FORECASTING GOVERNMENT BOND YIELD CURVE USING DYNAMIC NELSON-SIEGEL MODEL

by

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Kyiv School of Economics

Abstract

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Despite the presence of the variety of models that are used for yield curve forecasting, the application of the model that could be suitable for forecasting yield curve in Ukraine remains an open question. In this study, we investigate the forecasting performance of the Dynamic Nelson-Siegel (DNS) model by following the approach developed by Diebold and Li (2006). Our research showed that DNS is not superior to the random walk model in the context of long-term forecasting for the case of Ukraine. At the same time, the forecasting performance for shorter future horizons is revealed to be more promising. The findings also suggest that yield-curve factors are more potent in their effects on macroeconomic fundamentals than the effect of macroeconomic indicators on the future dynamics of yield-curve factors. This result provides a better characterization of the dynamic interactions between the macroeconomy and yield curve for policymakers when it comes to policy-related decisions.

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Chapter 1

INTRODUCTION

A yield curve that plots interest rates of bonds at different maturities plays an essential role for policymakers, investors, and other financial market participants. Being a transmitter of monetary policy (monetary policy instruments affect the entire term structure, which in turn determines the financing conditions of the economy), the yield curve is perceived to be a valuable source of information about future expectations. Despite numerous attempts to build accurate models that can be used in practice to forecast future movements of yield curves, there is no universal solution that can meet the wide variety of yield curve modeling demands. While macroeconomists try to build yield curve models for interest rate forecasting, the majority of investors are interested in bond pricing and use term structure modeling to assess risks and adjust portfolios at times of expected volatility or slowdown in the economy. This paper attempts to contribute to the existing literature by providing empirical evidence of the forecasting ability of the Dynamic Nelson-Siegel (DNS) model in the context of the term structure of interest rates in Ukraine. Moreover, the inclusion of macroeconomic factors provides a further step in the investigation of the possible interactions between the yield-curve factors and macroeconomic fundamentals in Ukraine.

There is abundant literature devoted to yield curve modeling. While Vasicek (1977), Cox et al. (1985), Hull and White (1990) are renowned for their contribution to the development of term-structure equilibrium models that focus on dynamic forecasting, Ho and Lee (1986) and Heath et al. (1992) were among the pioneers of no-arbitrage modeling that is revealed to be more accurate in determining cross-sectional properties of the term structure of interest rates. Since most of the models that were developed in the past century contain some limitations, more recent academic literature shifted in the direction of their resolution.

In this paper, we will apply the methodology developed by Nelson and Siegel (1987) and extended by Diebold and Li (2006). The rationale behind the choice of the model comes from the fact that the Dynamic Nelson-Siegel model, popularized by Diebold and Li (2006), combines the features of both no-arbitrage and equilibrium models, and, by using yields as inputs for the parametric curve equation, is capable of delivering more accurate out-of-sample forecasting results.

The purpose of this study is to assess the forecasting ability of the Dynamic Nelson-Siegel model and to investigate interactions of yields, real economic activity, and monetary policy in Ukraine. The questions of interest can be stated as follows: is forecasting performance of the Dynamic Nelson–Siegel model superior to the random walk model for both short-term and long-term horizons, and what are the links between yield-curve factors and macroeconomic fundamentals for the case of Ukraine?

The hypotheses that will be tested in this paper are: Dynamic Nelson-Siegel model has better forecasting ability as opposed to random walk; and there is bidirectional causality between yield-curve factors and macroeconomic fundamentals in Ukraine.

The data on discounted zero-coupon yields were retrieved from Bloomberg to estimate the parameters of the Nelson-Siegel model. The data about external macro-factors such as industrial production, target policy rates, and consumer price index changes were retrieved from the official website of the National Bank of Ukraine.

As for the implications of the research, the results can be of great importance for policymakers who can use the dynamic approach while setting up macroeconomic targets and policies. The results of the investigation of the macro-yields model may also provide a better characterization of the dynamic interactions between the macroeconomy and the yield curve, which can help policymakers to navigate policy-related decisions more effectively. At the same time, the forecasting approach evaluated in this study may be used as a powerful tool by investors and bond analysts to forecast yield curve movements with a high degree of precision.

The paper is organized as follows. In Chapter 2, a review of related literature is provided covering the evolution and recent evidence of term structure modeling and forecasting. Chapter 3 outlines the empirical methodology employed. In Chapter 4, a detailed description of the data utilized is provided. Chapter 5 sheds light on the estimation results. Chapter 6 contains conclusions, implications, and suggestions for future research.

Chapter 2

LITERATURE REVIEW

The idea of forecasting government bond yield curve is not a new one in the academic literature. Many scholars investigated the fitting performance of yield curves built based on the estimation of different models with various factors under analysis.

This literature review is divided into the following sections: (i) approaches to term structure modeling; (ii) Dynamic Nelson-Siegel model; (iii) macro-factor augmented Dynamic Nelson-Siegel model.

2.1. Approaches to Term Structure Modeling

In the second half of the last century, two approaches to term-structure modeling prevailed among academics and practitioners. Following no-arbitrage modeling of the yields, the term structure of interest rates is fitted at a particular point in time to eliminate arbitrage opportunities that may arise from the mispricing of assets caused by the changes in interest rates, which arbitrageurs can use to exercise abnormal profits (Diebold and Li 2006). The equilibrium approach to yields modeling involves using the so-called affine term-structure models, which assume that future dynamics of the term structure of interest rates is determined by some gradual developments in observed or unobserved factor (state variable) (Bolder 2001). Following the equilibrium approach, one models the pattern of dynamics for some instantaneous interest rate, which then can be used to derive yields at different maturities (Duffie and Kan 1995; Diebold and Li 2006). While no-arbitrage models are beneficial from the point of view of pricing derivatives and tend to be more accurate in determining cross-sectional properties of the term structure of interest rates, equilibrium models bring to the fore time-series properties and are evidenced to be more suitable when it comes to describing term-structure dynamics over specific periods.

The no-arbitrage approach was pioneered by Ho and Lee (1986), who used a binomial tree setting to model the arbitrage-free movement of the discounted bond yield curve (Hull and White 1993). The model developed by Ho and Lee (1986) was further extended by Heath et al. (1992), who proposed a new methodology to value interest rate sensitive contingent claims. In particular, the authors applied an arbitrage-free pricing model with a stochastic rate process to model the term structure of bonds and other contingent claims. The scholars were among the first to address the issue associated with inversion of the term structure and its relation to the market price of risk.

Vasicek (1977) was among the first who started to develop term-structure equilibrium models. Using a set of assumptions about spot interest rates and efficiency of the market, as well as an arbitrage argument about the expected rate of return on bonds, the scholar managed to derive an analytical solution to the bond pricing formula. The model proposed by Vasicek (1977) was of great significance among academics since it enabled to capture the mean-reverting process of short-term interest rates, a characteristic that is revealed to be distinctive for interest rates as financial prices.

In another study, Cox et al. (1985) provided an extension to the model designed by Vasicek (1977) by using an equilibrium intertemporal asset pricing framework to derive bond prices and track the relationship between expected future spot and past spot rates. The authors incorporated anticipations, the timing of consumption preferences, risk aversion, and investment alternatives as factors affecting the term structure of bond prices.

Hull and White (1990) extended both the model designed by Vasicek (1977) and the one proposed by Cox et al. (1985). In their study, Hull and White (1990) attempted to deduce the process followed by the short-term interest rates in the previous models from the term structure of interest rates and volatilities. A pattern derived by the scholars illustrated how one could use either Vasicek (1977) or Cox et al. (1985) models to value not only bonds but also any other interest-rate contingent claims.

Though equilibrium modeling is commonly used for forecasting purposes as opposed to no-arbitrage models that mainly focus on fitting performance of yield curves, these models are also subject to limitations that often result in poor forecasting performance. For example, in his study on term premia and interest rate forecasts, Duffee (2002) revealed that one of the failures of affine models in producing better forecasts than random walks was the dependence of risk compensation on the interest rate volatility. Inherent linearity and poor cross-sectional properties of equilibrium models are also among the features that limit their use as the tools for term structure forecasting (Bolder 2001).

