OPTION VALUATION: EMPIRICAL EVIDENCE FROM POLISH MARKET

by

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LIST OF ABBREVIATIONS

BHV Black model with historical volatility

BRV Black model with realized volatility

BIV Black model with implied volatility

GARCH Generalised Autoregressive Conditional Heteroscedasticity

BSM Black Scholes Merton

S&P Standard and Poor's

OP Overprediction

WIG Warsaw Stock Exchange General Index

BS Black-Scholes

HF High Frequency

MdAPE Median Absolute Percentage Error

ATM At the money

OTM Out of the money

ITM In the money

GBM Geometric Brownian Motion

TTM Time to maturity

IV Implied Volatility

ARPE Average relative percentage er

CHAPTER 1. INTRODUCTION

Options are one of the most dangerous financial instruments, one of the hardest to understand. For example, to start trading options on the Interactive Brokers, the average annual income should be at least $ 40,000 per year, and with experience in trading options at least 2 years. On the other hand, there are no such requirements for trading stocks on this platform. Since the risks are high, hence the income should be substantial as well.

Since options are complex, many investors have shunned them considering them to be sophisticated. Options have existed for about 40 years, but at the very beginning, even brokers did not properly understand how to use them correctly. There was even less understanding of how to approach the task of option valuation until Fisher Black, Myron Scholes and Robert Merton suggested their famous option pricing framework (Black and Scholes, 1973; Merton, 1973).

Now this is the most famous options pricing model. In 1997, Scholes and Merton were awarded the Nobel Prize for the creation of this model. Black did not live up to the award for 2 years. The model made several assumptions that were not true in the real world. For example, the volatility of the underlying security does not change over time. This model pushed financial engineers, economists, and mathematicians to study and improve it.

Heston (1993) extended the Black-Scholes-Merton model by assuming that the underlying asset's volatility is stochastic: the volatility dynamics is described via a separate stochastic differential equation. Adding Poisson jumps to the stochastic volatility model, it is assumed that this model will work better for unexpected price jumps, for example, reactions to unexpected events, news. Within the stochastic volatility model, plain vanilla options can be priced with a semi-closed-form valuation formula. This model has also been well studied and has been applied to various financial markets. We will show this later. This paper demonstrates the application of the Heston model to the Polish options market.

Why exactly the Polish market? Market participants use European type options. Many papers explore already established markets, such as the USA (Bates (1996), Bakshi, Cao, Chen (1997)) and evaluate models that are better suited to this market. In such markets, it was shown that the Black Scholes formula works worse than a stochastic volatility model or a stochastic volatility with Poisson jumps model. However, emerging markets cannot boast of such. Kokoszcynski, et al. (2010) studied the Polish market with high-frequency data for WIG20 index options. They evaluated 3 models (Black-Scholes-Merton variations) applying for futures contract where WIG20 index futures was the basis instrument, and it was determined that one of this model the best. Based on the previous paper, Kokoszczynski, Sakowski, Slepczuk (2017) defined that for the Japanese market (Nikkei 225 index options) the same model was the best among the other 5 models which included the Heston model.

We hypothesize that for developed markets, such as the US market, a stochastic volatility model (with or without jumps) works better than the Black-Scholes model. However, for developing markets a simpler model might be a better fit because there is less variability in the data, fewer valid contracts, low liquidity. It is also questionable that the developed Japanese market was taken by Kokoszczynski et al. (2017) to test the hypothesis of Kokoszcynski et al. (2010). We decided to continue studying the Polish market and checking what model is better among the BSM model, the Heston model, and the Heston model with jumps on daily frequency call options data. For investors in Eastern Europe, the Polish market is considered to be the most developed one, and we believe that these studies in options valuation will be useful for investors in emerging markets.

Since the market is emerging, this article also demonstrates data cleansing and preparing data for models with different contract volume. It also shows how stable the algorithms are. Estimation of model parameters on daily data shows the limits of our model parameters, their adequacy. All computations were done in Python programming language using Quantlib library for model evaluation, Scipy library for numerical method optimization and BeautifulSoup library for scrapping data.

CHAPTER 2. LITERATURE REVIEW

The first to make revolutionary research in the world of option models were Black, Scholes, and Merton. Black and Scholes in the "Pricing of Options and Corporate Liabilities" (1973) presented a mathematical interpretation of the option price and derived a formula. The model itself relied on several important assumptions that do not work in the real world.

Assumptions:

1. The option is European type

2. Markets are efficient (lack of arbitrage, market movement cannot be predicted)

3. Dividends are not paid. It is assumed that the shares do not pay either dividends or profits

4. Stock returns are normally distributed. The volatility doesn't vary with time

5. Interest rates are constant, which makes the underlying asset risk free

6. No transaction costs.

Merton (1973) interpreted mathematical terms into a more understandable language for investors and coined the term "Black – Scholes option pricing model".

It was later shown by Bates (2003) that some theoretical assumptions are not assumed in real data. But still, the formula is often used in comparison with other models in practice.

