MACROECONOMIC EFFECTS OF THE MINIMUM WAGE INCREASES IN AN ECONOMY WITH WAGE UNDERREPORTING

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Abstract

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I extend the standard New-Keynesian DSGE model to study the macroeconomic consequences of the minimum wage increase in an economy with wage underreporting. The model is calibrated and estimated for Ukraine. According to the model predictions, the larger is the degree of wage underreporting in the economy the less responsive is the economy to the minimum wage increase. When the degree of wage underreporting is large, the minimum wage increase: 1) affects production costs mostly through higher taxes, 2) has less strong negative effect on hours worked and 3) can lead to an increase in tax revenues.

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Chapter 1

INTRODUCTION

In this paper I study the macroeconomic effects of the minimum wage increase in the economy where workers and firms are engaged in wage underreporting activity. For this purpose I use the New Keynesian DSGE model framework which I extend to allow for presence of wage underreporting. Wage underreporting occurs when the firm and the worker agree to report the legally established minimum wage to the tax authorities instead of the true wage. In the economy in which wage underreporting takes place the observed share of minimum wage workers consists of the true minimum wage workers and of the workers engaged in wage underreporting. I look at the response of the model economy to the minimum wage increase under different shares of wage underreporting workers in the observed share of the minimum wage workers. I find that the larger degree of wage underreporting makes an economy less responsive to the minimum wage increase.

The model parameters are calibrated and estimated for the case of Ukraine since Ukraine is the country with the legally established nominal minimum wage and the issue of wage underreporting is quite relevant for Ukraine.

In the year 2017 Ukrainian government increased nominal minimum wage by approximately 132%, in year 2018 - by 16% (Figure 1). Such changes in legal minimum wage level led to concerns about possible adverse effects of such a policy including higher inflation, higher unemployment rate and higher costs of production.

In countries with the legal minimum wage regime the minimum wage imposes lower bound on the amount of the reported part of the worker's wage. Consequently, when the minimum wage level increases it also can affect firms and workers being engaged in wage underreporting.

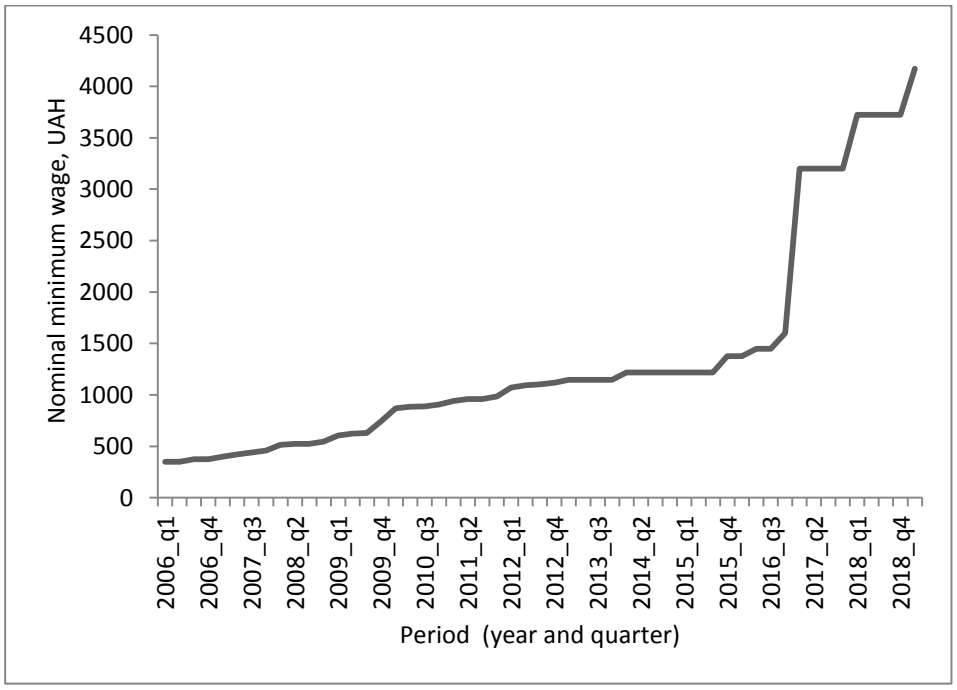

Figure 1. Nominal minimum wage dynamics in Ukraine

The indirect evidence of wage underreporting presence in Ukraine is 1) the large share of workers receiving the minimum wage and 2) the large degree of workers receiving "envelope" wages.

