OPTIMAL CURRENCY STRUCTURE OF DEBT FOR A SMALL OPEN ECONOMY

by

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ABSTRACT

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The paper studies the optimal currency structure of debt for a small open economy in the finite horizon setting. The model assumes the trade-off between the issuance of debt denominated in the foreign currency and debt denominated in the national currency: the risk of local currency depreciation and inflation when a government prefers denominating its debt in the national currency and the risk of default when it prefers denominating its debt in the foreign currency. The model assumes the opportunity of partial default on foreign currency debt: the government can choose any fraction of foreign currency debt that it can default on. The model concludes that in the deep recession for the government of a small open economy it is optimal to denominate most of its debt in national currency. However, the optimal currency structure of debt changes in the opposite direction as the economy approaches its long-run equilibrium.

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I would like to greatly thank Professor Sergii Kiiashko for his immense help and care given in the process of writing this thesis and his inspiration for the topic of government debt and default, which I have chosen as my future field of professional interest. Many thanks to Sergii for his great tolerance and liberal approach towards his students.

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GLOSSARY

Chapter 1

INTRODUCTION

Governments of small open economies always issue sovereign debt denominated in either foreign currency (commonly dollar or euro) or national currency. Both kinds of debt have their advantages and disadvantages. For example, dollar- or euro-denominated debt has always been associated with the decreased risk of exchange-rate devaluation, lower inflation risk, and increased level of foreign direct investments into the country. However, in many emerging market economies the dollarization of debt has been a major cause of debt-crises, in particular self-fulfilling roll-over crises. In addition, it should be noted that for many EM economies debt denominated in the foreign currency is a necessity because they cannot sell bonds denominated in their national currencies to foreign investors. The problem is often referred to as the “original sin" problem (Eichengreen, 2002). As of the sovereign debt denominated in the national currency, the main benefit is the absence of default risk. If the debt burden is relatively high, the government can always dilute its debt by printing money, as it causes an increase in the level of inflation and then depreciation of the local currency. However, the disadvantages of such an opportunity are the costs of inflation and depreciation, which cause various distortions in the economy, including the redistribution of national income. Therefore, the government should always take into account all the pros and cons of different kinds of debt before deciding how much should be borrowed in foreign and domestic currencies so that the risks associated with each kind of debt are minimized.

Ottonello and Perez (2016) derive two important facts about the formation of the government debt structure. First, they show that for many EM economies the structure of sovereign debt tends to be dominated by the debt denominated in the foreign currency: around two-thirds of sovereign debt of the economies they investigated appeared to be dollar- or euro-valued. Second, when there are economic booms, EM governments tend to denominate their debt mostly in the foreign rather than national currency. However, the empirical evidence in Ukraine is somewhat different. I have collected the data for Ukraine's government bonds for the period of 2017 – 2019. Below are presented the graphs describing the data for 2018 and 2019:

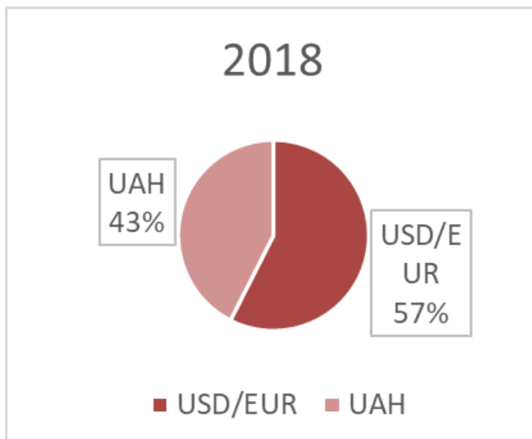

Figure 1. Ukraine's structure of debt in 2018

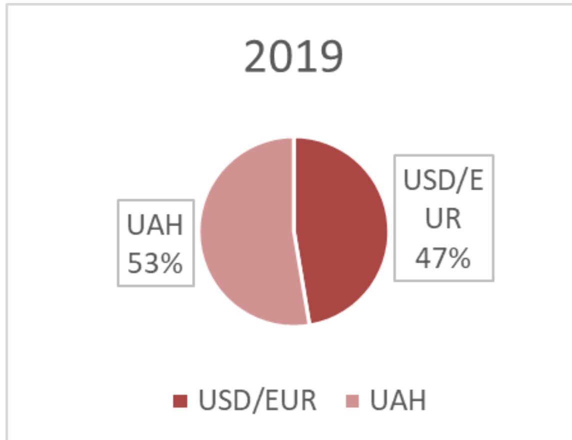

Figure 2. Ukraine's structure of debt in 2019

As can be seen in the diagrams that one year ago the sovereign debt was mostly denominated in the foreign currency (nearly 57%). Nowadays, the situation has completely changed: most of the debt nowadays is in the national currency. Many economists suggest various reasons for such a shift in government debt preferences. The two main causes are the government's willingness to reduce the risk of default and change in the level of exchange rate. This might be connected to an overdue level of external debt owed to IMF and other international investors and worth nearly 115 billion US dollars in 2019 (according to the data provided by the Ministry of Finance). According to the Ministry of Finance, nearly 84.3 percent of sovereign debt is denominated in the foreign currency (either dollar or euro) such that considering the fall in the level of foreign direct investments, distrust of the national currency, and relatively unstable situation in the country the risk of non-repayment is very high. As of the second reason, the recent appreciation of the national currency caused an increase in the real value of the debt denominated in domestic currency such that the government considered borrowing more in the domestic currency to be economically optimal.

In my paper, I answer the question: how much should be borrowed in both domestic and foreign currencies by the government of a small open economy so that both risks of default and inflation are minimized. In order to answer the question, I extend the model developed by Ottonello and Perez (2016). The main assumptions of the model are: the government lacks full commitment to its fiscal policies and fiscal trajectories, financial markets are incomplete, and the economy consists of two sectors: production of tradable and non-tradable goods. Every period, the government chooses a share of debt denominated in the foreign currency, which it defaults on. As a result, if the share is positive, a punishment is imposed and a particular level of debt that the government defaults on is required to be paid back next period by investors. In addition, every period the government chooses the optimal level of inflation in order to decrease the burden of debt denominated in the national currency. The government is benevolent and tries to maximize the utility of its households. At the beginning of the planning horizon, it chooses the optimal sequences of debt denominated in both foreign and national currencies, default shares, and inflation. The solution to the model is a set of policy and bond-pricing functions, which solve the respective Bellman equation.