Kokoszczyński, et ca. (2010) studied the variation of the Black-Scholes model. They estimated price for WIG20 index option using Black models for WIG20 index futures contract due to WIG20 index option and future contracts are expired on the same date. In this paper investigated 3 variations of Black model which differ in their volatility assessment:

Black realized volatility (BRV) – standard deviation of the returns (log returns) of the underlying asset with assuming mean zero.

Black implied volatility (BIV) – using implied volatility such as volatility that needs to be entered as a parameter of the option pricing model (in this case, the only one) to get the current option price.

Black historical volatility (BHV) – standard deviation of returns (log returns).

All three models were used for high-frequency data. This paper also describes the metric by which models are compared. As showed Bams (2009) even the best metric for comparison is a controversial issue. They decided to use the well-known BSM model for futures pricing (BHV model). The Black formula has a lot of common with Black–Scholes formula for stock options except that using discounted futures price instead of the spot price of the underlying. In this paper BRV model using different frequencies 10 seconds, 1 minute, 5 minutes, and 15 minutes. However, have been averaged only for 5 minutes and daily observations, where n = 1, 2, 3, 5, 10, 21. As a result, were studied the BRV model with an interval of 5 minutes and 6 BRV models with an interval of 5 minutes and an parameter of averaging n (1, 2, 3, 5, 10, 21).

So, there are models which were compared:

1. BRV with 10 seconds, BRV with 5 minutes, BRV with 5 minutes and parameter of averaging 5, BRV with 5 minutes and parameter of averaging 21, BHV, and BIV – for selecting the best model,

2. BRV with 5 minutes, BRV with 5 minutes and n different parameters of averaging (n = 1, 2, 3, 5, 10, 21) – for selection the best model.

Applying models to high-frequency data (10-seconds) for WIG20 index option, it was revealed that the best model is BIV. A little worse was BHV, and much worse BRV. Moreover, for different concepts of BRV models, results significantly differ.

The Black-Scholes model has the assumption that stock returns are normally distributed with known variance and mean. Also, there is no mean spot return in option-pricing equation; consequently, it cannot be summarized by allowing the mean to vary. But the assumption with volatility is dubious. Scott (1987), Hull and White (1987), and Wiggins (1987) added stochastic volatility to the Black-Scholes model. Melino and Turnbull (1990, 1991) determined that this approach quite good estimates the price of currency options. But all these papers have the disadvantage that to solve two-dimensional partial differential equations it is necessary to use numerical methods, it means there is no closed-form solution. Jarrow and Eisenberg (1991) and Stein (1991) investigated the model which assumes that volatility does not correlate with the spot asset. They used an average of Black-Scholes formula values over different paths of volatility. However, since the volatility does not correlate with spot returns, this approach cannot include important skewness effects that arise from such correlation. Heston (1993) introduced a stochastic volatility model (Heston model) that is not based on the BS formula and used it to derive a closed-form solution for the price of the European call option.

Kokoszczyński, Sakowski and Ślepaczuk (2017) investigated high-frequency data (5-minutes) for the Japanese Nikkei 225 index options. The motivation for this paper came from the paper by Kokoszczyński, et al. (2010). The authors explore 6 models: the Black model with different types of volatility processes (realized volatility with and without smoothing, historical volatility and implied volatility), the stochastic volatility model, and the GARCH (1, 1) model. The metrics by which quality was evaluated were Median absolute percentage error (MdAPE) $MdAPE = median\left(\frac{y_i - y_{model}}{y_i}\right), i = 1, .., N$ and Overprediction (OP) $P = \sum_{i=1}^{N} OP_i\ , where\ OP_i = 1\ if\ y_{model} > y_i\ and\ 0\ otherwise$, based on differences between theoretical and real options prices.

Options usually divide into three types ITM, ATM and OTM options. ITM call options mean that the current price of the underlying asset is more than the strike price. ATM call options mean that the current price of the underlying asset is identical to the strike price. OTM call options mean that the current price of the underlying asset is less than the strike price. ATM is more attractive for investors if they expect movement of the underlying asset. Contracts also can be of different duration. It calls Time to maturity (TTM) and means number of days to expiration of a contract.

A MdAPE metric indicated that the Black model with the implied volatility estimator (BIV) has the smallest average pricing errors for the majority of option classes. The Heston model produced slightly higher errors, but, on the other hand, was the best model for ATM options with TTM less than 60 days. The remaining models showed worse results. According to the OP metrics, the best model was BIV and the second best was Heston model. For put options, the results were similar for both metrics. Thus, this article confirms the result that the best model is BIV.

Very often, real data is faced with unpredictable leaps, which can talk about any news, failures. Merton (1976) introduces the Black Scholes model with Poisson jumps. Poisson jumps are responsible for unexpected information, for example, news. Nicola Gugole (2016) compared the Merton model and Black Scholes model by fitting log-return and volatility smile on S&P500. Merton's model turned out to be better. Bates(1996) combined Heston model and Merton model and obtained stochastic volatility model with Poisson jumps. Bates (1996), Bakshi, Cao, Chen (1997) showed that it works better than Heston model on developed market.

