestic expenditures on production
- Total output is exported and total consumption is imported, though agents are required to use some domestic money to acquire them, so they are constrained by a “liquidity-in-advance”
- Domestic and foreign money are imperfectly substitutable

Agents maximize utility $U(\bar{c}_t, c_t)$ subject to a budget constraint, thus the following optimization problem is solved.

$$\max \sum_{t=0}^{\infty} \beta^t U(\bar{c}_t, c_t) \qquad \text{Discounted utility} \tag{1}$$

s.t.

$$A_t = M_{d,t} + B_{d,t} + e_t(m_{f,t} + b_{f,t}) \qquad \text{Total wealth in period } t \tag{2}$$

$$\begin{aligned} \Delta A_t = & \qquad \text{Wealth flow from } t-1 \text{ to } t \\ & (1-\sigma)\bar{e}_t y + \sigma e_t y + \qquad \text{Income from exports} \\ & (i_{d,t-1}B_{d,t-1} + \bar{e}_t i_{f,t-1} b_{f,t-1}) + \qquad \text{Income from bonds} \\ & (e_t - e_{t-1})(b_{f,t-1} + m_{f,t-1}) - \qquad \text{Valuation effect} \\ & (\bar{c}_t + \rho_t c_t) \qquad \text{Consumption} \end{aligned} \tag{3}$$

Where M_t are money deposits in domestic or foreign currency, B_t domestic or foreign bonds with interest rate i_t, A_t is total wealth. Here also the price of a "legal" and "illegal" good is set $\bar{p} = p = 1$, so the unit of price is therefore an official exchange rate $\bar{e}$.

It is then normalized to reflect the real wealth by dividing by $\bar{e}_t$ to get $a_t = A_t/\bar{e}_t$ $m_t = M_t/\bar{e}_t$ $b_t = B_t/\bar{e}_t$. A black market premium is then defined as $\rho_t = e_t/\bar{e}_t$ which after rearranging yields following budget constraints:

$$a_t = m_{d,t} + b_{d,t} + \rho_t(b_{f,t} + m_{f,t}) \tag{4}$$

$$\begin{aligned} \Delta a_t = a_t - a_{t-1} = & [1 + \sigma(\rho_t - 1)]y + \\ & [i_{d,t-1} - \epsilon_t(i_{d,t-1} + 1)]b_{d,t-1} + \\ & [i_{f,t-1} + \Delta\rho_t]b_{f,t-1} - \\ & \epsilon_t m_{d,t-1} - \\ & (\bar{c}_t + \rho_t c_t) \end{aligned} \tag{5}$$

$$(\bar{e}_t \bar{c}_t + e_t c_t) = (m_{d,t})^{\delta}(\rho_t m_{f,t})^{1-\delta}$$

Where $\epsilon_t = \Delta\bar{e_t}/\bar{e_t}$ and the last equation represents a "liqudity-in-advance" constraint

A dynamic optimization problem is then solved for an optimizing sequence of inputs $\{\bar{c_t}, c_t, m_{f,t}, m_{d,t}, b_{f,t}, b_{d,t}\}_{t=0}^{\infty}$ to obtain following equilibrium conditions - a currency basket composition condition and an interest rate parity:

$$\frac{M_{d,t}}{e_t M_{f,t}} = (\frac{\delta}{1-\delta})[\frac{i_{f,t}}{(1-\epsilon_{t+1})i_{d,t}}] \tag{6}$$

$$(1-\epsilon_{t+1})i_{d,t} = \frac{i_{f,t} + \Delta\rho_{t+1}(1-\epsilon_{t+1})}{\rho_t} + \epsilon_{t+1} \tag{7}$$

A "buffer-stock" approach is used and a secondary optimization is performed to introduce forward-backward-looking components into the model. According to this approach agent after choosing the composition of his currency basket then chooses a short-run currency ratio, pursuing to not incur the costs of being out of long-run equilibrium. These costs may result, i.e., from the presence of regulation, prohibiting acquiring foreign currency, or from capital movement restrictions.