The model that I will focus on in this paper is developed by Diebold and Li (2006) who went beyond no-arbitrage and equilibrium models and used the Nelson-Siegel framework to forecast the yield curve by forecasting the factors of the curve.

2.2. Dynamic Nelson–Siegel Model

Nelson and Siegel (1987) developed a simple, parsimonious model that was capable of capturing critical features of yield-maturity relations and had valuable practical applications for both academics and practitioners. In the paper, the authors used the yield-maturities equation to impose structure on factors that define the shape of the curve. Using the data on T-Bonds from the Federal Reserve Bank of New York for the period between 1981 and 1983, Nelson and Siegel (1987) managed to show that their parsimonious representation was accurate enough in characterizing the shape of the term structure of the yield curve. The study conducted by Nelson and Siegel (1987) showed that modeled fitted curves were good predictors of the long-term United States T-bonds prices.

Since its introduction, the model designed by Nelson and Siegel (1987) has become widely used among scholars and financial market participants. Different variations of the model also appeared extending and modifying the standard model.

The core paper that this thesis will follow is by Diebold and Li (2006). In their study, Diebold and Li (2006) provided a dynamic extension of the framework developed by Nelson and Siegel (1987) by focusing on the out-of-sample forecasting of yields. The scholars used Fama–Bliss unsmoothed yields as inputs for their parametric curve equation to assess the fit of the three-factor model (Diebold and Li 2006). The authors also estimated autoregressive models for three dynamically evolving time-varying parameters, namely level, slope, and curvature of the yield curve to forecast the term structure of government bond yields. Compared to standard benchmarks, the empirical results of Diebold and Li (2006) are revealed to be more accurate (especially for long-term horizons), which points to the good fitting performance of the forecasted yield curve.

2.3. Macro-Factor Augmented Dynamic Nelson–Siegel Model

During the past decade, many scholars attempted to evaluate the links between the latent factors of the DNS model (level, slope, and curvature) and macroeconomic fundamentals such as inflation, benchmark interest rates, and real economic activity. As a result, a separate class of DNS models, known as macro-factor augmented Dynamic Nelson–Siegel models, started to gain popularity among academia.

One of such extensions of the original DNS framework can be found in the paper by Diebold, Rudebusch, and Aruoba (2006). The authors used the data on the U.S. Treasury yields and statistics on manufacturing capacity utilization, the federal funds rate, and annual price inflation to assess the presence of bidirectional causality between macro factors and yield factors such as level, slope, and curvature. The estimation results showed that the impact of macroeconomic fundamentals on yield-curve factors is more pronounced compared to the shocks coming from the yield factors. The analysis of impulse response functions illustrated the presence of a strong and persistent (positive) relationship between the slope factor and the identified macro variables (Diebold, Rudebusch, and Aruoba 2006). Moreover, the scholars found that an increase in inflation tends to raise the long end of the yield curve (level factor), which is also consistent with the expectations about long-term inflation levels (Diebold, Rudebusch, and Aruoba 2006).

In the study devoted to the analysis of the United Kingdom's monetary policy interaction with the real economy, Levant and Ma (2015) used the end of the month zero-coupon government yields data that spanned the period between January 1985 and December 2006 to investigate interactions between the term structure of interest rates and external macroeconomic factors such as industrial production, policy target rate, and inflation expectations. The scholars found that the yield curve's level and slope were related to inflation expectations and monetary policy, whereas the curvature factor was found to be more strongly related to economic activity in the UK. In another study, Nyholm (2015) suggested using a Rotated Dynamic Nelson-Siegel (RDNS) model to track the interaction of macro-financial factors with the term structure of interest rates. The author found that a new parametrization of the Dynamic Nelson-Siegel model that term premia generated by the model are economically meaningful, and macroeconomic variables are statistically significant but only for a certain period.

This thesis will contribute to the existing literature by providing empirical evidence of the forecasting ability of the DNS model in the context of Ukrainian government bond yield curve. Moreover, the inclusion of macro variables will enable to evaluate possible interactions between the term structure of interest rates and macroeconomic fundamentals in Ukraine.

C h a p t e r   3

METHODOLOGY

3.1. A Factor Model Representation

Diebold and Li (2006) factorization of the DNS yield curve model (a state-space framework) is a methodology that is commonly used by scholars to estimate the latent DNS yield curve factors (level, slope, and curvature) and all model parameters including the loading parameter λ (the exponential decay rate). The equation below represents the cross-section of yields at any point in time *(y_t)* and different maturities *(τ)* with time-varying parameters as interpreted by Diebold and Li (2006):

$$y_t(\tau) = \beta_{0t} + \beta_{1t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \quad (1)$$

where β_{0t} , β_{1t} , β_{2t} are time-varying level (long-term factor), slope (short-term factor), and curvature (medium-term factor). The parameter λ_t defines the exponential decay rate and also indicates the point at which the curvature factor (β_2) reaches its maximum value. As indicated by the authors, small values of the parameter govern the slow pace of decay (thus fit the curve better at longer maturities), whereas large values produce fast decay (fit the curve better at shorter maturities) (Diebold and Li 2006).

Following the standard approach developed by Nelson and Siegel (1987), the parameters of the model for each period *t* can be estimated using ordinary least squares model, which is a convenient and relatively simple way of estimating parameters of the yield curve compared to nonlinear optimizations. In the context of the Nelson-Siegel model, however, the parameter of exponential decay λ should be fixed at the level at which the curvature factor reaches its maximum. In our model, λ equals to 0.0609. After applying standard OLS procedure to the raw yields, one can obtain a time-series of estimates of the parameters and a set of residuals, the values of which are indicative of the fitting performance of the model.

3.2. Yield Curve Modeling and Forecasting

In this paper, a series of univariate AR (1) processes is applied to model and forecast yield-curve factors both in-sample and out-of-sample. The following is the AR (1) specification that is used to forecast yields:

$$\hat{y}_{t+h/t}(\tau) = \hat{\beta}_{0,t+h/t} + \hat{\beta}_{1,t+h/t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \hat{\beta}_{2,t+h/t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \quad (2)$$

where $\hat{\beta}_{1,t+h/t} = \hat{c}_i + \hat{y}_i\hat{\beta}_{it}$, $i = 1,2,3$, h is period forecast of the yield with maturity τ at time t, and and $\hat{c}_i$ and $\hat{y}_i$ are obtained by regressing $\hat{\beta}_{it}$ on an intercept and $\hat{\beta}_{i,t-h}$ (Diebold and Li 2006).

Following Diebold, Rudebusch, and Aruoba (2006), VAR (1) model is also an appropriate devise to capture the dynamic behavior of the yield curve factors. If the dynamic movements of $\beta_{0t}, \beta_{1t}, \beta_{2t}$ follow a vector autoregressive process of first order, then the model forms a state-space system, which has the following form:

$$\begin{pmatrix} \beta_{0t} - \mu_{\beta_0} \\ \beta_{1t} - \mu_{\beta_1} \\ \beta_{2t} - \mu_{\beta_2} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \beta_{0t-1} - \mu_{\beta_0} \\ \beta_{1t-1} - \mu_{\beta_1} \\ \beta_{2t-1} - \mu_{\beta_2} \end{pmatrix} + \begin{pmatrix} \eta_t \beta_0 \\ \eta_t \beta_1 \\ \eta_t \beta_2 \end{pmatrix}, \tag{3}$$

t=1,…., T. The measurement equation that links yields and yield-curve factors is as following:

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \dots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_2} \\ \dots & \dots & \dots \\ 1 & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_N} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \dots \\ \varepsilon_t(\tau_N) \end{pmatrix}, \tag{4}$$

which can be equivalently written in matrix form as

$$(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t, \tag{5}$$

$$y_t = \Delta f_t - \varepsilon_t. \tag{6}$$

In the context of optimization procedure, Diebold and Li (2006) require that the white noise transition and measurement disturbances to be orthogonal with respect to each other and to the initial state:

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right] \tag{7}$$

$$\mathrm{E}(f_0 \eta_t') = 0, \tag{8}$$

$$\mathrm{E}(f_0 \varepsilon_t') = 0. \tag{9}$$

Furthermore, the approach applied by Diebold and Li (2006) requires some additional assumptions formulated with respect to the *Q* and *H* matrices. In particular, it is assumed that covariance matrix *Q* is non-diagonal (state equation factor disturbances η_t are correlated), and covariance matrix *H* is diagonal (deviations of observed yields at various maturities are uncorrelated).