According to State Statistics Service of Ukraine, relatively large degree of workers is concentrated around the minimum wage level in Ukraine wage distribution. For instance, in year 2017 about 27.3-37.4% of workers were paid salaries close to the minimum wage level[1]. Even though this fact does not necessarily imply that about a third of wages are underreported in Ukraine, there is some evidence that there is a link between the share of workers receiving minimum wage and the degree of underreporting activity. For example, Tonin (2011) reports that there is a quite strong positive correlation between the share of minimum wage workers and the measure of shadow economy according to Schneider (2005) [2]. On figure 2 I replicate the exercise using share of minimum wage workers in the wage distribution of the country in year 2014 according to Eurostat and share of workers who receive envelope wage estimated by Williams (2013). The correlation between the variables on figure 2 is 0.45.

Common practice of paying "envelope wages" in Ukraine is supported by several studies. According to Williams (2007), 31% of workers were paid "envelope" wages in the year 2006. In addition, the recent poll by Head Hunter labor agency[3] finds that this practice has not changed and 29% of workers receive "envelope wages".

As stated in Lemos (2008), there are four channels through which the minimum wage increase affects the economy: aggregate demand, aggregate supply, input demand and input supply. When minimum wage goes up, the minimum wage workers are directly affected since their labor income increases. This stimulates demand for output of this group of workers. On the other hand, firms, which hire minimum wage workers, face higher labor costs. As a consequence, firms readjust their output, reset their prices in case

[1] According to ukrstat.gov.ua workers shares distribution by wage (share workers who were paid less than 4000 UAH in 2017)

[2] Reported correlation measure by Tonin is 0.279 and significant at 5% level

[3] https://kiev.hh.ua/article/20673

they have a monopoly power and also change the combination of inputs, decreasing the amount of hired minimum wage labor.

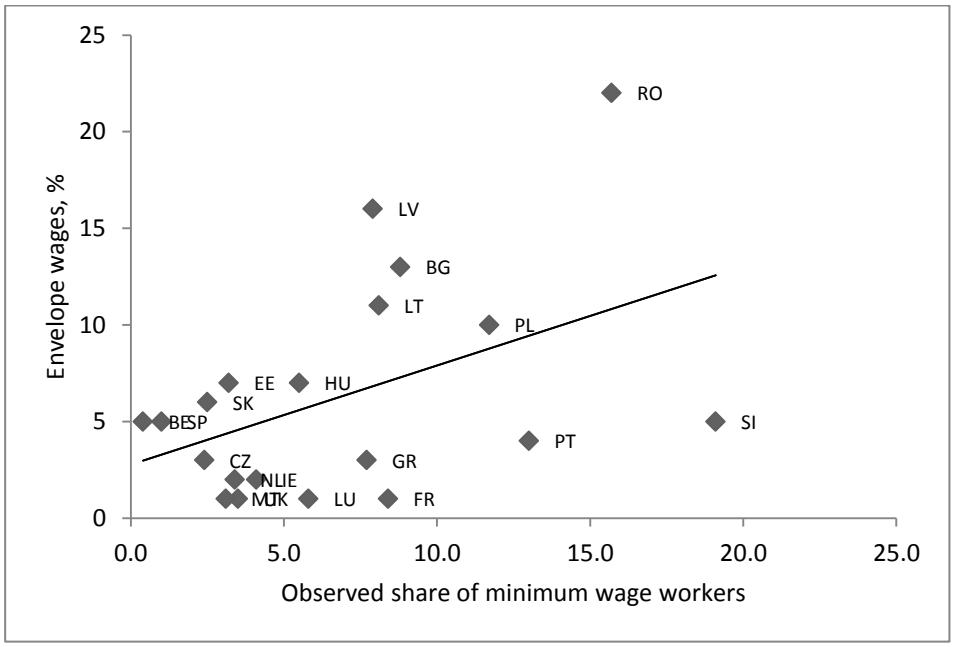

Figure 2. Share of minimum wage workers and % of envelope wages by country

However, in an economy with wage underreporting the minimum wage increase has different consequences. The disposable income of workers engaged in wage underreporting drops due to the fact that the larger part of their true wage becomes subject to taxation while firms face higher costs of labor input hired with underreporting. The costs of the true minimum wage labor increase more than the costs of labor hired with underreporting. Consequently, employment of the labor hired with underreporting will be less affected by the minimum wage increase than employment of the true minimum wage labor unless demand for underreporting labor is much more elastic with respect to changes in wage. Increase in the minimum wage in the economy with wage underreporting has less effect on employment comparing