To do that "a transaction cost" loss function is introduced

$$L = E_{t-1} \sum_{k=0}^{\infty} \beta^k \{\alpha_0 (q_{t+k}^d - q_{t+k})^2 + \alpha_1 (q_{t+k} - q_{t+k-1})^2\} \tag{8}$$

So for each period t agent optimizes a path $\{k, k+1, k+2, ...\}$, which converges to $q_{t+k}^d = ln(\frac{M_{d,t}}{e_t M_{f,t}})$ currency basket based on information available in $t-1$ period, so as to minimize the joint the cost of being out of equilibrium currency basket $\alpha_0 (q_{t+k}^d - q_{t+k})^2$ and cost of convergence, which is transaction itself $\alpha_1 (q_{t+k} - q_{t+k-1})^2$

The solution of this classical dynamic investment under uncertainty with quadratic loss function problem is given by Dynamic Macroeconomic Theory (Sargent 1987):

$$E_{t-1}q_t = \lambda_1 q_{t-1} + (1-\lambda_1)(1-\beta\lambda_1)\sum_{j=0}^{\infty}(\beta\lambda_1)^j E_{t-1}q_{t+j}^d \tag{9}$$

Where β is a discount factor and λ_1 is the first stable root of the stochastic Euler equation.

Assuming, that the actual currency ratio differs from the expected one only by a zero-mean, serially uncorrelated disturbance yields:

$$q_t = \alpha + \lambda_1 q_{t-1} + (1-\lambda_1)(1-\gamma\lambda_1)\kappa_1\sum_{j=0}^{\infty}(\gamma\lambda_1)^j E_{t-1}z_{t+j} + \mu_t \tag{10}$$

$$z_{t+j} = ln(\frac{\delta}{1-\delta}[\frac{i_{f,t+j}}{(1-\epsilon_{t+j+1})i_{d,t+j}}]) \tag{11}$$

$$q_t = ln(\frac{M_{d,t}}{e_t M_{f,t}}) \tag{12}$$

It can be seen, that the actual currency basket depends on a backward-looking component q_{t-1} and a forward-looking variable, a geometrically declining series of opportunity cost variables, from imposed rational expectations.

From which $ln(e_t)$ can be expressed and a direct regression equation obtained for further calibration.

To summarize this subsection: a DSGE-type model was developed based on a currency substitution framework to allow for endogenous black market exchange rate in presence of regulations, hence the possibility to mathematically peg the official exchange rate by setting $E_{t+1}\bar{e_{t+1}} = E_t\bar{e_t}$ and capital control in the form of transactions costs.

However, the most important aspect — a functional central bank is yet to be introduced in the model.

4.4. Calibration and econometric estimation

In the previous section, a theoretical regression equation for modeling black market exchange rate was obtained. There are several obvious restrictions that are faced:

Nonlinearity: The most obvious restriction comes in form of a nonlinear $(\beta\lambda_1)^j$ combination of coefficients that need to be calibrated. This restriction can be circumvented utilizing additive model architecture by first decomposing existing series into trend-seasonality-noise components and then modeling trend using GNLS (Generalized Nonlinear Least Squares) method, which allows nonlinear coefficients, with ARIMA residuals to eliminate autocorrelations. Then additionally random components can be modeled using the GARCH approach to account for volatility clustering, which is common in exchange rate data due to the presence of one-time significant events. At last seasonal terms can be modeled separately by fitting harmonic functions and then extrapolating them into future forecasts.

Infinite frontiers: Rational expectations extend infinitely, so it is only natural to limit its extent by data availability, which in our case is a maximum of 12 periods (month) ahead, as further data with monthly granularity will be used.

Expected future terms: Exchange rates are known to be inconsistent and quasi-effective; this means that agents think not about long-term relations, but iterate their short-term expectations, this is known as the bandwagon effect. So in this work, this will be achieved by iteratively imputing values between existing exchange rate data points and long-term reported expectations, which are available and known.

Regime Switching: Policy change can also be reflected by directly modifying $E_{t-1}z_{t+j}$ and allowing λ_1 to vary for different regimes.

Cointegrating factors: Money deposits and Bond holdings are directly related to the exchange rate. In this work, they will be modeled separately, so there is still some room for improvement in this area.

4.5. Modelling expectations

In this work, expectations are modeled by means of imputing values between existing exchange rate data points and long-term reported expectations using a rolling Clamann filter fitted on Arima.

First, a simple time series model reflecting historical changes in the exchange rate and market mood needs to be derived. To assess market mood Google Trends data is used to develop a sentiment index.