Though multivariate VAR (1) specification is considered to be inferior to the AR in the context of forecasting performance due to the potential for in-sample overfitting (Diebold and Li 2006), in this paper, the former is applied for two purposes. First, the VAR (1) specification is used to forecast yields in order to compare forecasting results with the ones generated by the AR (1) model. Second, following the methodology utilized by Diebold, Rudebusch, and Aruoba (2006), we use VAR (1) specification in the context of investigating the interactions between the factors of the Nelson-Siegel model and external macroeconomic factors.

The forecasting approach consists of the following steps. First, in-sample forecasting is performed by estimating the AR (1) models using an expanding window starting with 32 months. The predictions are made for 19 months for all maturities, as specified in Chapter 4. As in the case of in-sample forecasting, out-of-sample forecasting also includes an expanding window. This means that the process is started with a window of a certain size (window size R=32 months, as the in-sample data, ending at month t (starting at t-R+1)), and one goes step by step over time including an additional data point every time. The number of increments between successive rolling windows is 1 period, which means that the first rolling window includes observations for period 1 through R, the second one contains data for period 2 through R + 1, and so on.

For comparison, along with the AR(1) model, the VAR(1) and RW models are computed on the same subset of data. Since the models are based on parameters, the latter are forecasted for each time horizon (1,6,12). The rates in which we are interested at this stage are not all the maturities, but only some of them (those that are reflected in Chapter 5). Once parameters are forecasted, as per Diebold and Li (2006), one may translate them in forecasts of the rates following equation (1).

Further, the computations of the errors made are performed. Since there is one set of rates per horizon, at every time step one obtains a matrix of errors of the shape n x m, where n indicates horizons and m denotes the rates. The values of root mean-square-error (RMSE) are examined to assess the out-of-sample forecasting performance of the model. The formula for RMSE is as following:

$$RMSE_{model}(\tau) = \sqrt{\frac{1}{T-t_0}\sum_{t=t_0}^{T}(\hat{y}_t(\tau) - y_t(\tau))^2}\ , \tag{10}$$

where $\hat{y}_t(\tau)$ are the yields forecasted by the model, and $y_t(\tau)$ are the observed yields. The interval $[t_0, T]$ is indicative of the time horizons for which we make forecasts.

As a benchmark model, similar to the original paper, random walk model is used to give a base standard on predictive accuracy for each model (Diebold and Li 2006). The following is the equation for the standard random walk model:

$$\hat{y}_{t+h/t}(\tau) = y_t(\tau), \tag{11}$$

where $\hat{y}_{t+h/t}(\tau)$ is the forecast of the yield, and h is the horizon of the forecast.

3.3. The Yields-Macro Model Specification

Since both the Nelson-Siegel latent factors and macro-factors are present in a VAR framework, it provides the opportunity to conduct impulse response analysis of interactions between the factors of the Nelson-Siegel model and external macroeconomic factors such as the target policy rate (IR), industrial production (IKSO), and annual inflation rate (CPI). The inclusion of macroeconomic variables in the model requires an extension of the yields-only model with equations (5-7) being replaced with the following ones:

$$(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t, \tag{5'}$$

$$y_t = \Delta f_t - \varepsilon_t, \tag{6'}$$

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right], \tag{7'}$$

where $f_t{}' = (\beta_{0t}, \beta_{1t}, \beta_{2t}, CPI_t, IR_t, IKSO_t)$. It is worth noting that the inclusion of three macroeconomic variables to the state variables increases the dimensions of A, μ, η_t and Q. In particular, Δ is now of the dimension N x 6 with the three rightmost columns containing zero values. Such a form ensures that the loading of the yields is carried out on the yield curve factors only, not macroeconomic variables (Diebold and Li 2006).

When it comes to assessing the dynamics of the yields-macro system, impulse response functions with 95 percent confidence intervals are used. To evaluate the links between macroeconomic variables and yield factors, and vice versa, four groups of impulse responses are taken into account, namely macro responses to yield curve shocks (and macro responses to macro shocks), and yield curve responses to macro shocks (and respective yield curve responses to yield curve shocks). As a metric for analyzing yield curve and macro interactions, variance decompositions for both the yields-only and the yields-macro models are provided at different forecast horizons.

C h a p t e r 4

DATA DESCRIPTION

4.1 Yield-Only Model

To estimate the macro-factor augmented Dynamic Nelson-Siegel model, the monthly data on zero-coupon yields were retrieved from the Bloomberg for the period between Jan. 2015 and Nov. 2019. The zero-coupon yield curves of domestic sovereign bonds of Ukraine in UAH are illustrated in Figure 1. The continuously compounded spot yields are calculated by the National Bank of Ukraine using the Nelson-Siegel parametric model based on data on transactions with domestic sovereign bonds of Ukraine on the stock exchange and OTC markets of Ukraine over the last five business days. The National Bank of Ukraine uses these zero-coupon yield curves to estimate the fair value of Ukraine's domestic sovereign bonds.

The maturities under investigation are the 3, 6, 9, 12, 24, 36, 48, 60, 84, 96, and 120-month. Though it is not a strict requirement of the model to have fixed maturities, Diebold and Li (2006) suggest the latter would significantly simplify the forecasting process. From the graph, one can observe that the yields are smoothed as opposed to the raw data, and a large degree of temporal variation in the curve's level is present as we go from the beginning of the sample period.

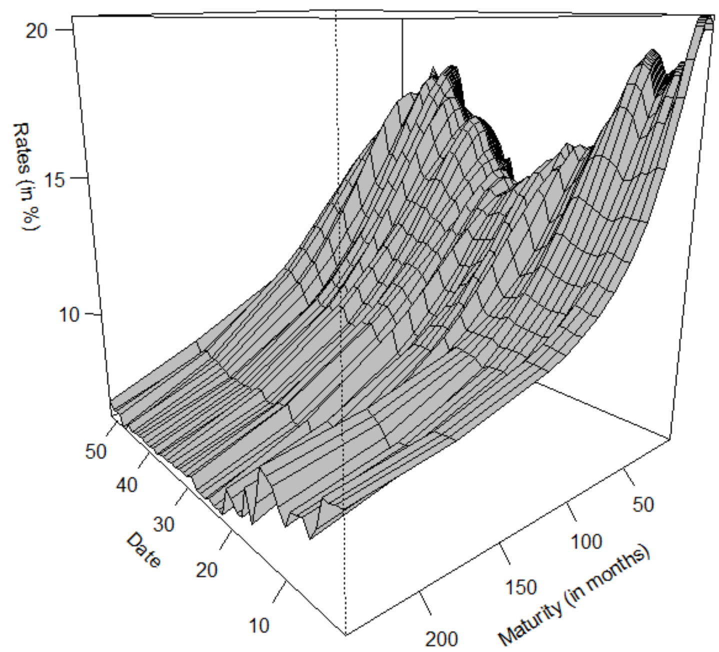

Figure 1. Zero-coupon yield curves of domestic sovereign bonds of Ukraine in UAH for different maturities (the sample period is 2015:08 – 2019:11, months)

The descriptive statistics for the monthly yields at different maturities are presented in Table 1. From the table, one can observe that the typical yield curve is downward sloping as we move from shorter maturities to longer ones. Also, it is worth noticing that the rates with longer maturities are less volatile than those with shorter maturities.

Table 1. Descriptive statistics for monthly yields at different maturities (the sample period is 2015:08 – 2019:11)

Maturity (months)	Mean	Std. dev.	Minimum	Maximum
3	16.00	2.29	10.58	19.99
6	16.15	2.18	10.37	20.39
12	16.22	1.96	10.00	20.10
24	15.79	1.56	9.55	17.99
36	14.99	1.28	9.37	17.04
48	14.09	1.09	9.34	16.20
60	13.20	0.93	9.37	15.09
72	12.36	0.77	9.40	13.92
84	11.61	0.63	9.40	12.97
96	10.94	0.52	9.37	12.09
120	9.83	0.46	9.02	10.72

Since the yields that we use as inputs in our model are derived from the parameters of the Nelson-Siegel model estimated by the National Bank of Ukraine, one could have used the parameters that are reported by the NBU as inputs for forecasting. However, to ensure the validity of the model used and the assumptions that accompanied the process of estimating the parameters of the Nelson-Siegel model, we reproduced the parameters using package `YieldCurve` in R software. Rates and maturities were used as arguments to estimate coefficients of the Nelson-Siegel model. In such a way, reproducing the parameters helped in 'double-checking' the quality of inputs.