The structure of this paper is organized as follows. In the second chapter, I review the literature on the currency composition of sovereign debt. In the third chapter, I explain the underlying model of partial default and its main assumptions. In the fourth chapter, I describe the calibration and estimation of the model. In the fifth chapter, I discuss the main results. The last section concludes.

Chapter 2

LITERATURE REVIEW

The question regarding the optimal currency structure of sovereign debt has not been scrutinized in the literature extensively. According to Eichengreen (2003), the reason for this topic being overlooked by many economists for a long period of time is the fact that debt currency denomination has always been one of the primary concerns of financial engineers only. Eichengreen states that the currency denomination of sovereign debt is extremely important for both empirical and theoretical studies in economics, as this is one of the key determinants of the stability of output, credit rating of the country, volatility of country's international and domestic capital flows, and the level of interest rates, which is an important issue for effective monetary policy.

One of the papers connected to the topic of debt currency denomination is by Arellano, Bai, and Mihalache (2018), which is the first paper in the economic literature combining the frameworks of the risk of sovereign debt and monetary policy aimed at inflation targeting. In the paper, the authors develop a New Keynesian model in order to study how monetary policy conducted by an interest rate rule and default risk arising from debt issued by the government are intertwined. In their model, the authors make conventional assumptions about a government and households. The government is benevolent and issues only long-term bonds denominated in the foreign currency and transfers all the collections from these issuances to the households. Also, the government lacks full commitment to its debt repayments, which is why it can default on its entire debt in the equilibrium. Default is commonly associated with a fall in productivity, which is accompanied by a reduction in the level of consumption and increased inflation. In addition, the authors impose standard N-K assumptions about the presence of domestic and imported goods, the presence of final and intermediate goods producers, and price rigidity. Households provide labor to both sectors of the economy.

In the model, pricing frictions encountered by firms and monetary policy of the Central Bank cause monetary frictions resulting in inefficiencies connected with input allocations and increased volatility of inflation. Moreover, the risk of default intensifies monetary frictions and makes it harder for the Central Bank to conduct inflation targeting. The authors measure these monetary frictions using the so-called monetary wedge, which comes from the New Keynesian Phillips Curve.

The main conclusions that the authors derive from the model are the following. First, when the risk of default is present, the monetary transmission is changed and monetary policy requires higher volatility of nominal interest rates. Second, in the environment of price rigidities, the increase in the risk of default amplifies monetary frictions through the fall in the level of consumption and exports, which in turn weakens government incentives to accumulate debt and reduces interest rate spreads.

The paper by Aguiar and others (2016) on coordination and crises in monetary unions has changed the economic belief about the choice of a monetary union by the country with a high level of debt denominated in the currency of a union chosen (basically, foreign currency debt – as in the previous paper – is considered). The authors make a model, where they assume that monetary policy in a monetary union is conducted by a single monetary authority (e.g. the Central Bank) and fiscal policy is conducted by national governments that lack commitment to their debt payments. They concluded that the country with a high level of debt should prefer being a member of the monetary union with an intermediate mix of countries with high-level debt and low-level one rather than a member of the monetary union with all countries having a low level of debt. The reason for this is the following. If the country that is a member of the second type of monetary union incurs high-level debt and faces the probability of default, other members that incur low-level debt and do not face such a prospect may not be willing to increase the level of inflation in the union in order to decrease the burden of debt held by the first country. Therefore, the risk of default for such a country is higher. However, if the country is a member of the first type of monetary union, then half of the members that also incur high-level debt will prefer a higher level of inflation. As a result, the risk of default for such a country, in this case, is lower.

Another important paper is by Arellano, Bai, Kehoe, and Ramanarayanan (2019), which differs from the previous papers in that it studies the relationship between government debt denominated in national currency and government commitment to its future monetary and fiscal policies. The lack of commitment, in this case, determines how inflation incentives influence the optimal maturity structure of sovereign debt and the relationships between amounts of debt, maturity of bonds, and inflation.

As in Lucas and Stokey (1983), the authors assume the joint determination of both monetary and fiscal policies in an economy without capital. Besides, the authors assume that the government can finance its expenditures through labor income taxes, issuance of long-term and short-term bonds, and seignorage from the creation of money. These assumptions raise the possibility for the government to inflate away sovereign debt outstanding by simply printing money. However, inflation has its costs in the form of consumption distortions both within and across time periods. In addition, the incentives to inflate away long-term debt are higher than the incentives to inflate away short-term debt, as an increase in the level of prices through time periods compounds such that the present value of long-term debt falls more than the present value of short-term one. On the contrary, long-term debt has its benefits for the government in that if the government spending increases, this causes inflation and interest rates to rise. As a result, interest rates rise as well and long-term bond prices fall. Given the assumed trade-off between costs and benefits of long-term debt, the authors derive the optimal maturity structure of government debt. Overall, they make two models: a social planner model – in order to find optimal maturity structure of nominal government debt under full commitment to monetary and fiscal policies, and Markov perfect competitive equilibrium – in order to account for the lack of commitment. The authors find that under the full commitment to both economic policies for the government it is optimal to issue long-term bonds. However, when the government fully lacks commitment to its future economic policies – then for it the issuance of only short-term bonds is optimal. The model matches empirical evidence of most OECD countries.

Despite a number of papers studying debt denominated in a single currency, there are some that study both kinds of debt denomination. Bleany and Ozkan (2011) in their paper study the connection between the currency denomination of sovereign debt and exchange rate regime. They make the conclusion that the choice of exchange rate regime depends on the credibility of the exchange rate peg, which in its turn is influenced by the levels of real and nominal debt. Basically, if the exchange rate peg is not fully credible, then the government will likely transit to the floating exchange rate regime as the costs of pegging will be higher.

A closely related paper to the topic of research is by Artus (2003). The author makes a theoretical model and shows that it is possible to have a lower devaluation risk but higher interest rates when debt is in foreign currency (in comparison to when it is in the domestic currency) because of a relatively higher risk of default. He also concludes that risks and costs are higher if the debt is denominated in foreign currency. Therefore, the government should prefer debt denominated in the national currency.

The seminal paper studying the currency denomination of debt is by Araujo and others (2013). The authors assume two possibilities of the denomination of debt: in the local currency and in the common currency (of some monetary union). The authors conclude that for the government it is optimal to denominate its debt in the common currency when the refinancing risks are highly correlated in all the union members. In the contrary case, it is optimal to hold its debt in the local currency.