This article enables to identify how well the models perform on daily data for WIG20 index options. Unlike previous articles, we do not use data on futures contracts, for an underlying asset we use WIG20 index. It also demonstrates a data cleaning method for models by choosing the valid number of purchased contracts Volume per each day. The purpose of this article is to choose the best model among the three others Black-Scholes model, Heston and Heston with jumps model for daily, low-frequency data. From previous articles, it can be confirmed that Black-Scholes model works better than Heston and Heston with jumps.

As was said Black-Scholes formula is the beginning of research in the option pricing model. It can be deduced from Geometric Brownian Motion (GBM) equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

$W_t, t \geq 0 -$ Brownian motion

$$S_t = S(0)e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t}$$

All derivation explained in Fisher Black, Myron Scholes (1973).

The final formula for call option is:

$$c = S_t N(d_1) - Ke^{-rt}N(d_2)$$

$$where:\ d_1 = \frac{ln\frac{S_t}{K} + (r_f + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

c –call option price, S – underlying price, K – price, r_f –risk-free interest rate, t – time to maturity, N – normal distribution

In the Black-Scholes formula, the only unobservable variable is σ, which is the volatility of future stock returns. To estimate σ, a GARCH model for past stock returns or past option prices can be used (in the later case, the resulting volatility is called implied volatility).

To extract implied volatility from observed option prices, a numerical procedure should be used, since the Black-Scholes option pricing formula is highly nonlinear (though monotone) in parameter σ. The Newton-Raphson is frequently used in the literature for this purpose. As Kokoszcynski et al. (2017) show, a model with implied volatility works better than a model based on GARCH or historical volatility. So we use this approach for daily frequency data with estimating implied volatility and predict the next day.

Heston model (1993) is not based on Black-Scholes formula and it assumes that the dynamics of underlying asset price S_t and its volatility V_t are given by the following set of differential equations:

$$dS_t = \mu S_t dt + S_t \sqrt{V_t} dW_t^1$$

$$dV_t = k(\theta - V_t)dt + \sigma \sqrt{V_t} dW_t^2$$

$$dW_t^1 dW_t^2 = \rho dt$$

Additionally, it is assumed that $V_t, t \geq 0$, is a mean-reverting process, with long memory expected value θ and mean-reverting coefficient k. There are five unknown parameters: mean reversion k, long-run variance θ, current variance V_t, correlation ρ, and volatility of volatility σ. Additionally, if $2k\theta > \sigma^2$, the volatility motion is always above zero, which could ensure the volatility to be positive (see Cox et al. (1985)).

There is also analytical solution of Heston model which provided in Yiran Cui, Sebastian del Bano Rollin, Guido Germano (2016).

Defined by $C^*(K_i, T_i)$ the market price of a call option with a strike K_i and maturity $T_i, C(\theta, K_i, T_i)$ the price computed via the Heston (1993) analytical formula with unobserved parameter vector $\theta = [V, \sigma, \rho, k, \theta]^T$. Defined the residuals for n options:

$$r_i(\theta) = C(\theta, K_i, T_i) - C^*(K_i, T_i), \qquad i = 1, \dots, n$$

Vector of residuals $r(\theta) = [r_1(\theta), r_2(\theta), \dots, r_n(\theta)]^T, r(\theta) \in R^n$

There was used for market price $C^*(K_i, T_i) = \frac{Ask_{price} + Bid_{price}}{2}$

Calibration of the Heston model is an inverse problem in the nonlinear least-squares form:

$$\min_{\theta \in R^n} f(\theta) \tag{1}$$

In our case $m = 5$ and $f(\theta) = \frac{1}{2}\, r(\theta)^T r(\theta)$.

Since we have more than 5 observations within each data subsample, the calibration problem is overdetermined. Solution problem (1) see in Yiran Cui, Sebastian del Bano Rollin, Guido Germano (2016). Heston model is also very popular since there is a closed-form solution for the price of European type call options.

$$C(S_t, V_t, t, T) = S_t P_1 - K e^{-r_f(T-t)} P_2, where$$

$$P_j(x, V_t, T, K) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty Re\left\{\frac{e^{i\phi \ln(K)} f_j(x, V_t, T, \phi)}{i\varphi}\right\} d\phi$$

$$f_j(x, V_t, T, \phi) = \exp\left\{r\phi i r_\varphi + \frac{a}{\sigma^2}\left[(b_j - \rho\sigma\phi i)\tau - 2\,ln\left(\frac{1 - ge^{dr}f}{1 - g}\right)\right]\right.$$
$$\left. + \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}\left(\frac{1 - e^{dr}f}{1 - ge^{dr}f}\right) + i\phi\, ln(S_t)\right\}$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = k\theta, b_1 = k + \lambda - \rho\sigma, b_2 = k + \lambda$$

After determining the parameters of the Heston model the formula above can be used to find the theoretical price of the call options.