In the beginning keywords for Google Trends data are selected by choosing the most correlated series to the exchange rate from the EU Economic Lexicon Dictionary (Barbaglia et al. 2022) dictionary. The Algorithm is as follows:

1. Download all Google Trend data for all words in the dictionary
2. Run a mowing window cross-correlation analysis on the SVI index vs. monthly aggregated black market exchange rate for all words present, select the most correlated (anti-correlated) lag for each word
3. Select the top 15 most correlated (anti-correlated) words and transform the SVI series in the lagged version of itself before adding to other SVIs.
4. Sum across SVIs and languages

$$\Delta SVI_{i,t} = ln(SVI_{i,t}) - ln(SVI_{i,t-1}) \tag{13}$$

$$SI_t = 0.75(\sum_{i=1}^{15} \Delta SVI_{i,t} - \sum_{j=1}^{15} \Delta SVI_{j,t})_{UA} + 0.15(\sum_{i=1}^{15} \Delta SVI_{i,t} - \sum_{j=1}^{15} \Delta SVI_{j,t})_{RU} \tag{14}$$

Where SVI is a Search Volume Index reported by Google, SI is an artificial sentiment index weighted for the case of Ukraine for 75% searches in Ukrainian and 15% in russian for the 15 most correlated and anti-correlated searches. Figure 1 below shows an example of Cross-Correlation analysis for one keyword and exchange rate series. This time there is no significant correlation present, however, it is not the case for most words.

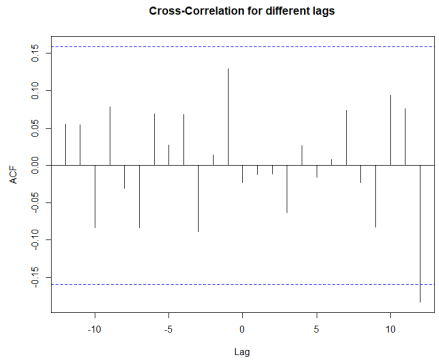

Figure 1: Cross-Correlation example for black market exchange rate series and SVI for one keyword

As the maximum(minimum) correlation is selected across different lags it may be so, that some keywords and their associated SVIs are simultaneously most correlated and most anticorrelated due to their periodic nature causing them to come in and out of phase with exchange rate series, as can be seen in the Figure 2.

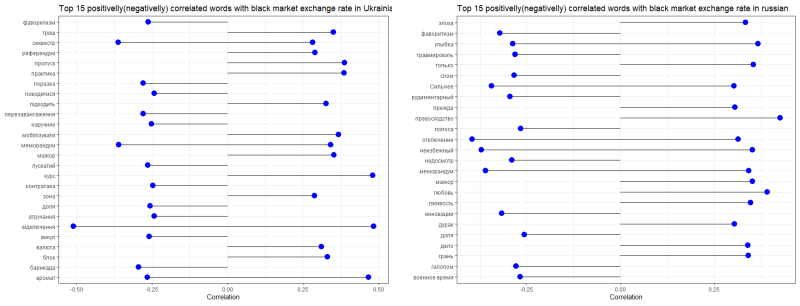

Figure 2: Words selected for construction of SI and their correlations

After completing this algorithm, the following SI, Figure 3 is created.

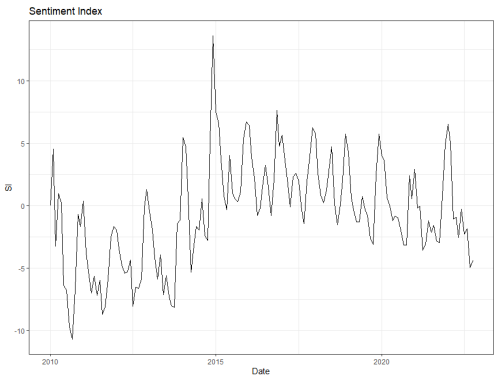

Figure 3: Sentiment Index for Ukraine

SI highly resembles the exchange rate path, hence the selection process, and displays a strong monthly seasonal pattern persistent throughout all estimation time, which is demonstrated by a scalogram in the Firure 4 formed by a continuous wavelet transform (Morlet form) of SI series.

The next step is to use the SI series to model short-term exchange rate expectations based on the rolling Kalman filter imputation with.

As a Kalman filter is a state-space model, a regression for $E_t e_t$ with ARIMA errors is taken as a state equation and a discontinuous time series of past exchange rates plus one point 12 periods ahead in the future — reported expectation is observation series, which allows to make a one step ahead imputation and then refit the whole state model to include this point (hence

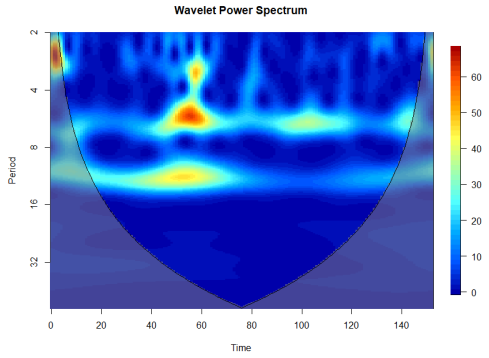

Figure 4: Wavelet Power Spectrum (Scalogram) for Sentiment Index

simulate rolling expectations effect) and repeat until imputed series converges to the last reported point.

$$\begin{aligned} E_t e_t =& \alpha_0 + \alpha_1 e_{t-1} + ... + \alpha_k e_{t-k} + \\ &+ \beta_0 + \beta_1 SI_{t-1} + ... + \beta_k SI_{t-k} + \nu_t \end{aligned} \tag{15}$$

This procedure allows the creation of a robust framework for modeling expectations 12 months ahead for every data point.