The paper by Bohn (1990) studies debt denominated in the foreign currency as a hedging device "subject to stochastic shocks to output." In the model, the government is able to reduce uncertainty in domestic consumption and wealth by issuing debt denominated in both currencies provided that the country's GDP is uncorrelated with domestic and foreign inflation. The author concludes that debt denominated in the foreign currency is desirable compared to that denominated in the domestic currency.

Of all the papers, the one that is directly connected to my topic of research is by Ottonello and Perez (2016). The authors find what currency composition of sovereign debt an emerging market economy should have. The economy, in their model, consists of two manufacturing sectors: the one producing tradable goods and the other one producing non-tradable goods. In fact, the authors answer the question that I have raised, though the conclusion is pretty general and is not applicable for some emerging market economies including my country – Ukraine. Moreover, there are significant differences between the model by Ottonello and

Perez and mine. The first difference is the introduction of partial default in my model as in Arellano, 2019. In their underlying model, the authors do not assume the opportunity of default at all. According to one of their assumptions, the government, based on the current levels of debt in foreign currency, debt in national currency, production of tradable goods, and the level of prices formed at the previous period of time, makes the fiscal decision regarding additional borrowing in both currencies and the monetary decision regarding the level of inflation sufficient for inflating away an optimal amount of already accumulated debt denominated in the national currency. In my underlying model, I allow the government to default on a part of its debt denominated in the foreign currency. If the partial default happens, the country pays a share of its output as a penalty. The higher is the fraction of debt the government defaults on, the higher is the share of output it pays as a penalty. In addition, a share of debt in the foreign currency, which the government refused to service, is added to the stock of debt in the foreign currency for the next period (as in Arellano, 2019). The second difference is the methodological approach. My model is finite-horizon with time-dependent value functions. The model by Ottonello and Perez is infinite-horizon with one time-independent value function (and Bellman equation, respectively). The authors conclude that for an emerging market economy it is optimal to borrow more in the foreign currency rather than in the national currency. In my model of partial default, calibrated for Ukraine, I expect the converse result.

The paper by Bohn (1990) studies debt denominated in the foreign currency as a hedging device "subject to stochastic shocks to output." In the model, the government is able to reduce uncertainty in domestic consumption and wealth by issuing debt denominated in both currencies provided that the country's GDP is uncorrelated with domestic and foreign inflation. The author concludes that debt denominated in the foreign currency is desirable compared to that denominated in the domestic currency if GDP in both countries grows at the same rate and domestic inflation is uncertain.

C h a p t e r   3

METHODOLOGY

3.1 Description of the model

In the model it is assumed that the country is a small open economy and, therefore, the level of interest rates in it is strictly determined by the international flows of capital. As in Ottonello and Perez (2016), the economy consists of two manufacturing sectors: the one producing tradable goods and the other one producing non-tradable goods.

The government is benevolent, so that it chooses finite sequences of debts in the foreign currency, $\{b^*_{t+1}\}_{t=0}^{T-1}$, and the national currency, $\{b_{t+1}\}_{t=0}^{T-1}$, and also the sequence of shares of debt in the foreign currency that it defaults on, $\{d_t\}_{t=0}^{T}$, and the sequence of inflation rates, $\{\pi_t\}_{t=0}^{T}$, so as to maximize the welfare of its households. The planning horizon is finite. The problem is given by:

$$max_{\{b^*_{t+1}, b_{t+1}\}_{t=0}^{T-1}, \{d_t\}_{t=0}^{T}, \{\pi_t\}_{t=0}^{T}} \; E_0\{\sum_{t=0}^{T} \beta^t \{u(c_t) - l(\pi_t)\}\} \qquad (1)$$

with $c_t = C(c_{T,t}, c_{N,t})$ and given b^*_0, b_0, P_{-1} (the level of prices at period -1), and $y_{T,1}$ (production of the tradable output at period 1). *u(.)* is the utility function of consumption and *l(.)* is the disutility function of inflation, and β is the discount coefficient. In this case, $c_{T,t}$ and $c_{N,t}$ are, respectively, levels of consumption of tradable and non-tradable goods. It is assumed that $c_{N,t} = y_{N,t}$, that is, all non-tradable goods produced in the country are fully consumed. C(.) is a continuous function, which is monotonically increasing in both arguments. The sequences of both kinds of debt finish with the pre-last period T – 1 as, according to one of the assumptions of the model, the government pays back its entire debt in the last period T and no additional borrowing in either currency happens.

As in Ottonello and Perez (2016), bonds can have maturity higher than one period. If the government issues a bond denominated in either foreign or national currency at a given period of time, it promises to pay δ units of consumption the next period, δ^2 units at the period after the next period and so on. In the case of a one-period bond δ = 1, that is, the government fully returns face value of the bond (and some interest accrued) the next period.

Both debt denominated in the foreign currency and debt denominated in the national currency satisfy the following equations:

$$b_{t+1}^* = \delta * b_t^* + (1-\delta) * \kappa * d_t * b_t^* + i_t^* \quad (2)$$

$$b_{t+1} = \delta * b_t + i_t \quad (3)$$

where i_t^* and i_t are net debt issuances at period t, and κ is the fraction of defaulted debt at period t that the international investors demand to be compensated at period t+1.

As in Arellano (2019), if the government declares partial default d_t at a given period of time t, then the country is penalized through the seizure of the share of tradable output (e.g., due to sanctions): $y_{T,t}\Psi(d_t, y_{T,t})$, where $\Psi(d_t, y_{T,t}) \leq 1$ for all values of d_t and $y_{T,t}$ (basically, it is the share of output, which remains after the confiscation). The remaining part of the tradable output $y_{T,t}\Psi(d_t, y_{T,t})$ is assumed to be fully consumed.

Price of a bond that is denominated in the foreign currency is determined by the default policy of the government at a given period of time. Price of a bond that is denominated in the national currency is determined by the inflation policy of the government at a given period of time.

The inverse exchange-rate function (mapping into the units of foreign currency per one unit of national currency) depends on the current price level and relative consumption of tradable goods: $r\ (P_t, \frac{c_{T,t}}{y_{N,t}})$. All the variables (except for the prices of bonds denominated in the national currency) are expressed in terms of the foreign currency.