Merton (1976) proposed a jump-diffusion for the stock price model. The stock price process is divided into two parts: "normal vibrations", modeled by a standard Brownian motion, and "abnormal vibrations", resulting from firm-specific factors or new information, modeled by a jump process. Bates(2002) combined Heston and Merton models.

$$dS_t = \mu S_t dt + S_t\sqrt{V_t}dW_t + S_t d\left(\sum_{i=1}^{N(t)}(Y_i - 1)\right)$$

$$dV_t = k(\theta - V_t)dt + \sigma\sqrt{V_t}dW_t^2$$

$$dW_t^1 dW_t^2 = \rho dt$$

Where $\mathrm{N(t)}$ is a Poisson process with parameter λ and $\{Y_i, Y_i > 0\}$ which is a sequence of independent identically distributed (i.i.d.) random variables (sequence of jump size). If we assume $\lambda = 0$ and $V = const$, then the stock price returns have the same dynamics as those in the BSM approaches. If $V = const$ and $\lambda \neq 0$ as in the BS model case, we can get a closed-form solution for the Merton model. In particular, we have

$$S_t = S_0 \exp\left\{\sigma W_t + \left(\mu - \frac{1}{2}\sigma^2\right) t\right\} \prod_{i=1}^{N_t} Y_i, \qquad 0 \leq t \leq T,$$

where $V_i = \log Y_i \sim N(\eta, \delta)$.

So we have 3 diffusion parameters λ, η, δ. And also we have an unknown five parameters. Totally: 8 parameters to estimate.

To estimate these parameters:

$$\theta = [V, \sigma, \rho, k, \theta, \lambda, \eta, \delta]^T = \underset{\theta \in \Theta}{\operatorname{argmin}} \sum_{i=1}^{N} \left(\Sigma_i - \Sigma_i'(\theta)\right)^2, \qquad (2)$$

Where $\Sigma_i = \Sigma(K_i, T_i, C_i)\ and\ \Sigma_i'(\theta) = \Sigma\big(K_i, T_i, C(\theta)\big), K - strike\ of\ option\ i, T_i - maturity\ of\ option\ i, C_i - option\ price, C(\theta) - estimating\ option\ price.$

For market option price was used average between bid and ask prices. Solution problem (2) with the Multinomial Maximum Likelihood approach is described in Gugole (2016).

Using Python libraries Quantlib for building models and Scipy for numerical optimization methods was found solutions to these problems. The algorithm employed for determining the parameters of the models was tested in the following way. Using random inputs with different strike prices and Heston model values, IV and Call option prices were estimated. After that calibration method with initial values of Heston model was used to define the final parameters of Heston model and compared the results with input values of this model. The conclusion made from such experience was that if the initial values are close to the true values, then the fitted values will also be close to the true values. Otherwise, the algorithm is likely to reach a non-optimal value.

There are many different metrics to measure the quality of models. For example, the percentage of overprediction, median absolute percentage error, root mean square error. Since we do not have a lot of daily data and after clearing them even less, thus to measure a quality Average relative percentage error were used:

$$ARPE = \frac{1}{n}\sum_{k=1}^{n}\frac{|C^{true} - C^{est}|}{C^{true}}$$

We are scrapping the following option data from stooq.com using BeautifulSoup library in Python: Ask, Bid, TTM, Premium, Volume(number of contracts purchased), Symbol, Strike, Spot price, IV, Historical volatility (Volatility). By this time, we have daily data on options from 19-05-2020 to 24-07-2020. In total, we got 32 business days. After that, the data are prepared and ready to be used in models. The following restrictions are imposed via the query: Volume more than 2, Bid, and Ask is not equal to zero. After that was investigated the ratio between ask and bid prices.

Figure 1. The ratio of ask and bid prices for each option

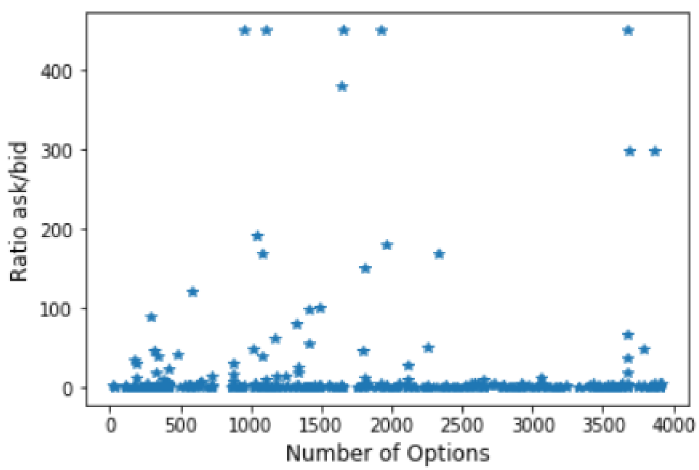


Figure 1. shows that there are options with ask price higher than bid price more than 400 times. These options are invalid for evaluating. To choose the optimal ratio at which the option will be considered valid, options were cleaned from outliers in the context of options with different TTM. After that was an investigated number of contracts per each day. Outliers were meant options which are higher than