to the economy where all the observed minimum wage workers are the true minimum wage workers. Since employment is less affected by the minimum wage increase, the government can potentially enjoy higher tax revenues. Higher tax revenues can affect the economy through the increase in government spending.

In addition, nominal minimum wage policy has important implications in an environment with nominal rigidities. If there is positive trend inflation, which is a reasonable assumption for the most of the countries including Ukraine, inflation "eats up" the real value of the nominal minimum wage causing the government to review the nominal minimum wage value again. If a government increases nominal minimum wage in an economy with sticky prices, real minimum wage immediately goes up. This increase leads to a temporarily higher real wages of the true minimum wage workers and to the lower real wages of the workers engaged in wage underreporting. The word "temporarily" is used to emphasize that positive inflation will eventually bring the real wages back to the steady state level. However, in the short run, since real income of households is affected, their consumption and saving decisions change affecting aggregate demand. The temporarily higher real labor costs, on the other hand, affect supply side of the economy and the tax revenues. The equilibrium outcome, according to the model discussed in the next sections, is that inflation rises and output drops.

Further discussion proceeds as follows. Chapter 2 reviews the relevant literature. Chapter 3 presents the model description and the description of the model extensions. Chapter 4 describes calibration and estimation of the model parameters. Chapter 5 presents the discussion of the results. Chapter 6 concludes.

Chapter 2

LITERARURE REVIEW

There are three main streams of literature related to the research question: minimum wage literature, informal economy literature and relevant monetary economics literature. The number of studies addressing the link between the minimum wage and informal activities is very limited since the conditions under which this link arises are observed only in a small set of countries. In this section I provide an overview of the papers looking separately into the effects of minimum wage increase and at the informal economic activities and review the studies relating minimum wage to informality. I conclude the review of the literature by featuring the monetary economics articles, which are relevant for modeling choices, calibration and estimation of the model.

Various effects of the minimum wage changes have been a subject of interest to the researchers for several decades. According to the predictions of the theory of the perfectly competitive labor markets, employment should drop when the minimum wage increases. But many empirical studies, starting from Card and Krueger (1995), show that employment is not affected very strongly by the minimum wage shocks. For instance, Card and Krueger (1995) estimate the effects of minimum wage increase in the US and find that employment is positively affected. Dickens et al. (1999) study the employment consequences of minimum wage increase in the UK and find that employment was not affected. The most popular theory attempting to explain lack of response of employment to minimum wage shock features presence of monopsony power on the labor market (Dickens et al., 1999; Dube, Lester and Reich, 2011). However, Aaronson and French (2006) in their study of the US fast-food industry challenge this theory by showing that monopsony power is not a key factor in explaining the lack of employment response to the minimum wage increase.

When minimum wage goes up, the costs of the firm increase. If firms pass these higher costs on prices, consumers become negatively affected by higher inflation. The survey paper by Lemos (2008) provides overview of the estimated effects of the minimum wage increase on prices and concludes that on average the effect of the minimum wage increase across studies is moderately positive. For instance, in the US the 10% minimum wage increase is associated with 0.4% price increase on average across studies. Lemos (2005) conducts a separate study for Brazil and finds that in Brazil 10% increase in minimum wage is associated with 3.5% increase in prices. Aaronson (2001) looks into prices in the restaurant industry in the US and finds that the minimum wage increase is passed onto consumers through higher prices.