The dynamic problem given by (1) can be rewritten in the following form:

$$V_t\ (s_t) = max_{b^*_{t+1}, b_{t+1}, d_t, \pi_t}\ u\Big(C\big(c_{T,t}, y_{N,t}\big)\Big) - l(\pi_t) + \ \beta E_{y_t}\{V_{t+1}(s_{t+1})\} \quad (4)$$

subject to:

$$c_{T,t} = \ y_{T,t}\ \Psi\big(d_t, y_{T,t}\big) - (1 - d_t)b^*_t - r(P_t, \frac{c_{T,t}}{y_{N,t}}) * b_t + q^*_t(b^*_{t+1}, b_{t+1}, y_{T,t})[b^*_{t+1} - \ \delta * b^*_t - (1-\delta) * \kappa * d_t * b^*_t] + q^{\sim}_t(b^*_{t+1}, b_{t+1}, y_{T,t}) * [b_{t+1} - \ \delta * b_t] \quad (5)$$

$$q_t^*(b_{t+1}^*, b_{t+1}, y_{T,t}) = \frac{1}{R} E_t\{1 - \pi_t(b_{t+1}^*, b_{t+1}, y_{T,t+1}) + \left(\delta + (1-\delta) * \kappa * d_t(b_{t+1}^*, b_{t+1}, y_{T,t+1})\right) * q_{t+1}^*(b_{t+1}^*(.), b_{t+1}(.), y_{T,t})\} \tag{6}$$

(price of a bond denominated in the foreign currency);

$$q_t^{\sim}(b_{t+1}^*, b_{t+1}, y_{T,t}) = \frac{1}{R} E_t\{1 - d_t(b_{t+1}^*, b_{t+1}, y_{T,t+1}) + \delta * q_{t+1}^*(b_{t+1}^*(.), b_{t+1}(.), y_{T,t})\} \tag{7}$$

(price of a bond denominated in the national currency);

$$P_t = \pi_t P_{t-1} \tag{8}$$

(aggregate price level at period t);

for t=0,…,T-1.

The argument s_t is the vector of state variables ($b_t^*, b_t, y_{T,t}, P_{t-1}$). R is the gross international interest rate paid by a riskless security. In addition, the output is assumed to follow AR (1) process in logs given by:

$$\log(y_t) = \rho \log(y_{t-1}) + \varepsilon_t \tag{9}$$

where the error term is assumed to be normally distributed with some values of mean and variance. The discount coefficient β lies in the interval (0,1).

For the last period T, the value function is defined as follows:

$$V_T\ (s_T) = max_{\pi_T, d_T}\ u\left(C\left(c_{T,T}, y_{N,T}\right)\right) - l(\pi_T) \qquad (10)$$

subject to:

$$c_{T,T} = \ y_{T,T} - (1 - d_T)b_T^* - r(P_T, \frac{c_{T,T}}{y_{N,T}})^* b_T \qquad (11)$$

$$P_T = \ \pi_T P_{T-1} \qquad (12)$$

In this case, the argument s_T is the vector of state variables ($b_T^*, b_T, y_{T,T}, P_{T-1}$).

3.2 Solution to the model.

The solution to the model is the set of functions $\{V_t, q_t^{\sim}, q_t^*, b_{t+1}^*(.),\ b_{t+1}(.), d_t(.), \pi_t(.)\}_t$, where (for each period t=0,…,T-1) the value functions V_t and V_{t+1} solve the above-given Bellman equation, given respective policy functions $b_{t+1}^*(.),\ b_{t+1}(.), d_t(.),$ and $\pi_t(.)$; V_T solves the Bellman equation for the final period T given two policy functions $\pi_T(.)$ and $d_T(.)$. Bond-pricing functions $q_t^{\sim}$ and q_t^* (for each period t=0,…,T-1) solve equations II and III described above.

The algorithm for the solution of the model is as follows. In the first step, grids of equally spaced points are made for all four state variables, inflation, and default share. The algorithm starts from solving the Bellman equation for period T. For all possible combinations of chosen grid values a maximum of the utility function over all values of π_T and d_T is found. Each such value will be the approximation of the respective value of the value function for period T at each respective combination of state values. In this way, the approximation of the value function for the final period is received. Moreover, argument of the maximum will be the approximated value of the policy functions for inflation $\pi_T(.)$ and default share $d_T(.)$ (for each respective combination of state values).

In the second step, Markov probability transition matrix for the AR (1) process is approximated using Tauchen's approximation method.

In the third step, for period T-1, for each combination of state values, a maximum over all values of choice variables is found: for each combination of state values and each combination of choice values (given state values), there exist values of the value function for period T for different levels of tradable output $y_{T,T}$; using Markov probability transition matrix, the expected value of the value function is calculated and then the value of the expression as in the right-hand side of the above-written Bellman equation is calculated. Given each combination of state values, a maximum of such expressions over all values of choice variables is selected. The maximum found is the approximated value of the value function for period T-1 given each combination of state values. Therefore, the value function for period T-1 is approximated. Moreover, the arguments of maximum for respective choice variables (given each combination of state values) are approximated values of respective policy functions on given values of state variables for T-1: $b_T^*(.)$, $b_T(.), d_{T-1}(.), \pi_{T-1}(.)$. Also, the consumption policy function is approximated for each combination of state values by plugging respective state values and values of policy functions (instead of choice variables) in the respective expression for consumption. Using Markov transition matrix the approximated values of bond-pricing functions for bonds in the foreign currency and in the national currency are calculated: expected values are calculated in the way identical to the one in which expectation of the value-function for period T is calculated; then the expected values are discounted by the gross international interest rate R.

In the last step, the algorithm proceeds to further steps, which are identical (in terms of the approximation of value function and other functions) to step 3, described above (for periods t=0,…,T-2). Approximations to respective policy functions and bond-pricing functions for bonds denominated in the national and foreign currencies are calculated.

Chapter 4

CALIBRATION AND ESTIMATION

The model has been solved for two periods: the government chooses one-period bond in the first period and returns it in the second one ($\delta = 1$). The calibration is made for Ukraine. The punishment function and exchange rate function are defined as in Arellano et. al. (2018) and Ottonello and Perez (2018) respectively:

$$\Psi\left(d_t, y_{T,t}\right) = \left(1 - \varphi 0 * d_t{}^{\gamma}\right) * \left(1 - \varphi 1\_hat * \left(y_{T,t} - y^*\right)\right) \qquad (13)$$

with

$$\varphi 1\_hat = \begin{cases} \varphi 1, for\ y_{T,t} > y^*\ and\ d_t > 0 \\ 0, otherwise \end{cases} \qquad (14)$$

and

$$r\left(P_t, \frac{c_{T,t}}{y_{N,t}}\right) = \frac{1}{\alpha * P_t} * \left(\frac{c_{T,t}}{y_{N,t}}\right)^{1-\alpha} \qquad (15)$$

The functions of utility from consumption and disutility from inflation are defined as in Ottonello and Perez (2018):

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{16}$$

$$l(\pi) = \frac{\psi^{inf}}{2}(\pi - \pi^*)^2 \tag{17}$$

where σ is the constant relative risk aversion coefficient of the utility function, ψ^{inf} is the cost of inflation parameter, and π^* is the level of inflation preferred by the people.