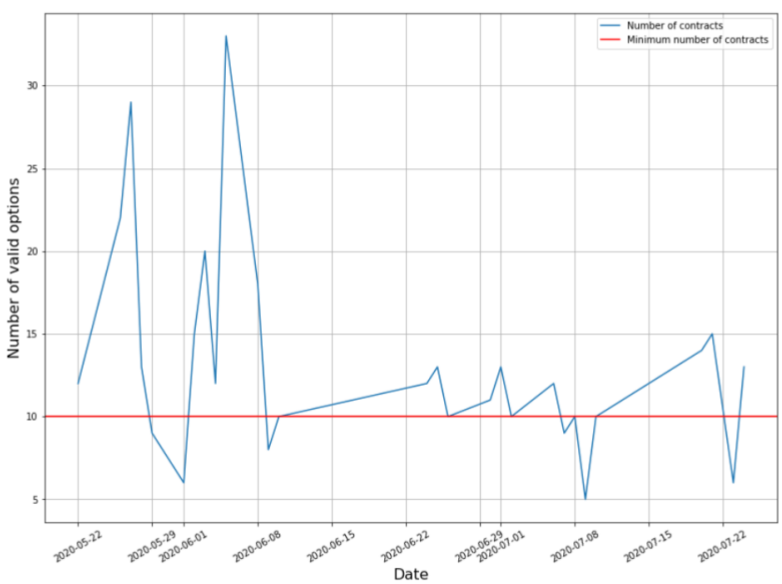
$$q_3 + (q_3 - q_1) * 1.5, \qquad where\ q_1 - first\ quartile, q_3 - third\ quartile$$

Table 1. Descriptive statistics of call options, in PLN

	Ask	Bid	IV	Volume, units
Mean	69.34	53.20	0.26	25.39
Median	32.28	17.22	0.25	5.00
Std	89.68	82.50	0.07	72.35
Min	0.10	0.01	0.00	1.00
Max	724.25	694.25	1.03	845.00
25%	8.51	3.50	0.23	2.00
75%	98.80	69.30	0.28	18.00

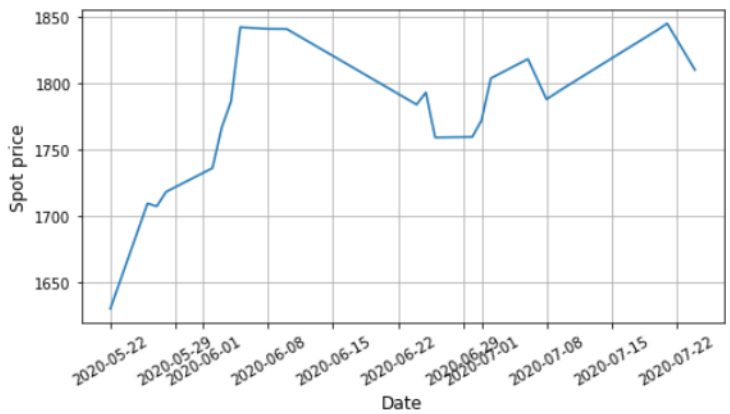
As can be seen from the quantiles, the options market volume is low. Also, we have two options with zeros IV and it means that there were no changes in price.

Figure 2. Number of contracts per each day



10 option contracts for each day were selected as the threshold number. As a result, 22 days were obtained for estimating the parameters of the models.

Figure 3. WIG20 index dynamics



This chart shows the spot price by day on which the models were built. It is evident that the spot prices are following an upward trend in May and stabilized in the next two months. Suppose this is due to the stabilization of the situation with the COVID19. Since the situation with the COVID19 after the lockdown began to improve, the government began to soften the conditions for staying in public places, and accordingly, some enterprises resumed work. The economic situation began to improve.

Table 2. Number of contracts by Maturity and Moneyness

Maturity (in days)				
Moneyness(K/S)	7-30	30-50	50-120	120-333
0.81-0.95	14	1	1	7
0.95-1.0	34	3	13	7
1.0-1.05	55	15	16	10
1.05-1.1	42	10	22	11
1.1-1.47	12	3	28	23

Table 3. Average prices by Maturity and Moneyness

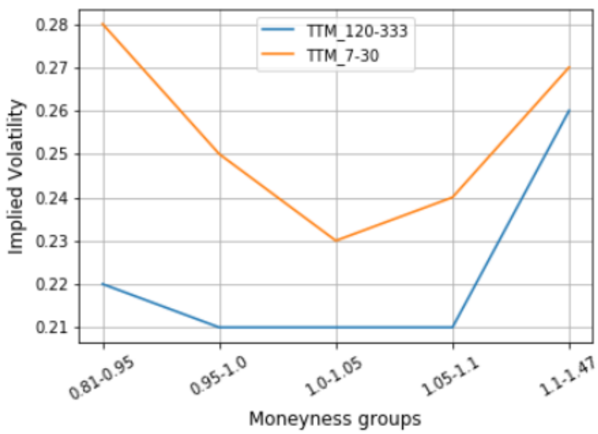
Maturity (in days)				
Moneyness(K/S)	7-30	30-50	50-120	120-333
0.81-0.95	182.55	121.70	198.65	244.84
0.95-1.0	67.95	108.07	109.59	167.36
1.0-1.05	21.92	34.52	54.20	100.82
1.05-1.1	5.42	11.81	31.38	86.85
1.1-1.47	2.44	3	14.85	22.19

Table 4. Average IV by Maturity and Moneyness

Maturity(in days)				
Moneyness(K/S)	7-30	30-50	50-120	120-333
0.81-0.95	0.28	0.22	0.27	0.31
0.95-1.0	0.25	0.21	0.25	0.29
1.0-1.05	0.23	0.21	0.23	0.25
1.05-1.1	0.24	0.21	0.22	0.26
1.1-1.47	0.27	0.26	0.23	0.24

Tables 3-4 represent the average prices of ITM, OTM, and ATM options which are increasing with a higher duration of an option contract. Moreover, for each group of maturity, it is evident that the IV is decreasing to some level in line with increasing Moneyness and then the reversal of its direction follows. This effect is called a volatility smile and raises concerns against the assumption of constant volatility in the BSM model.