In Figure 2, we plot observed yield curve constructed using the parameters estimated by the NBU and fitted yield curve built based on the parameters estimated by our model in R software for some selected date. One can observe that our curve is concurrent with the one constructed using the parameters estimated by the NBU. The residual plot depicted in Figure 3 also indicates a good fit of the reproduced estimators.

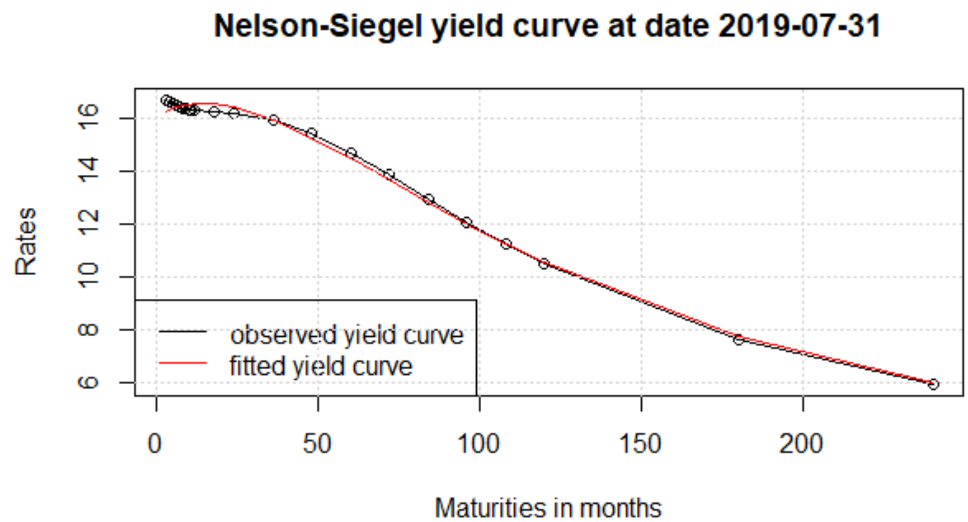

Figure 2. Selected fitted (model-based) yield curve

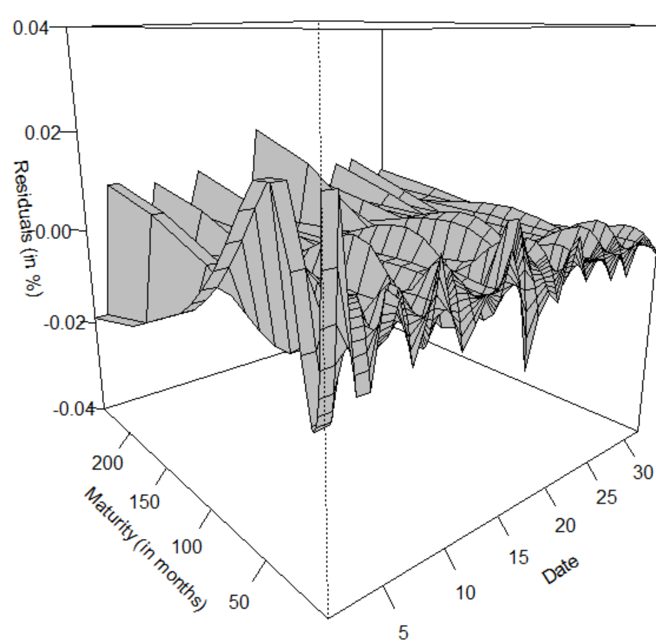

Figure 3. Yield curve residuals from Nelson–Siegel yield curves fitted month-by-month (2015:08 – 2019:11)

Descriptive statistics for the estimated factors are presented in Table 2. The values of the mean, standard deviation, minimum and maximum, as well as the autocorrelation for estimated parameters (factors) of the model at displacements of 1 and 12 months are reported in Table 2. The data from the autocorrelations of the three factors confirm the stylized fact, as noted by Diebold and Li (2006) about stronger persistence of level factor (β_0) compared to slope and curvature factors (β_1 and β_2 respectively). However, one can also observe that such persistence tends to fade away as the number of months increases (already at a displacement of 12 months in our case).

Table 2. Descriptive statistics, estimated factors (percentage-based data)

Parameter	Mean	Std. Dev.	Min	Max	MAE	RMSE	ϱ(1)	ϱ(12)
β_0	4.899	1.618	2.418	7.405	2.213	5.151	0.763	-0.063
β_1	10.350	1.197	8.138	12.354	3.217	10.416	0.673	-0.329
β_2	17.988	2.148	13.674	20.957	4.241	18.112	0.629	-0.373

4.2 Macroeconomic Variables

As for the macro-factor augmented Nelson-Siegel model, the data on production, monetary policy-related interest rate, as well as changes in consumer price index (CPI), are used as the macro risk factors that are typically found in the related literature (Diebold and Li (2006); Levant and Ma 2016; Nyholm 2015). Data on the three macro-factors come from the NBU database, and their descriptive statistics are reported in Table 3.

Table 3. Descriptive statistics for macroeconomic variables (the sample period is 2015:08 – 2019:11)

Variable	Mean	Std. dev.	Minimum	Maximum
IR	16.86	3.15	12.50	27.00
CPI	15.71	12.23	5.10	52.80
IKSO	2.21	4.40	-8.09	17.95

The Index of Key Sectors Output (IKSO) was used as a proxy for the industrial production variable. The IKSO is the index of economic activity (calculated by the NBU) that covers activities such as agriculture, industrial production (mining, processing, the supply of electricity, gas, etc.), construction, trade turnover (wholesale and retail) and transport. Following the methodology of the NBU, the IKSO is a good GDP predictor, given that it covers more than 50% of the economy. The graphical representation of the changes in IKSO over the analyzed period is showed in Figure 4.

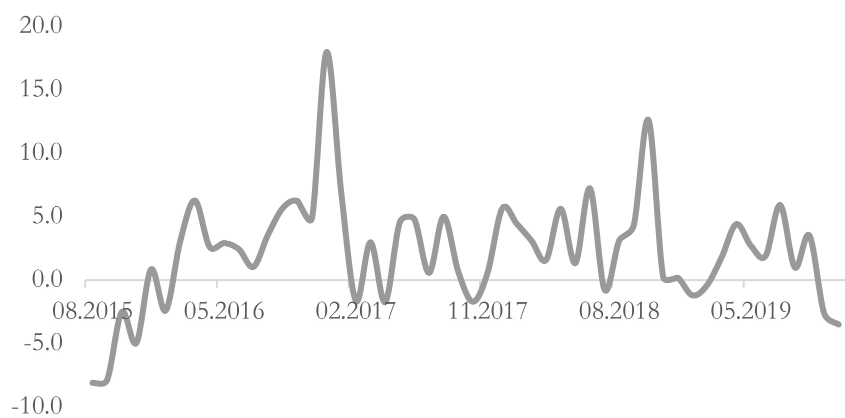

Figure 4. IKSO, to corresponding month of the previous year, % (the sample period is 2015:08 – 2019:11, months)

The second macro variable that is incorporated in our model is Consumer Price Index (CPI), which is the 12-month percent change in the prices paid by consumers for a market basket of consumer goods and services. As for the policy rate, the target interest rate set by the NBU is used as another macroeconomic variable.