In order to solve the two-period model, the simplification is made. Value functions for all periods are defined as dependent on the ratio of debt denominated in the national currency to the price level at the previous period of time (deflated nominal debt). Basically, for the last period the one obtains:

$$V_2\ (b_2^*, b_2/P_1, y_{T,2}) = max_{\pi_2, d_2}\ u\Big(C\big(c_{T,2}, y_{N,2}\big)\Big) - l(\pi_2) \tag{18}$$

subject to:

$$c_{T,2} = \ y_{T,2} * \Psi\big(d_2, y_{T,2}\big) - (1 - d_2)b_2^* - r(\pi_2, \frac{c_{T,2}}{y_{N,2}}) * \frac{b_2}{P_1} \tag{19}$$

where:

$$P_2 = \pi_2 P_1 \tag{20}$$

Value function for the first period is defined in the similar way:

$$V_1\left(b_1^*, b_1/P_0, y_{T,1}\right) = max_{b_2^*, b_2^{defl}, \pi_1, d_1}\, u\Big(C\big(c_{T,1}, y_{N,1}\big)\Big) - l(\pi_1) + \beta * E_{y_{T,2}}\{V_2\left(b_2^*, b_2^{defl}, y_{T,2}\right)\} \tag{21}$$

subject to:

$$c_{T,1} = y_{T,1} * \Psi\big(d_1, y_{T,1}\big) - (1-d_1)b_1^* - r(\pi_1, \frac{c_{T,1}}{y_{N,1}})* \frac{b_1}{P_0} + q_1^*\big(b_2^*, b_2^{defl}, y_{T,1}\big) *[b_2^* - b_1^*] + q_1^{\sim}(b_2^*, b_2^{defl}, y_{T,1})*[b_2^{defl} - b_1^{defl}] \tag{22}$$

where:

$$P_1 = \pi_1 P_0 \tag{23}$$

$$b_2^{defl} = \frac{b_2}{P_1} \tag{24}$$

$$b_1^{defl} = \frac{b_1}{P_0} \tag{25}$$

$$q_1^*\left(b_2^*, b_2^{defl}, y_{T,1}\right) = \frac{1}{R} E_1\{1 - d_2(b_2^*, b_2^{defl}, y_{T,2})\} \quad (26)$$

(price of a bond denominated in the foreign currency)

$$q_1^{\sim}(b_2^*, b_2^{defl}, y_{T,1}) = \frac{1}{R} E_1\{1 - \pi_2(b_2^*, b_2^{defl}, y_{T,2})\} \quad (27)$$

(price of a bond denominated in the national currency)

For simplicity, I assume that debt in both currencies is zero in the initial period. This means that there is no stock of debt to default on at the first period, so clearly the optimal value of d_1 is 0. As there is no trade-off between costs of default and costs of inflation in the initial period, I assume that the government chooses inflation at the socially-optimal level π^*. It follows that at the initial period of time, the government chooses only the optimal levels of debt both in foreign and national currencies: b_1^* and b_1^{defl}.

In the table below, the values of parameters calibrated for Ukraine are given. After the table, the calibration process is described.

Table 1. Values of the parameters

Parameters	Names	Values
σ	CRRA	7
α	Share of tradable output in households' consumption	0.65
β	Discount factor	0.9606
ψ^{inf}	Cost of inflation parameter	2
π^*	Socially-optimal level of inflation	0.05
R	Gross international risk-free interest rate	1.04
$y_{N,1}$	Non-tradable output at period 1	1
$y_{N,2}$	Non-tradable output at period 2	1
$\varphi 0$	Default cost parameter	0.035
$\varphi 1$	Default cost parameter	0.106
γ	Default cost parameter	1.4
y^*	Default cost parameter	0.933

Value of the discount coefficient is 0.9606 and has been taken from Bondarenko, 2018. Any deviation from this value makes outcomes of the model worse in terms of statistical moments described below. The production of non-tradable output at period 1 and 2 is assumed to be on its potential level, so the values of $y_{N,1}$ and $y_{N,2}$ are set to one. The value of π^* is 0.05, as it is the optimal long-run inflation target chosen by the National Bank of Ukraine. Values of other parameters have been calibrated for Ukraine such that the six key statistical moments (given in the table below) generated by the model match those observed empirically as close as possible. Standard deviation of expected inflation was estimated based on the quarterly year-over-year data taken from the website of Ukrstat for 2006 through 2016. Correlation between exchange rate and expected inflation was estimated based on statistical data taken from the websites of the NBU and Ukrstat, respectively, for years 2010 through 2019 (annual data). Due to lack of data, I assume that expected inflation for 2010 through 2019 is approximately equal to the level of current inflation in respective years (adaptive inflationary expectations). The level of expected inflation taken for the calibration of the model is the one forecast by the National Bank of Ukraine for the current year of 2020. Correlation between total debt and gross domestic product was estimated based on the data taken from the websites of the Ministry of Finance and Ukrstat, respectively, for 2010 through 2019 (annual data). Correlation between exchange rate and total debt was estimated based on the data taken from the websites of the NBU and the Ministry of Finance, respectively, for 2010 through 2019. Correlation between expected inflation and total debt was estimated based on the data taken from the websites of the National Bank of Ukraine and the Ministry of Finance, respectively, for 2010 through 2019. Correlation between spread and GDP was estimated based on the confidential data provided by the NBU and data taken from the website of Ukrstat, respectively, for 2010 through 2019. In all cases, GDP is expressed in real terms – in fixed prices of the year of 2016. In the model, correlations with tradable output and total GDP are the same as production of non-tradable goods is assumed to be constant: 1 (actually, it is likely for many small open economies, which are dependent on commodities trading, including Ukraine).

AR(1) process was estimated empirically using the data for GDP taken from the website of Ukrstat for 2001 through 2019 (the time series for GDP had been preliminarily detrended using Hodrick-Prescott filter). The level of output for each year was normalized by potential GDP (as is the case in the model). The method of approximation is by George Tauchen with the assumption about the normal distribution of the log of output following AR(1) process (conditional on its lagged value).

Table 2. Statistical moments.