So the imputation is mostly non-uniform and non-linear as can be seen in Figures 5 and 6, where there is a black market exchange rate (black), 12-month ahead reported expectations (green), and imputed values in-between (grey) all stacked up vertically for a corresponding month.

4.6. Regime switching

To account for regime-switching derived interest rate parity is used for periods when central bank interest rate enforcing effectiveness drops to model market interest rates due to the lack of data. As for the expected change in the official exchange rate, a simple rule is used $\epsilon_t = 0$ for the pegged exchange

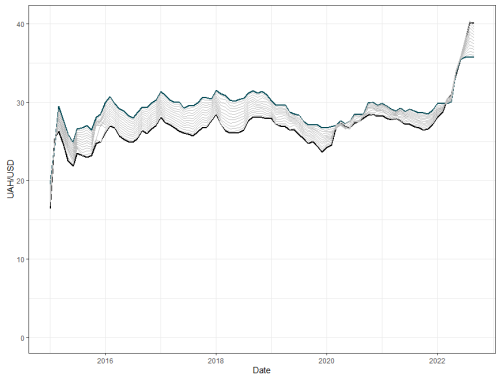

Figure 5: Example of rolling Kalman imputed values

rate, $\epsilon_t = \Delta e_{t+j}/e_{t+j}$, which equates official exchange rate with the black market one, and for a newly adopted policy with adjustment-peg, a switching rule is used.

$$z_{t+j} = ln(\frac{\delta}{1-\delta}[\frac{i_{f,t+j}\rho_{t+j}}{i_{d,t+j} + \Delta\rho_{t+j+1}}]) \quad \text{for pegged exchange} \quad (16)$$

$$z_{t+j} = ln(\frac{\delta}{1-\delta}[\frac{i_{f,t+j}}{(1 - \Delta e_{t+j+1}/e_{t+j+1})i_{d,t+j}}]) \quad \text{for floating exchange} \quad (17)$$

$$z_{t+j} = ln(\frac{\delta}{1-\delta}[\frac{i_{f,t+j}}{(1 - \Theta(\epsilon_{t+j+1}))i_{d,t+j}}]) \quad \text{for adjustment-pegged exchange} \quad (18)$$

Where $\Theta(\epsilon_{t+j+1})) = \epsilon_{t+j+1}$ for $\rho_{t+j+1} \leq 0.2$ and
$\Theta(\epsilon_{t+j+1})) = \Delta e_{t+j+1}/e_{t+j+1}$ for $\rho_{t+j+1} \geq 0.2$