Figure 5. Consumer price index (to corresponding month of the previous year, %) (the sample period is 2015:08 – 2019:11, months)

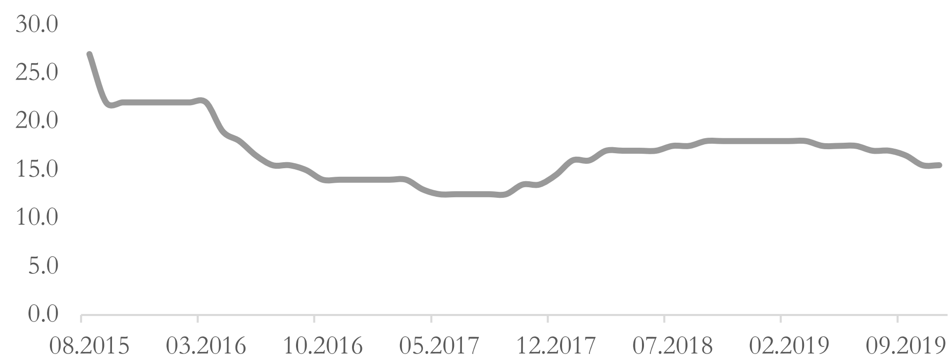

Figure 6. Target interest rate dynamics, % (the sample period is 2015:08 – 2019:11, months)

Chapter 5

RESULTS

5.1 In-Sample Forecasting Performance

The results that are present below include the estimation of the Dynamic Nelson-Siegel model using zero-coupon yields at different maturities (the sample period is 2015:08 – 2019:11). All the estimations were performed using R software with the YieldCurve library being the one that is used to model and estimate the yield curves. These results include the estimation of factor loadings of the model, in-sample and out-of-sample forecasting, as well as yields-macro interactions.

Figure 7 shows the computed autocorrelations of parameters and residuals after estimating the AR (1). The AR (1) models were estimated using an expanding window starting with 32 months (which accounts for around 60% of the data sample). The predictions are made for 19 months for all maturities. The estimates of the beta coefficients show highly persistent own dynamics with own-lag coefficients of 0.77, 0.68, and 0.63 for $\beta_{0t}, \beta_{1t}, \beta_{2t}$ respectively. From the graphs, it can be inferred that the model in-sample performs well since the autocorrelations are relatively small. When compared to the results of the core paper by Diebold and Li (2006), it can be seen that the autocorrelation plots are aligned, which also points to the well in-sample fit.

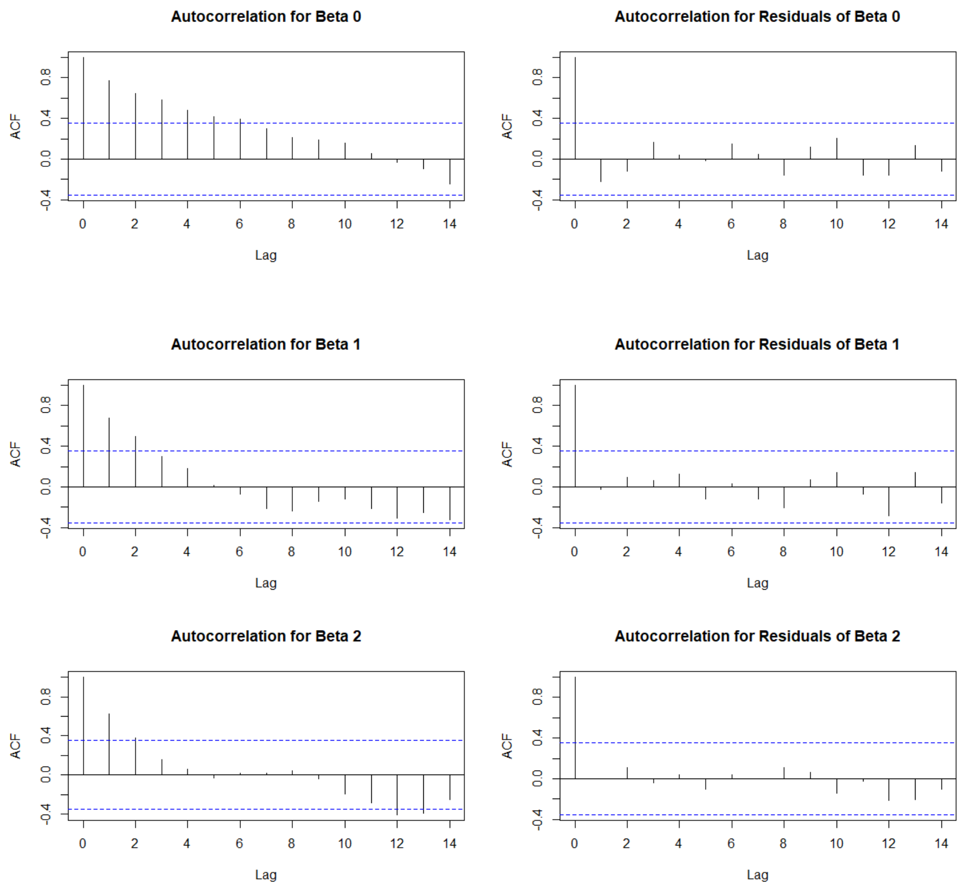

Figure 7. Autocorrelations and residual autocorrelations of level, slope and curvature factors.

5.2 Out-of-Sample Forecasting Performance

As for the out-of-sample forecasting, it was carried out using AR (1), VAR (1), and random walk models for three different forecast horizons h: 1, 6, and 12 months. It is also worth mentioning that the curve fitting and forecasting of the yields are performed using R software.

Tables 4–6 present the root mean square errors (RMSE) of the 1-, 6-, and 12-month forecasts. The bold values in the tables indicate the smallest RMSE in the context of the alternative models. From the tables, RMSE comparison at various maturities reveals the results that contradict the outcomes found by Diebold and Li (2006). In particular, RMSE calculated for random walk model tend to be smaller as we increase our forecasting horizon. Notably, at the rates with longer maturities, random walk appears to be the best model for yield curve forecasting. Matters improve, however, in our case, when we consider shorter forecasting horizons such as 1-month-ahead forecasting reported in Table 4. In the table, RMSEs for VAR (1) model are smaller compared to the random walk model, though only for the rates with maturities up to 36 months.

Table 4. Out-of-sample 1-month-ahead forecasting results (N = 19 individual forecasts)

Model Name	Maturity (τ)	Mean	Std. Dev.	RMSE	$\rho(1)$	$\rho(12)$
Nelson–Siegel with AR(1) factor dynamics	3	-0.123	0.917	0.901	0.661	-0.356
	6	-0.009	0.930	0.905	0.770	-0.408
	12	-0.171	0.934	0.925	0.729	-0.391
	36	-2.246	0.708	2.349	0.225	-0.020
	60	-3.396	0.851	3.496	0.570	-0.120
	120	-3.159	1.017	3.311	0.740	-0.216
Nelson–Siegel with VAR(1) factor dynamics	3	0.176	0.854	**0.849**	0.522	-0.264
	6	0.234	0.823	**0.835**	0.694	-0.382
	12	-0.010	0.810	**0.789**	0.699	-0.411
	36	-2.201	0.731	**2.313**	0.273	-0.027
	60	-3.354	0.988	3.490	0.641	-0.161
	120	-3.088	1.230	3.312	0.767	-0.243
Random walk	3	2.613	1.221	2.871	0.730	-0.221
	6	3.136	1.234	3.358	0.767	-0.242
	12	3.612	1.227	3.804	0.759	-0.262
	36	2.743	0.935	2.890	0.509	-0.163
	60	1.954	0.719	**2.076**	0.477	-0.107
	120	2.404	0.368	**2.430**	0.619	-0.104

Table 5. Out-of-sample 6-month-ahead forecasting results (N = 19 individual forecasts)

Model Name	Maturity (τ)	Mean	Std. Dev.	RMSE	$\rho(1)$	$\rho(12)$
Nelson–Siegel with AR(1) factor dynamics	3	-0.870	1.902	2.029	0.765	-0.232
	6	-0.519	1.882	1.886	0.788	-0.231
	12	-0.321	1.758	1.724	0.785	-0.223
	36	-1.731	1.095	2.027	0.557	-0.080
	60	-2.588	0.756	2.689	0.497	0.123
	120	-1.989	0.479	2.042	0.723	-0.070
Nelson–Siegel with VAR(1) factor dynamics	3	0.346	2.450	2.386	0.720	-0.249
	6	0.532	2.339	2.316	0.755	-0.248
	12	0.483	2.078	2.060	0.776	-0.245
	36	-1.321	1.095	**1.691**	0.573	-0.157
	60	-2.241	0.570	2.307	0.316	0.065
	120	-1.622	0.451	1.679	0.225	-0.186
Random walk	3	1.276	1.428	**1.877**	0.728	-0.218
	6	1.120	1.465	**1.802**	0.760	-0.224
	12	0.609	1.452	**1.526**	0.752	-0.224
	36	-1.579	1.068	1.884	0.506	-0.094
	60	-2.233	0.798	**2.362**	0.455	0.065
	120	-1.071	0.373	**1.130**	0.725	0.061