Statistical moment	Model	Data
Corr. (exchange rate, expected inflation)	0.85	0.95
Expected inflation (2020)	0.053	0.063
Corr. (total debt, GDP)	-0.99	-0.83
Corr. (txchange rate, total debt)	0.97	0.79
Corr. (expected inflation, total debt)	0.72	0.63
Corr. (spread, GDP)	-0.67	-0.54

The above-described backward induction algorithm is applied in order to find the set of functions $\{V_t, q_t^{\sim}, q_t^{*}, b_{t+1}^{*}(.),\ b_{t+1}(.), d_t(.), \pi_t(.)\}_t$ that will solve the model.

In order to approximate the intervals for state and choice variables, the following discretization of intervals was applied: 51 points were taken for both kinds of debt in the interval [0, 0.25], 9 points were taken for tradable output of the second period in the interval [0.8, 1.2], 41 points were taken for tradable output of the first period in the interval [0.7, 1.3], 61 points were taken for the share of debt the government defaults on in the interval [0, 0.3], and 61 points were taken for inflation in the interval [0, 0.3].

Chapter 5

ESTIMATION RESULTS

Below are given the graphs of the optimal government debt structure, expected inflation, and expected default shares for different states of the economy. Inflation is expressed in decimals (not 1+inflation rate). The output considered in the solution to the model is tradable one: in small open economies, where a large proportion of GDP belongs to the trade of commodities (including Ukraine), shocks in the tradable sector mostly influence the country's economy. Tradable output is normalized: 1 means it is on its potential level of production, 0.9 means the production of tradable output is 10% below its potential level and so on.

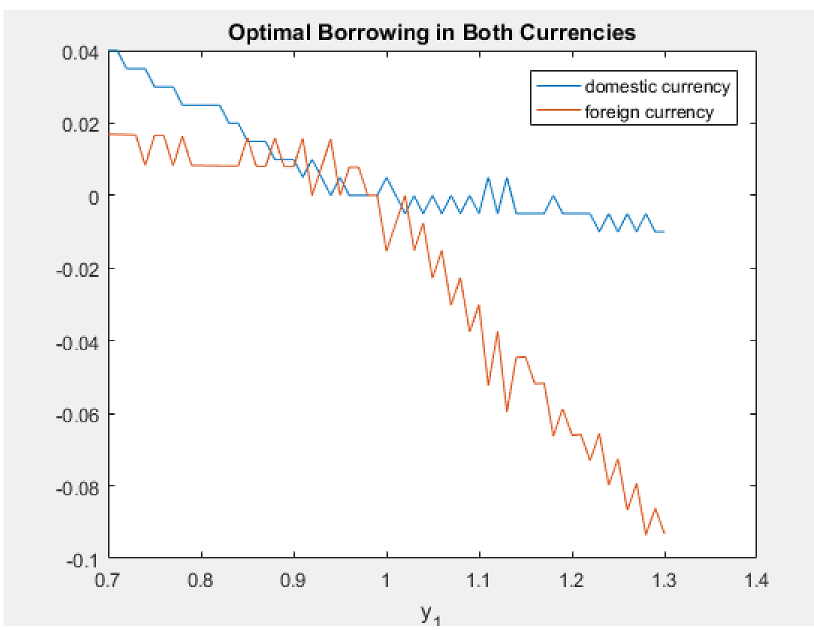

Figure 3. Optimal currency structure of sovereign debt (both debts are in the national currency)

As can be seen in the figure, when the economy is in the deep recession (due to fall in the level of tradable output), then for the government of a small open economy it is optimal to borrow more in the national currency. For example, when normalized GDP is 0.8 (the economy is 20% below its long-run equilibrium), the optimal ratio of debt denominated in the national currency to the one denominated in the foreign currency is approximately 3.125. However, when GDP moves closer towards its long-run equilibrium (1), this ratio gradually falls. For example, when GDP is 0.93, the ratio is approximately 0.71. Intuitively, the reason is the costs of debt-financing: when the economy is in the prolonged recession, foreign currency debt is much harder to be financed as inflation increases and exchange rate depreciates. In addition, in the deep recession the interest rate spread, as can be seen in the figure given below, increases, which makes foreign currency debt even more expensive to be financed. Domestic currency debt is, conversely, much cheaper for the government to finance in the situation of deep recession as the costs of inflation are, as compared to the costs of partial default on the debt denominated in the foreign currency, less looming. In addition, both increase in the level of inflation and exchange rate depreciation due to fall in the level of output make domestic currency debt cheaper to finance (in real terms).

When the economy is in the long-run equilibrium, it is optimal to lend money in the foreign currency and borrow in the national currency. The ratio of debt denominated in the national currency to credit denominated in the foreign currency in this state of the economy is -0.32 (as lending in the model goes with the negative sign).

When the economy is in the state of overheating, it is optimal (mostly) to lend money in both currencies (or no lending in either national or foreign currency is optimal). As can be seen in the figure, the higher is the level of GDP in the state of overheating, the more lending to other countries should be committed in the foreign currency. The reason is as follows. When GDP grows above 1 (in normalized terms), the national currency experiences (as shown in the figure below) appreciation and no inflation is optimal. This makes lending in the foreign currency more profitable (in real terms) than lending in the national currency, so more credit should be given in the foreign currency.

Below is given the table showing ratios of debt denominated in the national currency to that denominated in the foreign currency for some states of the economy. Debt is, as tradable output, normalized by the potential level of tradable GDP.

Table 3. Debt ratios for different levels of GDP

Level of GDP (normalized)	Ratio of debt denominated in the national currency to debt denominated in the foreign currency
0.74	4.19
0.84	2.48
0.93	0.64
1	-0.32

When output is 0.74,0.84, or 0.93, then borrowing in both currencies in the ratios given is optimal. However, when output is 1, it is optimal to give a credit denominated in the foreign currency and issue debt denominated in the national currency in the ratio given (the negative sign is explained above).