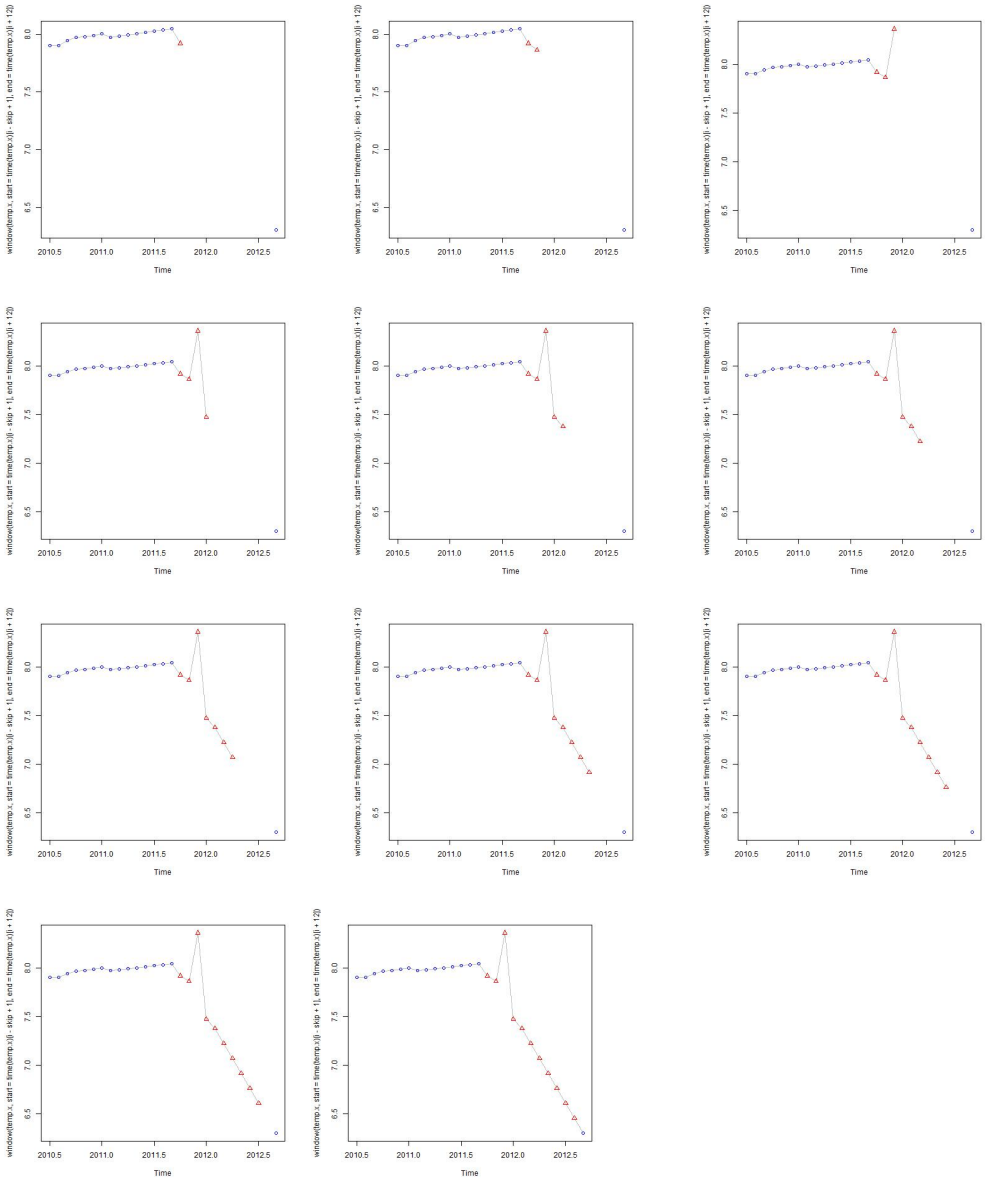

Figure 6: Singe instance of rolling Kalman filter imputation for a series

Chapter 5.

RESULTS

Estimated regression coefficients using Wolfram Mathematica GNLS capability for the following equation:

$$q_t = \alpha + \lambda_1 q_{t-1} + (1-\lambda_1)(1-\gamma\lambda_1)\kappa_1 \sum_{j=0}^{\infty} (\gamma\lambda_1)^j E_{t-1} z_{t+j} + \mu_t$$

With μ_t being ARIMA-errors, γ a future discount factor, and λ an Euler equation characteristic root.

	$\hat{\beta}$	$\hat{\sigma}$	$P(\|T\| > \|t\|)$
λ	−0.91	0	0
κ	0.12	1.7×10^{-11}	$0.\times10^{-3}$
γ	1.0	1.8×10^{-7}	$0.\times10^{-3}$
α	1.3×10^{4}	5.2×10^{-18}	$0.\times10^{-3}$

Figure 7: Coefficients for black market exchange rate regression

All coefficients are significant for a given confidence level, with p-values reported essentially being a machine-precision number.

As $|\lambda| < 1$, being a stable root of a stochastic equation, current policy yields stable results, meaning there is no reason to expect explosive exchange rate path behavior. This also means, that under current policy there is no "herding" behavior present among currency holders, which implies that there is no speculative attack on reserves present or pending delaying balance of payments crisis.

However, as a further improvement, a proper optimal policy analysis needs

to be made namely by an introduction of acting non-observant National Bank, actively intervening in the currency markets.

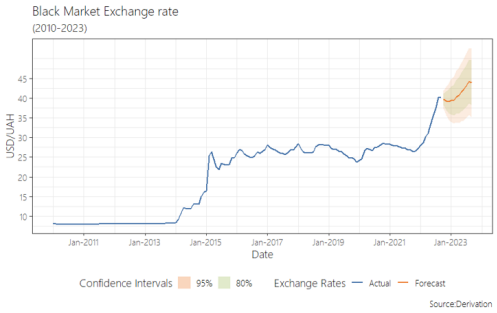

Figure 8: Results of black market exchange rate modeling

As a result of modeling the exchange rate by means of the dynamic currency substitution model, adopted for parallel markets, we can conclude, that the parallel market exchange rate will continue to rise, though official exchange rate corrections may slow it down.

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