Though our results contradict the evidence found by Diebold and Li (2006), many scholars also reached similar outcomes that do not confirm the superior forecasting performance of the Nelson-Siegel model. According to Duffee (2002), random walk forecasts dominate the ones produced by affine models similar to DNS due to the dependence of risk compensation on the interest rate volatility. In the paper written by Molenaars, Reinerink, and Hemminga (2013), the scholars also found that DNS is not capable of delivering convincing forecasting results when compared to the random walk model. Their conclusion was backed by the hypothesis about "the short-lived success of the model." In other words, the authors claimed that the model is valid for only a limited period of time (Molenaars, Reinerink, and Hemminga 2013). The evidence that supports this hypothesis can be found in the papers by Mönch (2008), Reschenhofer and Stark (2019), who point to the fact that robust forecasting performance of the Dynamic Nelson-Siegel model popularized by Diebold and Li (2006) may come from the choice of forecast period that scholars utilized for forecasting. Similar to our case, this rationale can partly explain the matter of fact that at some points in time the model outperforms RW, whereas at other periods it underperforms (Mönch 2008).

Table 6. Out-of-sample 12-month-ahead forecasting results (N = 19 individual forecasts)

Model Name	Maturity (τ)	Mean	Std. Dev.	RMSE	$\rho(1)$
Nelson–Siegel with AR(1) factor dynamics	3	-0.560	1.542	**1.548**	0.575
	6	-0.022	1.449	1.355	0.602
	12	0.355	1.330	**1.294**	0.606
	36	-1.330	1.314	1.811	0.450
	60	-2.437	0.990	2.607	0.417
	120	-1.812	0.291	1.832	0.485
Nelson–Siegel with VAR(1) factor dynamics	3	-0.124	2.174	2.037	0.411
	6	0.329	2.015	1.914	0.437
	12	0.578	1.813	1.792	0.451
	36	-1.316	1.683	2.051	0.366
	60	-2.461	1.331	2.758	0.319
	120	-1.832	0.490	1.888	0.262
Random walk	3	-0.781	1.480	1.589	0.573
	6	-0.057	1.408	**1.318**	0.603
	12	0.636	1.324	1.392	0.611
	36	-0.274	1.381	**1.321**	0.470
	60	-1.014	1.079	**1.431**	0.448
	120	-0.035	0.324	**0.305**	0.547

The forecast defects may, in fact, come from a variety of sources, some of which could be avoided. First and foremost, one should point to the fact that the yields that were used as inputs in the model are retrieved from Bloomberg, which uses the data from the NBU. Since the National Bank of Ukraine uses the Nelson-Siegel model to build yield curves, we fail to incorporate unsmoothed zero-coupon discounted yields as inputs for our model. Also, as noted by Diebold and Li (2006), pricing errors, which arise due to some bonds' illiquidity, may affect the results. Adding additional variables addressing this issue could potentially improve the outcome.

To check the hypothesis about potential defects related to the data inputs, following similar methodology, we also constructed forecasting tables for the yield curves based on the Canadian zero-coupon yields (the yields were not constructed using the Nelson-Siegel model). Tables 11-13 in Appendices show that out-of-sample forecasting results are significantly improved in absolute and relative terms for AR (1) and VAR (1) model for all forecasting horizons. It is worth noticing that the results of the Nelson-Siegel VAR (1) factor dynamics model also outperform the random walk model for 12-month-ahead forecasting (RMSE are lower). Though this model specification is believed to be inferior to AR (1) processes (Diebold and Li 2006), in our example, it is revealed to be superior to both AR (1) and RW models.

5.3 Macro-Yield Model Estimation Results

In addition to the above analysis of the forecasting performance of selected models, this paper also investigates the interactions between macroeconomic fundamentals such as key policy rate, inflation, and real economic activity and yield-curve factors. Following the methodology described in Chapter 3, we managed to produce VAR estimates of the yields-macro model, as well as to track interactions between the macro variables and yield-curve variables using impulse responses and variance decompositions of the yields-macro model.

Similar to the approach by Diebold, Rudebusch, and Aruoba (2006), Table 7 reports the estimates of the parameters of the yields-macro model, where each row presents coefficients from the transition equation for the respective state variable. In the table, bold entries denote parameter estimates significant at the 5 percent level, and standard errors are reported in parentheses (Diebold, Rudebusch, and Aruoba 2006). One can observe that while many off-diagonal elements seem to be insignificant, the coefficients of our interest appear to be jointly significant (except for the IKSO variable).

Table 7. Yields-macro model parameter estimates VAR Parameters

Parameters	β_{0t-1}	β_{1t-1}	β_{2t-1}	CPI_{t-1}	IR_{t-1}	$IKSO_{t-1}$	μ
β_{0t}	**0.55** (0.25)	-0.23 (0.22)	-0.02 (0.06)	0.00 (0.02)	0.21 (0.17)	0.03 (0.04)	0.89 (2.18)
β_{1t}	0.17 (0.31)	**1.06** (0.27)	0.01 (0.07)	0.01 (0.03)	-0.12 (0.21)	0.00 (0.04)	0.31 (2.70)
β_{2t}	0.35 (0.71)	0.18 (0.61)	**0.49** (0.16)	-0.04 (0.06)	-0.22 (0.48)	0.06 (0.11)	9.52 (6.08)
CPI_t	**1.10** (0.51)	0.78 (0.43)	**0.32** (0.12)	**0.92** (0.04)	**-0.89** (0.34)	-0.09 (0.07)	-3.93 (4.34)
IR_t	**0.38** (0.15)	**0.51** (0.13)	**-0.07** (0.03)	**0.05** (0.01)	**0.43** (0.10)	**0.05** (0.02)	2.00 (1.30)
$IKSO_t$	0.44 (0.93)	-0.00 (0.80)	-0.23 (0.22)	-0.08 (0.08)	-0.25 (0.63)	0.20 (0.14)	10.34 (7.99)

TABLE 7 – Continued (estimated Q matrix)

Parameters	β_{0t}	β_{1t}	β_{2t}	CPI_t	IR_t	$IKSO_t$
β_{0t}	0.01	-0.01	0	0	0	0
β_{1t}	-0.01	0.02	-0.01	0	0	0
β_{2t}	0	-0.01	0.08	0	-0.01	0.04
CPI_t	0	0	0	0.04	0	0
IR_t	0	0	-0.01	0	0	0
$IKSO_t$	0	0	0.04	0	0	0.15

5.4 Impulse Response Functions and Variance Decompositions

After conducting an impulse response analysis, we obtained mixed results. In particular, while some of the interactions are in line with macroeconomic theory and outcomes of Diebold, Rudebusch, and Aruoba (2006), some responses such as one of the yield-curve factors to the shock in IR remained muted. Table 8 provides a brief comparison of our results with Diebold, Rudebusch, and Aruoba (2006) and Figure 8 illustrates the responses of macro factors to shocks in yield-curve factors and vice versa.

