Kyiv School of Economics

Admission Exam in Mathematics

Spring 2019

General instructions (read carefully!):

• You should NOT open the exam before your proctor says so.

• The exam has 10 problems. All problems will be weighted equally.

• You have 80 minutes for this exam.

• The answer to each problem is a number or a short expression. Write down your answers in the Answer sheet. However, please, provide, in the exam book, detailed explanations of how the answers have been attained.

• In the case of a wrong answer, a partial credit may be given based on your explanations.

• Please, write legibly (readably).

• Cheating on any exam automatically invalidates all your admission tests!

• You can use the back of any page for your draft notes.

YOUR NAME_____

Answer Sheet

1._____

2._____

3._____

4._____

5._____

6._____

7._____

8._____

9._____

10._____

1. Evaluate the following integral

$$\int_1^3 (x^2 + 1)x dx$$

2. Compute the second derivative of the following function:

$$\frac{\ln x}{x}$$

3. Find the inverse of the following matrix

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 3 \\ -1 & 1 & 0 \end{bmatrix}$$

4. Compute

$$\lim_{x \to 0} \frac{(2x-2)^3 + 8}{x^2 + x}$$

5. Find the derivative of the function at $x = 1$

$$f(x) = \frac{\sqrt{x}}{x^2 + 1}$$

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6. At a car park there are 100 vehicles, 60 of which are cars, 30 are vans, and the remainder are trucks. If every vehicle is equally likely to leave, find the probability of a car leaving second if either a truck or van had left first.

7. The maximum number of calculators that an instrument company can sell on any given day is 800. The $150 calculator requires 2 hours of production time, and the $100 calculator requires 1 hour of production time. The number of available production hours on any given day is 1200. How many calculators of each type should be produced so as to maximize its daily revenues?

8. Let X be uniformly distributed over $(0, 1)$. Find the variance of X, $\mathrm{Var}X$.

9. A committee of 4 is selected from a group of 4 men and 3 women. If the selection is made randomly, what is the probability that the committee consists of 3 men and 1 woman.

10. Let X be a random variable exponentially distributed with some parameter $\lambda > 0$. That is, its probability density function is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} \text{ if } x \geq 0 \\ 0 \text{ if } x < 0 \end{cases}$$

Calculate its expected value.

Extra page