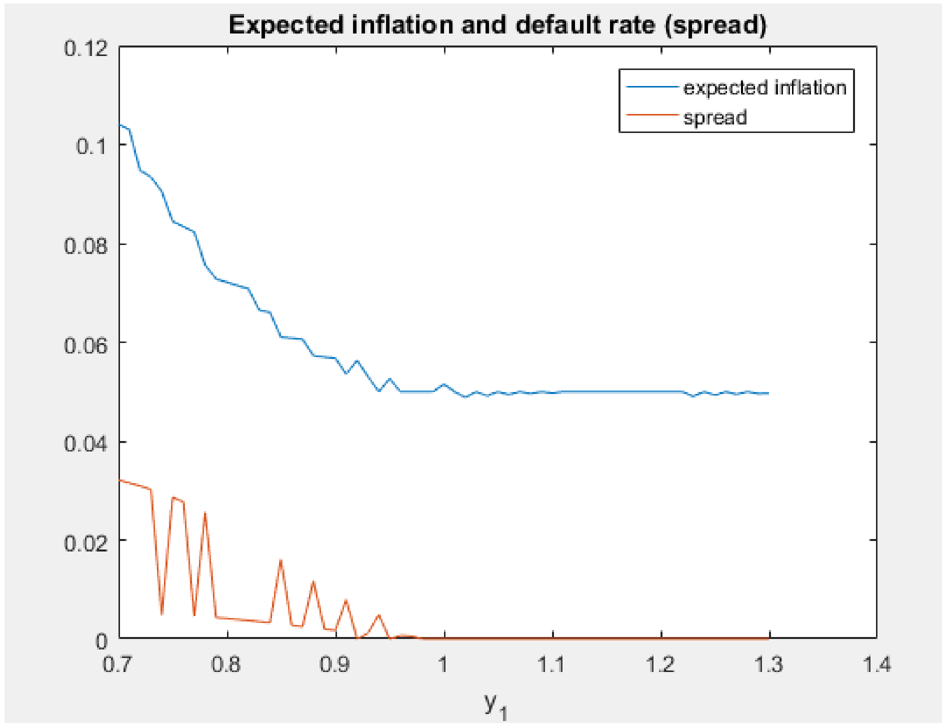

Figure 4. Expected inflation and spread.

The model, as was partially mentioned, also predicts the behavior of expected inflation and interest rate spreads (expected default share) in the situation when the economy is in the recession. When output falls, both increase: the first one because production falls and the government is, at the same time, incentivized to inflate its debt denominated in the national currency away in order to decrease debt burden, and the second one because the government is incentivized to default on a higher share of debt denominated in the foreign currency in order to, as before, reduce its debt burden. Interestingly, in the situation when output falls, the level of expected inflation increases more than the level of spread because the exchange rate depreciation and government incentives to inflate debt denominated in the national currency away put higher pressure on the level of inflation in the country than government incentives to default on a part of its debt denominated in the foreign currency put on interest rate spread (expected default share).

Also, in the figure below one can see the behavior of expected exchange rate. The expected exchange rate shown is expressed as units of foreign currency per one unit of national currency. As output grows, the expected exchange rate goes up as the higher level of output is associated with the higher level of consumption of domestic tradable goods resulting in the relative appreciation of the national currency (demand for foreign currency falls). In addition, as output grows, expected inflation falls, resulting in even higher level of expected exchange rate.

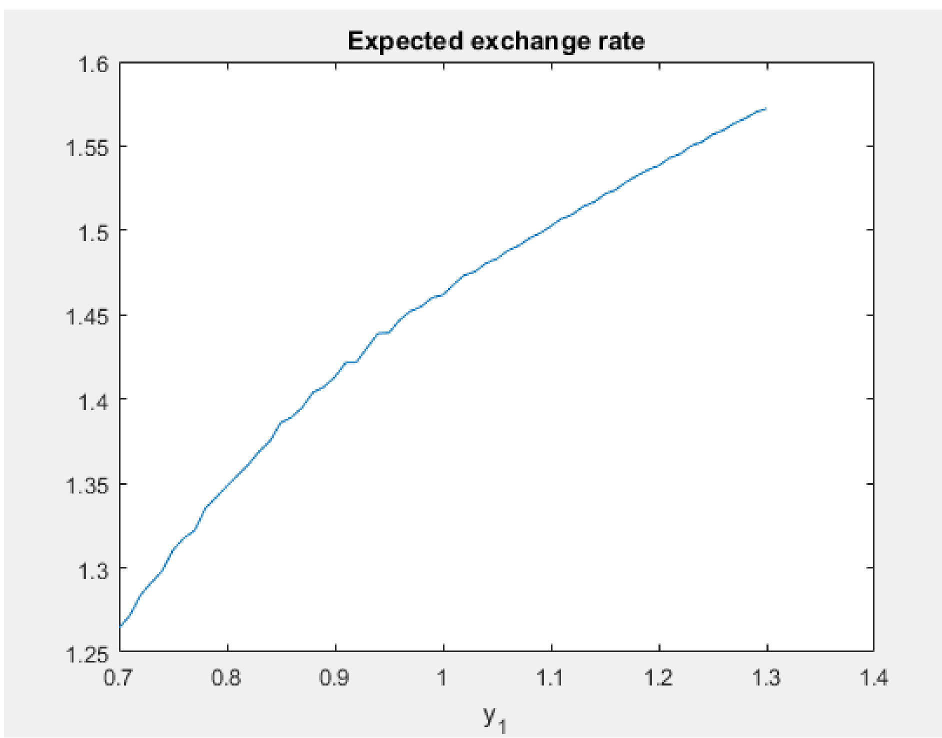

Figure 5. Expected exchange rate

In the last figure the one can see the behavior of the prices of bonds denominated in both currencies when output changes. The blue line corresponds to the prices of the bond denominated in the foreign currency and the orange line corresponds to the prices of the bond denominated in the national currency.