Table 8. Comparison of impulse responses of our model and Diebold, Rudebusch, and Aruoba (2006) (DB = Diebold)

Impulses→ Responses ↓	β_0	β_1	β_2	IR	CPI	IKSO
β_0	β_0 reacts to its own shocks as in DB, but less persistently.	In line with DB, negligible, even though in DB we see some relationship, total flat in our case.	More or less in line with DB, where it is less negligible.	Different from DB, negligible.	Slightly different from DB, it is negligible.	Different from DB, it is negligible.
β_1	Not as DB: strong negative reaction to shocks in beta1.	In line with DB, there is a light positive response, decreases over time.	Response in line with DB, negligible.	Different from DB, negligible.	Response in line with DB, negligible.	Slightly different from DB, it is negligible.
β_2	Response in line with DB, negligible.	Different from DB: strong negative response, it fades away over time.	In line with DB, strong response in short term shocks, decreasing over time.	More or less in line, here it is negligible, slightly stronger in DB.	Response in line with DB, negligible.	Slightly different from DB, it is negligible.
IR	Response in line with DB, negligible.	In line with DB, initial timid response which decreases over time.	Response in line with DB, negligible.	Much lighter response than DB.	Response in line with DB, negligible.	Different from DB; it is negligible.
CPI	In line with DB, even stronger response.	Different from DB: strong negative, persistent response.	Different from DB: strong response, which remains persistent.	Different direction of the response from DB, as it is negative and less persistent.	Response in line with DB, strong and persistent.	Different from DB; it is negligible.
IKSO	Response in line with DB, negligible, but decreasing over time (contrarily to DB).	More or less in line with DB, quite negligible, in DB slightly more persistent.	Different from DB: positive response turning negative and neutralizing over time.	Stronger response in short term shocks, different from DB.	Different from DB, it is much more negligible.	In line with DB, short term strong response then absorbed.

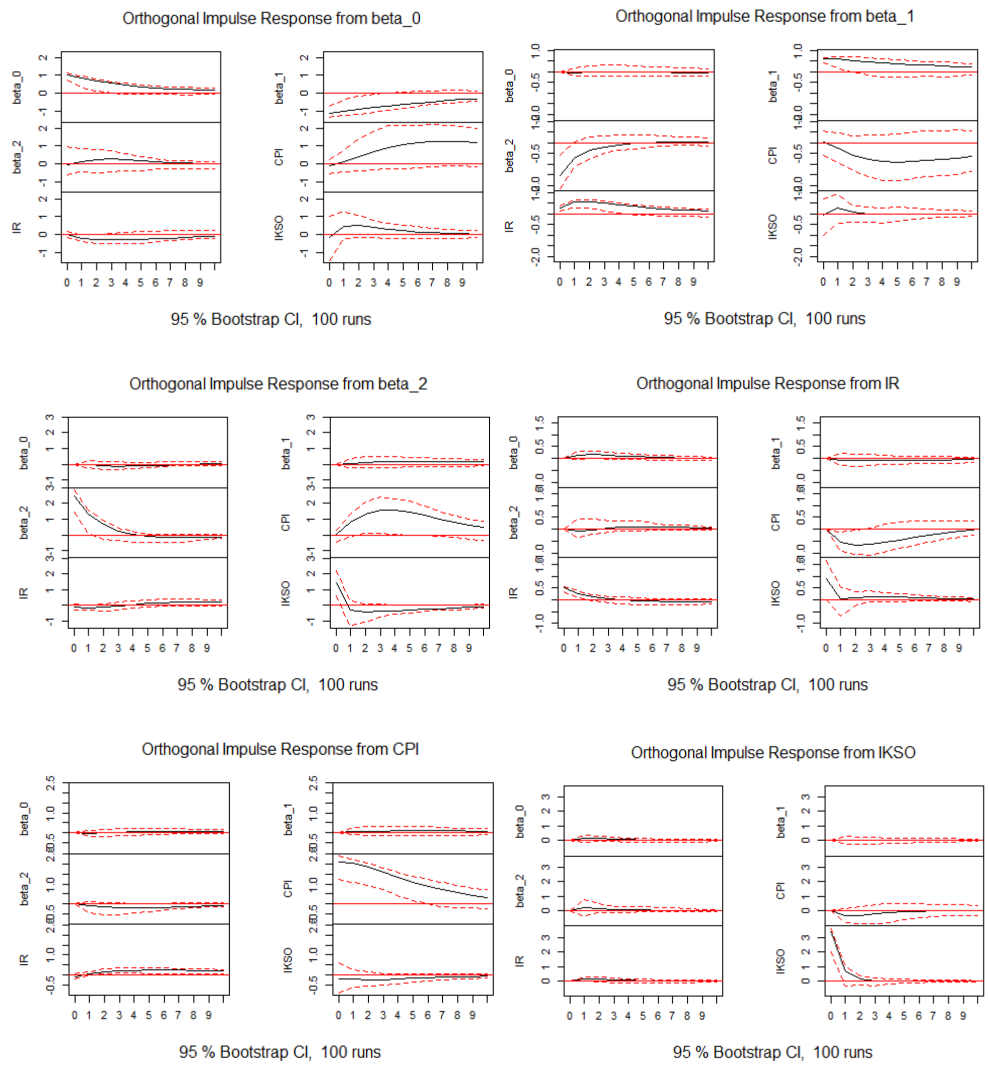

Figure 8. Impulse responses of the yields-macro model

Following Diebold, Rudebusch, and Aruoba (2006), variance decompositions were used to analyze macro and yield curve interactions. In Table 9, variance decompositions of the parameters/yields at respective forecast horizons are shown. From the table, one can observe that very little of the variation in rates is driven by the macro factors, which suggests that a great portion of idiosyncratic variation that is unrelated to macroeconomic variables is present in the yield curve (Diebold, Rudebusch, and Aruoba 2006).

Table 9. Variance decompositions, yields

	Horizon	β_0	β_1	β_2	CPI	IR	IKSO
β_0 Yields-only model	1	1.00	0.00	0.00	-	-	-
	12	0.99	0.01	0.00	-	-	-
	60	0.97	0.03	0.00	-	-	-
Yields-macro model	1	1.00	0.00	0.00	0.00	0.00	0.00
	12	0.93	0.00	0.02	0.01	0.02	0.01
	60	0.92	0.01	0.02	0.01	0.02	0.01
β_1 Yields-only model	1	0.74	0.26	0.00	-	-	-
	12	0.69	0.31	0.00	-	-	-
	60	0.66	0.34	0.00	-	-	-
Yields-macro model	1	0.78	0.22	0.00	0.00	0.00	0.00
	12	0.72	0.24	0.03	0.01	0.01	0.00
	60	0.70	0.24	0.04	0.02	0.01	0.00
β_2 Yields-only model	1	0.00	0.28	0.72	-	-	-
	12	0.00	0.32	0.68	-	-	-
	60	0.00	0.33	0.67	-	-	-
Yields-macro model	1	0.00	0.28	0.72	0.00	0.00	0.00
	12	0.02	0.26	0.69	0.02	0.00	0.01
	60	0.02	0.25	0.69	0.02	0.00	0.01

Table 10 reports the variance decompositions for the macroeconomic variables. From the table, one can infer that the yield-curve factors do predict substantial movements in CPI (though for the horizons of 12 and 60 months only), and key policy rate (IR).

Table 10. Variance decompositions, macroeconomic variables

	Horizon	β_0	β_1	β_2	CPI	IR	IKSO
CPI	1	-	-	-	1.00	0.00	0.00
Macro-only model	12	-	-	-	0.76	0.21	0.02
	60	-	-	-	0.69	0.28	0.03
Yields-macro model	1	0.00	0.00	0.00	1.00	0.00	0.00
	12	0.22	0.11	0.25	0.37	0.04	0.01
	60	0.29	0.13	0.22	0.32	0.03	0.00
IR	1	-	-	-	0.03	0.97	0.00
Macro-only model	12	-	-	-	0.02	0.95	0.03
	60	-	-	-	0.03	0.94	0.03
Yields-macro model	1	0.00	0.22	0.04	0.03	0.71	0.00
	12	0.16	0.46	0.10	0.15	0.12	0.02
	60	0.15	0.41	0.14	0.17	0.11	0.02
IKSO	1	-	-	-	0.00	0.00	0.99
Macro-only model	12	-	-	-	0.01	0.01	0.99
	60	-	-	-	0.01	0.01	0.99
Yields-macro model	1	0.00	0.00	0.14	0.00	0.05	0.80
	12	0.05	0.01	0.17	0.02	0.05	0.72
	60	0.05	0.01	0.17	0.02	0.05	0.72

Overall, the analysis of variance decompositions suggests that the impact of the yield curve on the macro variables are more critical than the effects of the macro variables on the yield curve. A possible explanation for this outcome can be the fact that macroeconomic variables are rough approximations for the sample and specification of our model.