Figure 4. IV-Moneyness. Volatility smile



The figure above shows a nice volatility smile for the groups with TTM 7-30 and 120-333 days. The volatility smile shows higher volatility for OTM and ITM options than ATM options. Thus, the Black-Scholes model assumptions that volatility is constant over time and underlying asset returns have log-normal distributions are incorrect in the real world. The volatility smile was first inscribed after the stock market crash in 1987. That day was called Black Monday and was described by Tim Metz(2003).

Figure 5. IV-Moneyness

Figure 5 represents that for group with TTM 50-120 days we have downward trend and for group with TTM 30-50 we have almost volatility smile.

Wibor 3-Month interest rate which is the reference rate for interbank unsecured borrowing is employed as a risk-free rate.

CHAPTER 5. RESULTS

5.1. Tests of Fit and Parameter Estimates

Was reviewed the data from 19-05-2020 to 24-07-2020 and after the cleaning data was obtained 22 days. The first model that was investigated is the Heston model. Using the input data `Expiration Dates`,` Strike prices`, `IV for Call options`,` risk-free rate`, `dividend rate`, `Spot prices`, initial value. To measure the quality of the models, the metric Average relative percentage error was selected.

The Nonlinear least-squares method was used to solve the Market Implied Volatility Estimate problem. After converting the data into classes, a result was obtained for each of the twenty-two days. Initial values for the parameters were the next :

$$[\theta, k, \sigma, \rho, V] = [0.2, 0.2, 0.5, 0.1, 0.02]$$

Table 5. Descriptive statistics of the Heston model estimated parameters

	θ	k	σ	ρ	V
Mean	11.08	12913.59	1268.11	-0.26	8.82
Median	0.09	18.30	7.58	-0.36	0.08
Std	30.60	26475.52	2425.86	0.36	26.35
Min	0.00	0.00	0.0	-0.71	-4.44
Max	122.10	90106.56	8253.36	0.87	111.40
25%	0.06	0.13	0.52	-0.48	0.05
75%	0.22	5389.72	1507.12	-0.12	1.16

We see that parameter estimates have a huge standard deviation. It follows that the daily data are too different from each other either the programmed model is highly sensitive to minor changes. The quality and comparison of models are shown in Figures 3 and 4.

The next model in which parameters were evaluated is the Heston with Poisson Jumps (Bates model). The input remains the same, except for initial values.

$$[\theta, k, \sigma, \rho, V, \eta, \delta, \lambda] = [0.02,0.2,0.5,0.1,0.01, 0.1, 0.1, 0.2]$$

For this model, were faced with the problem of the estimated parameter going out of bounds. For example, the estimated parameter $|\rho| > 1, \sigma < 0, V < 0$. In such cases, the value was rounded to the nearest possible.

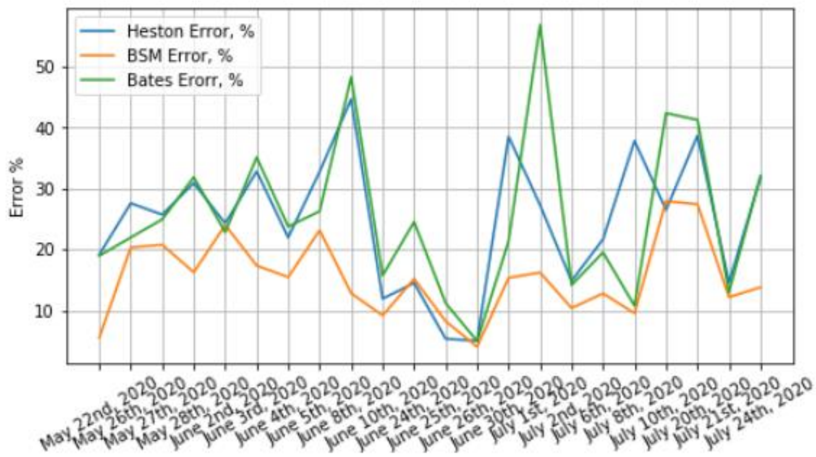
Table 6. Descriptive statistics of the Heston with jumps model estimated parameters

	θ	k	σ	ρ	V	η	δ	λ
Mean	0.17	50.62	2.86	-0.47	0.08	-0.42	0.26	0.81
Median	0.08	1.19	0.45	-0.90	0.08	0.13	0.01	0.27
Std	0.27	197.12	10.04	0.72	0.00	2.18	0.84	1.06
Min	0.01	0.14	0.00	-1.00	0.08	-9.38	0.00	0.10
Max	0.08	862.60	44.26	1.00	0.08	0.24	3.72	3.68
25%	0.03	0.84	0.02	-1.00	0.08	-0.01	0.00	0.20
75%	0.15	2.58	1.23	0.10	0.08	0.18	0.13	0.83

Based on descriptive statistics (Table 6), it can be seen that the same parameters of the kappa, sigma model has less standard deviation than in the Heston model. Also, parameter V is constant for all days.