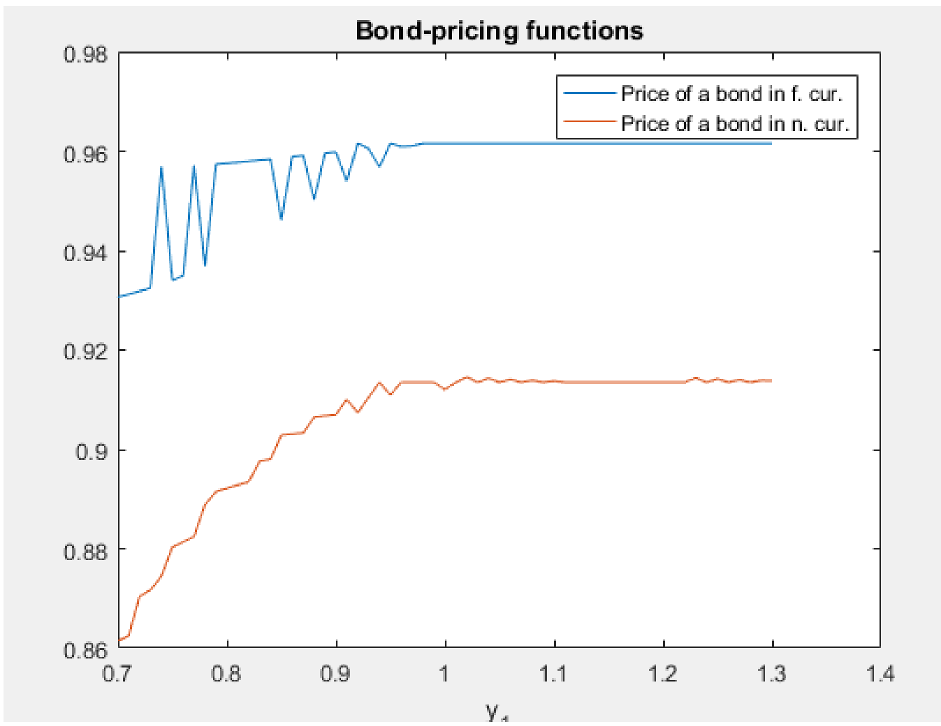

Figure 6. Prices of bonds denominated in foreign and national currencies

In the two-period model, the bond-pricing policy functions are defined as follows:

$$q_1^*\left(b_2^*(b_1^*, b_1^{defl}, y_{T,1}), b_2^{defl}(b_1^*, b_1^{defl}, y_{T,1}), y_{T,1}\right) =$$
$$\frac{1}{R}E_1\{1 - d_2(b_2^*(b_1^*, b_1^{defl}, y_{T,1}), b_2^{defl}(b_1^*, b_1^{defl}, y_{T,1}), y_{T,2})\}$$

(28)

(pricing function of a bond in the foreign currency)

$$q_1^{\sim}(b_2^*(b_1^*, b_1^{defl}, y_{T,1}), b_2^{defl}(b_1^*, b_1^{defl}, y_{T,1}), y_{T,1}) =$$
$$\frac{1}{R}E_1\{1 - \pi_2(b_2^*(b_1^*, b_1^{defl}, y_{T,1}), b_2^{defl}(b_1^*, b_1^{defl}, y_{T,1}), y_{T,2})\}$$

(29)

(pricing function of a bond in the national currency)

In my case, both b_1^* and b_1^{defl} are equal to zero, as I assume that no debt is issued in the initial period. As output grows, the price of the bond denominated in the foreign currency increases. The reason is fall in the level of expected default rate. The same is the logic with the bond denominated in the national currency: when output grows, the price of the bond increases as the level of expected inflation falls.

Chapter 6

SENSITIVITY ANALYSIS

I have conducted a sensitivity analysis in order to find out how the model responds with respect to change in the value of the parameters. Below is given the table showing how much the statistical moments change when values of some of the parameters change in a given direction. The tables are given for the parameters to which the model is the most sensitive (small changes are analyzed).

Table 4. Change in moments when the value of σ changes.

Statistical moment	Increase by 1.5	Decrease by 1.5
Corr. (exchange rate, expected inflation)	0.8413	0.8621
Expected inflation (2020)	0.0535	0.0526
Corr. (total debt, GDP)	-0.9898	-0.9854
Corr. (exchange rate, total debt)	0.9728	0.9656
Corr. (expected inflation, total debt)	0.7069	0.7120
Corr. (spread, GDP)	-0.7194	-0.6008

When the value of sigma changes, spread responds the most. Basically, depending whether agents have a steeper utility function (with MPC higher at every respective point) or lower (with MPC lower at every respective point), the government chooses different levels of default share in order to maximize overall utility.

Table 5. Change in moments when the value of β changes.

Statistical moment	Increase by 0.02	Decrease by 0.02
Corr. (exchange rate, expected inflation)	0.8455	0.8631
Expected inflation (2020)	0.0524	0.0540
Corr. (total debt, GDP)	-0.9888	-0.9850
Corr. (exchange rate, total debt)	0.9706	0.9646
Corr. (expected inflation, total debt)	0.7090	0.7111
Corr. (spread, GDP)	-0.6395	-0.6490

The most responsive in this case is exchange rate (two respective correlation coefficients change most). The reason is that households smooth their consumption differently when they discount their future in different ways (if the coefficient is higher, they value future higher and vice versa). Therefore, depending on what state of the economy is, they will have quite different consumptions (under different values of the discount coefficient). As the real consumption enters exchange rate function, exchange rate will change quite as well.

Table 6. Change in moments when the value of α changes.

Statistical moment	Increase by 0.05	Decrease by 0.05
Corr. (exchange rate, expected inflation)	0.8597	0.8573
Expected inflation (2020)	0.0536	0.0528
Corr. (total debt, GDP)	-0.9907	-0.9863
Corr. (exchange rate, total debt)	0.9750	0.9674
Corr. (expected inflation, total debt)	0.7385	0.7150
Corr. (spread, GDP)	-0.5892	-0.6305

When alpha changes, spread and expected inflation respond the most. The reason is, the lower is the share of tradable goods consumed, the lower is the utility from consuming one unit of tradable output. As a result, higher level of inflation and higher default share can be introduced in order to make the utility higher. Reverse happens when the value of the parameter increases.

Chapter 7

CONCLUSIONS

Overall, the government of a small open economy, when the opportunity of partial default is introduced, should prefer issuing most of its debt denominated in the national currency, if the economy is in the deep recession (e.g. due to fall in the level of manufacturing of commodities caused by economic shocks in the tradable sector). The reason is the fact that when the economy is in the deep crisis, then the costs of financing debt denominated in the foreign currency are higher than costs of financing debt denominated in the national currency due to increased inflation and exchange rate depreciation. In addition, interest rate spread increases, which makes foreign currency debt even harder to finance. Moreover, in the deep recession costs of default, as estimated by the model, are higher than costs of unwilling inflation, which incentivizes the government to issue more debt denominated in the national currency (as it can always be inflated away at a lower level of losses). According to the estimation made in the model, the average ratio of debt in the national currency to that in the foreign currency when the economy is in the recession is nearly 1.8 (over all respective states of the economy).

However, when the economy is overheating, lending in both currencies is optimal (or no lending and borrowing in either currency are optimal). However, as output grows, more money should be lent in the foreign currency. The reason is that the higher is the level of output when the economy overheats, the higher is the level of exchange rate (foreign currency per unit of national currency). As a result, lending in the foreign currency becomes more profitable, so more credit should be in the foreign currency. The average ratio of credit in the national currency to that in the foreign currency when the economy is overheating is nearly 0.1 (over all respective states of the economy).

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