Chapter 6

CONCLUSIONS

This study investigates the forecasting performance of the Dynamic Nelson-Siegel (DNS) model. We follow the approach applied by Diebold and Li (2006), which suggests using a parametric model that places strict structure on factor loadings (level, slope, and curvature) of the yield curve. Though the emphasis in this paper is put on univariate modeling with AR (1) factor dynamics, the random walk model is also used as an alternative benchmark model. Moreover, vector autoregressive VAR (1) counterpart of the AR (1) model was examined despite the evidence of the poor forecasting results generated by this model (Diebold and Li 2006).

Following Diebold and Li (2006), the theoretical and empirical frameworks of the DNS were first introduced. Though it was revealed that equilibrium modeling is commonly used for forecasting purposes as opposed to no-arbitrage models that mainly focus on fitting performance of yield curves, these models are subject to limitations that often result in poor forecasting performance. Diebold and Li (2006) went beyond no-arbitrage and equilibrium models and used the Nelson-Siegel framework to forecast the yield curve by forecasting the factors of the curve. We applied the same approach as discussed by Diebold and Li (2006) to the Ukrainian government bond yield data covering the period from 2015:08 up to 2019:11.

Our study provided evidence that the DNS model for Ukrainian data was inferior in terms of forecasting performance compared to the random walk model for all but short-term horizons. The results were in contrast with those of Diebold and Li (2006), who found that the DNS model produces yield curve forecasts that are superior to a random walk, especially for the long-term forecasting horizons. Despite this fact, extensive literature suggests that our findings are not uncommon. In particular, the time period of the data sample is hypothesized to be the factor that explains why the Diebold and Li (2006) model produced relatively good forecasts of the yield curve.

The second part of our research is focused on the incorporation of both yield factors (level, slope, and curvature) and macroeconomic variables (target interest rate, CPI, and IKSO). The state-space representation of the DNS allowed testing the hypotheses regarding dynamic interactions between the macroeconomy and the yield curve. Impulse response analysis and variance decompositions of yields and macroeconomic indicators were performed to assess the links between these variables.

The study found that there is weak evidence of macroeconomic effects on the future yield curve. In contrast, there is more robust evidence of the influence of yield curve factors on future macroeconomic developments – the results that contradict the findings obtained by Diebold, Rudebusch, and Aruoba (2006). However, academic literature devoted to the bidirectional causality between macroeconomic fundamentals and yield-curve factors also appeared mixed. While Diebold, Rudebusch, and Aruoba (2006) found more persuasive evidence of the macro effects on the future dynamics of yield-curve factors, our results are in line with the findings of Estrella and Mishkin (1998), and Stock and Watson (2000), who argue that asset prices are useful predictors of inflation, real output growth, and other macroeconomic indicators. Among other factors, the scholars point to the importance of the geography and time period of the data under analysis in producing accurate forecasting results (Stock and Watson 2000).

When it comes to the implications of the research, several issues need to be considered. First, as our research showed, the Nelson-Siegel model is not superior to an alternative random walk model in the context of long-term forecasting horizons. At the same time, the forecasting performance of the model for shorter future horizons is more promising and is worth using by policymakers for forecasting yield curves. Second, our results revealed that yield-curve factors are more potent in their effects on macroeconomic fundamentals than the effect of macroeconomic indicators on the future dynamics of yield-curve factors. This result provides a better characterization of the dynamic interactions between the macroeconomy and the yield curve that policymakers should be mindful of when it comes to policy-related decisions.

Finally, it is worth highlighting that there are several limitations that our research potentially faces, including issues associated with data collection and cleaning and period under analysis. Among the directions of future research, one may consider utilizing unsmoothed discounted zero-coupon yields to assess better the fitting performance of the DNS to the observed yield curves. As suggested by the literature, such an approach may improve the forecasting performance of the model, both in the short-term and long-term periods. Also, the time period of the data sample appears to be a critical issue in our research. Therefore, replicating the analysis over another period may produce more promising results.

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APPENDIX

FORECASTING PERFORMANCE OF DNS (CANADIAN DATA)

Table 11. Out-of-sample 1-month-ahead forecasting results (Canada, N = 19 individual forecasts)

Model Name	Maturity (τ)	Mean	Std. Dev.	RMSE	$\rho(1)$	$\rho(12)$
Nelson–Siegel with AR(1) factor dynamics	3	-0.139	0.149	**0.201**	0.668	-0.219
	6	-0.218	0.181	**0.280**	0.717	-0.129
	12	-0.277	0.233	**0.358**	0.744	-0.038
	36	0.049	0.350	0.344	0.771	-0.041
	60	0.255	0.410	0.474	0.769	-0.110
	120	0.443	0.479	0.644	0.753	-0.203
Nelson–Siegel with VAR(1) factor dynamics	3	-0.130	0.164	0.206	0.654	-0.144
	6	-0.227	0.189	0.292	0.706	-0.056
	12	-0.313	0.230	0.385	0.712	0.040
	36	-0.035	0.346	**0.338**	0.721	0.008
	60	0.160	0.427	0.446	0.727	-0.088
	120	0.344	0.537	**0.625**	0.717	-0.192
Random walk	3	-1.170	0.132	1.177	0.710	-0.123
	6	-1.285	0.117	1.290	0.689	0.018
	12	-1.345	0.176	1.355	0.800	-0.218
	36	-0.654	0.335	0.730	0.873	-0.403
	60	-0.055	0.382	**0.375**	0.872	-0.434
	120	0.651	0.406	0.762	0.861	-0.455

Table 12. Out-of-sample 6-month-ahead forecasting results (Canada, N = 19 individual forecasts)

Model Name	Maturity (τ)	Mean	Std. Dev.	RMSE	$\rho(1)$	$\rho(12)$
Nelson–Siegel with AR(1) factor dynamics	3	-0.442	0.154	0.466	0.663	-0.261
	6	-0.443	0.197	**0.482**	0.712	-0.218
	12	-0.378	0.253	**0.449**	0.716	-0.121
	36	0.211	0.342	0.391	0.710	-0.032
	60	0.516	0.371	0.628	0.707	-0.060
	120	0.784	0.413	0.879	0.726	-0.103
Nelson–Siegel with VAR(1) factor dynamics	3	-0.369	0.208	**0.420**	-0.115	-0.106
	6	-0.440	0.215	0.487	-0.207	-0.053
	12	-0.478	0.248	0.534	-0.056	0.047
	36	-0.048	0.306	**0.299**	0.081	0.084
	60	0.237	0.395	0.449	0.263	-0.016
	120	0.523	0.689	0.845	0.430	-0.070
Random walk	3	-1.189	0.031	1.189	0.457	-0.061
	6	-1.294	0.099	1.297	0.651	-0.152
	12	-1.338	0.186	1.350	0.632	-0.139
	36	-0.613	0.314	0.683	0.655	-0.137
	60	0.000	0.350	**0.337**	0.665	-0.159
	120	0.721	0.375	**0.806**	0.687	-0.186

Table 13. Out-of-sample 12-month-ahead forecasting results (Canada, N = 19 individual forecasts)

Model Name	Maturity (τ)	Mean	Std. Dev.	RMSE	$\rho(1)$
Nelson–Siegel with AR(1) factor dynamics	3	-0.669	0.085	0.674	0.556
	6	-0.610	0.079	0.615	0.475
	12	-0.466	0.090	0.474	0.109
	36	0.305	0.091	0.317	-0.442
	60	0.697	0.101	0.704	-0.255
	120	1.054	0.136	1.062	0.025
Nelson–Siegel with VAR(1) factor dynamics	3	-0.496	0.047	**0.498**	0.262
	6	-0.446	0.030	**0.447**	-0.391
	12	-0.334	0.080	**0.342**	-0.251
	36	0.246	0.144	**0.281**	0.128
	60	0.481	0.140	0.498	0.064
	120	0.643	0.140	**0.656**	0.034
Random walk	3	-1.096	0.022	1.096	0.322
	6	-1.133	0.025	1.134	-0.292
	12	-1.106	0.063	1.108	-0.333
	36	-0.327	0.093	0.338	-0.371
	60	0.245	0.106	**0.265**	-0.194
	120	0.887	0.161	0.900	0.171