The last model that was investigated was the BSM model. The initial values are not needed for this model in Quantlib library.

Figure 6. Model errors for each day, %



It can be seen that in most cases (19/22 cases) the BSM model outperforms the Heston model and Heston model with jumps. The Heston model with jumps in most cases works similarly to the Heston model. The Heston with jumps model stands out from the rest at one point (first of July).

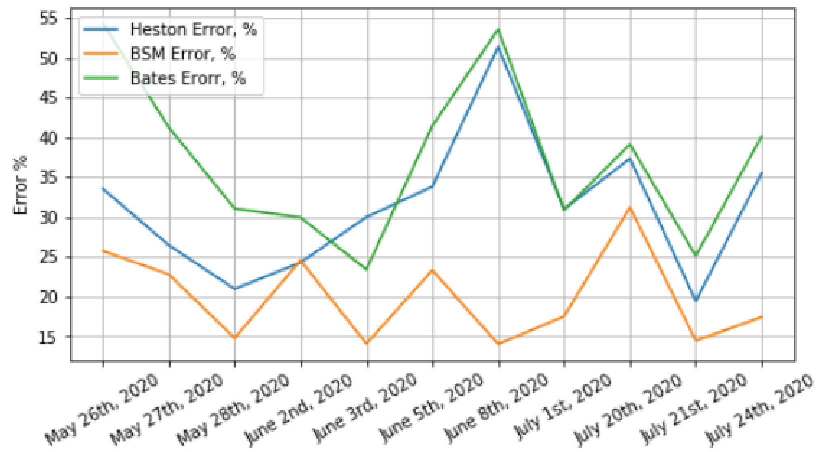
Table 7. Average error, %

	BSM	Heston	Heston with jumps
Average Error, %	15	24	25

5.2. Tests of Fit and Parameter Estimates for Volume more than 6

Contracts with a volume of more than six were investigated to decrease the difference between ask and bid prices and to increase the number of ATM, ITM contracts. Was investigated 11 days.

Figure 7. Model errors for each day with Volume > 6, %



It can be seen that the general situation has not changed. BSM model still has fewer errors in 10/11 cases.

Table 8. Average error for Volume > 6 (contracts), %

	BSM	Heston	Heston with jumps
Average Error, %	20	31	37

All three models, on average, for data with Volume > 6 showed a higher error than for the data with Volume > 2.

5.3. Comparing days with the highest error and the smallest error.

After comparing the days in which the Heton model had the smallest error and the largest error, the conclusion was drawn:

1. The average between ask and bid prices has a higher correlation with implied volatility(0.82) for the day with the smallest error than for day with the highest error (0.12).

2. Less number of options for the day with the smallest error.

3. The high percentage of short term options (80%) for the day with the smallest error.

CHAPTER 6. CONCLUSIONS AND RECOMMENDATIONS

Options are non-linear financial instruments based on the underlying asset. Options enable the buyer of the option to buy or sell, depending on the type of the option contract. The holder of the option undertakes to execute the transaction. Unlike futures contracts, options contracts enable the holder the opportunity to buy or sell an asset. Call options provide an opportunity to buy for a specific price within a specified time frame. Put options provide the ability to sell at a specified price at a specified time. This article covered the WIG20 index call options. Options are considered the most difficult financial instruments. This is because it is difficult to estimate the fair price of these contracts. There are a huge number of models for estimating prices. In this paper, three models were examined: BSM model, Heston model, Heston with Poisson jumps model (Bates model). BSM model is one of the first model that estimated fair price of option contracts. BSM model is based on several assumptions one of that is the volatility of the underlying asset is constant over time. BSM model has only one unknown parameter. Heston model or stochastic volatility model is the model where underlying asset price and volatility follows stochastic differential equations. Hence this model solved the problem with constant volatility. There are already five unknown parameters. Adding the Poisson jumps to Heston model we make this model more complex moreover, these jumps are responsible for unexpected news, and number of uknown parameters is already eight.

The application of these models to the Polish options market is due to the fact the Polish market is considered one of the largest in Eastern Europe; moreover, the most recent study of models for assessing the fair price of options was in 2010. In comparison with paper Kokoszcynski et al. (2010), other models for evaluating call options that were applied to daily data were demonstrated in this paper. Moreover, Kokoszcynski et al. (2010) evaluated 3 models (Black-Scholes-Merton variations) and applying them to the futures contract where WIG20 index futures was a basis instrument. Based on the previous paper, Kokoszczynski, Sakowski, Slepczuk (2017) defined that for the Japanese market (Nikkei 225 index options) the same model was the best among the other 5 models which included the Heston model. The daily data of call options were obtained using the BeautifulSoup scraping library from the website stooq.com. Since the Polish market is a developing market, data cleaning was needed. Namely in this work, the method of data cleaning using the proportion of Bid and Ask prices and applying 3σ approach for filtering outliers are demonstrated. For the risk-free rate was chosen Wibor 3 months. The minimum number of options per day was selected 10. Heston model and Heston model with Poisson jumps per each day more often gave higher error than BSM model. Options with a higher volume of purchased contracts (7 and higher) were also investigated. Heston and Heston with Poisson jumps models were again worse than Black-Scholes model. But it was also noticed that, on average, all three models showed a higher error than on all data (with a contract volume of 3 and higher). The numerical methods that were used to evaluate the parameters of the Heston model and Heston with Poisson jumps model will be investigated more thoroughly because some parameters in some cases seemed unlikely. To evaluate models, the Quantlib library was chosen, for numerical optimization methods Scipy, which used Nonlinear least-squares. It was found that the Heston and Heston with jumps models are very sensitive to the choice of the initial parameters vector. It was also investigated on test data created manually with parameters for the Heston model and then applying the parameter calibration algorithm.

There are a lot of figures in this article that describe the data. Including the plot of Implied Volatility - Moneyness. Moneyness is the ratio of the strike price to the spot price. The figure 4 shows a nice volatility smile for the groups with TTM 7-30 and 120-333 days. And it means higher volatility for OTM and ITM options than ATM options. And this again confirms the fact that the assumptions in the Black Scholes model that volatility is constant over time and underlying asset returns have log-normal distributions are incorrect.

There was also a comparison of the data with the largest error and the data with the smallest error. The day with the smallest error has a higher correlation between average ask, bid prices and implied volatility for call options, a fewer number of options(in 2 times), higher percent of short term options (80%, 22 days to maturity).

This paper will be of interest to investors and brokers for the proper pricing of options. This study gives an understanding of what stage the options market is in Poland, demonstrates the use of models that work well in a developed market (USA). It is also worth noting that pricing options using the BSM model were often better than the Heston model and Heston with Poisson jumps model. For developed countries, the opposite is true. Heston model and Heston with jumps model are very sensitive for the initial value of parameters and also are highly nonlinear. Since the Polish market is a developing market thus the number of valid options after data cleaning per each day is small. And the complex model for this such of the market works worse than the Black-Scholes model for the majority of days.

For further work, there are still multiple things to focus on:

1. First of all, it is crucial to investigate the stability of the solution of the Heston model equation and how to decrease sensitivity to initial values of parameters. Because depending on the different initial conditions, the evaluation of some parameters is very different from each other. Also to confirm or reject the hypothesis that Black models work better than the Heston model for an emerging market, models should be tested on data from a similar options market (e.g. Hungary).

2. Secondly, it would be interesting to evaluate the put options and compare the results. In this article, only daily data are used and the parameters are evaluated daily. It would also be interesting to do parameter estimation on weekly data and using others numerical optimization methods. For

numerical optimization methods was used nonlinear least-squares method. There are many other optimization techniques, such as differential evolution for finding the global minimum.

3. At last, it might be of interest to an average agent who has no access to the risk-free rate borrowing or faces significant transaction costs to have the models estimated using somewhat relaxed assumptions.

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APPENDIX

Figure 7. Ask and Bid prices for options Volume > 2, %

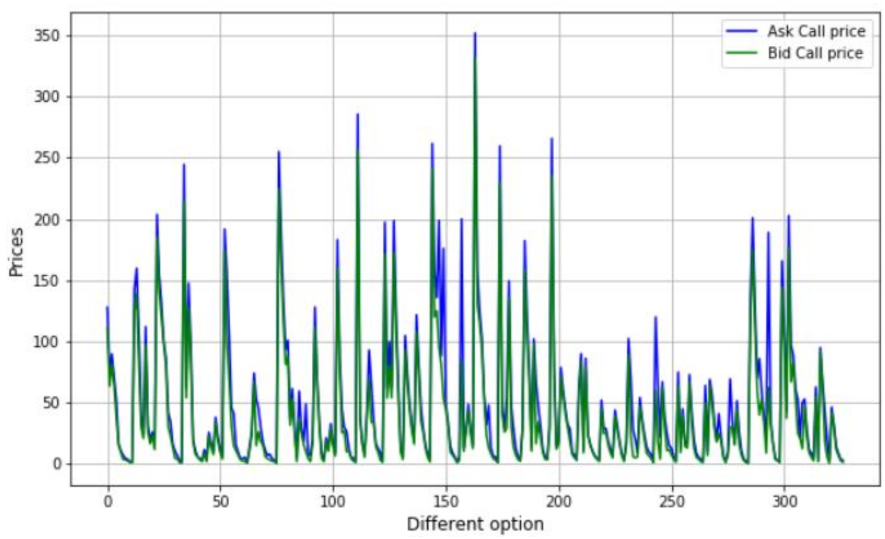

Figure 8. Wibor 3 month interest rate, %