Informal economy is generally studied in the literature in the broader sense than wage underreporting. Informal or shadow economic activity is any activity which is not criminal but covered from the tax authorities in order to reduce tax expenses. The size of shadow economies around the world is estimated by Schneider (2005) who finds that the size of shadow economy is the largest in the transition countries (about 40%). Wage underreporting is a particular type of informal activity which is closely related to paying so called "envelope wages". "Envelope wage" is the wage paid in envelope in cash with the purpose of reduction of the tax burden. Williams (2009) provides estimate of share of "envelope wage" workers for EU member countries. He shows that "envelope wage" practice is prevalent in East-Central European countries where about 11% of formal employees receive part of their salary unofficially. In context of Ukraine Williams (2007) estimates the share of "envelope wage" workers in Ukraine and finds it to be about 31%. Orsi et al. (2014) estimates the size of underground economy in Italy with help of neo-classical DSGE framework. The model includes two sectors: formal and underground. Firms in the underground sector don't pay taxes but each period face the probability of being audited. Author concludes that the size of the shadow economy is strongly affected by the tax rate. Presence of shadow economy has important

macroeconomic implications which were investigated by several authors. Castillo and Montoro (2010) look into inflation dynamics in the presence of informal labor markets using the NK DSGE model with labor market search frictions. They find that the presence of the informal economy diminishes the effect of demand shocks. Busato and Chiarini (2002) investigate business cycle implications of shadow economy with help of two sector DSGE model and find that the presence of the informal sector presents smoothing opportunities for households. Cesaroni (2014) addresses the link between the allocation of labor between formal and informal sectors and the downward nominal wage rigidities in the formal sector. The empirical studies addressing the cyclicality of formal and informal employment find that informal employment is pro-cyclical see for instance Bovi (2007) - hence the informal economy also affects business cycle.

Minimum wage is a special type of the downward nominal wage rigidity. The effect of the downward nominal wage rigidity was addressed by, for instance, Benigno and Ricci (2011), Schmitt-Grohe and Uribe (2013). As noted in Schmitt-Grohe and Uribe (2013), in the environment with positive trend inflation, the downward nominal wage rigidity loses its power. Nominal minimum wage, on the other hand, remains distorting even under positive inflation since government increases it every other period. Presence of high trend inflation makes government to review the nominal minimum wage level frequently. Presence of price rigidities creates a link between nominal minimum wage and real minimum wage. There are two most common ways of modeling price rigidities in the DSGE literature: staggered contracts by Calvo(1983) and price adjustment costs by Rotemberg(1982). Presence of trend inflation influences the region of determinacy of the DSGE model, see Ascari (2012). Ascari shows that under positive trend inflation the region of determinacy with Rotemberg-type price adjustment costs expands. With Calvo-type price stickiness positive trend inflation may lead to absence of unique steady state equilibrium.

Legally established minimum wage imposes lower bound on wage reported to the tax authorities. Firms engaged in paying "envelope wages" in countries with the minimum wage regime usually report the minimum wage to the tax authority and pay the rest of the worker's salary "in envelope". The link between the minimum wage regime and "envelope wages" was addressed in study by Tonin (2011). The author points out that a large observed share of minimum wage workers could be an indicator of underreporting activity, when firms and workers choose to report legal minimum wage instead of the true wage to tax authorities in order to cut tax and social security contributions costs. Such a situation arises when the detection of tax evasion is not perfect, for instance, due to weak institutions. Under such conditions minimum wage increase affects the economy also through the "fiscal channel", as the attractiveness of wage underreporting activity drops and the tax revenues increase. On the empirical side Tonin (2011) looked into the change in consumption of those Hungarian households, which should have been positively affected by the minimum wage increase. He found that consumption actually dropped following the minimum wage hike, which may indicate the presence of wage underreporting in the Hungarian economy. Also Tonin (2011) points, interestingly, that in the economy with underreporting the response of employment to the minimum wage increase is also lower even if labor markets are perfectly competitive. Since firms in the economy are different in terms of size, they face different risks of being audited by tax authorities. Small firms are less likely to be audited and, consequently, have more incentive to engage in underreporting activities. Feldina and Polanec (2012) looked into the employment effects of the minimum wage increase in Slovenia for firms of different sizes and found out that employment in small firms is less affected by the minimum wage increase, which may, again, point to underreporting of earnings by small firms.

There are only a few studies incorporating the minimum wage into the framework of DSGE models. Heberer (2010) added the minimum wage into the standard dynamic general equilibrium model and found that the economy was generally hurt by the minimum wage. Porter and Vitek (2008) looked into the business cycle effects of introducing the minimum wage in the Hong Kong SAR. They found that the economy was generally more volatile under the minimum wage regime, since binding minimum wage reduced the flexibility of labor markets. Glover (2018) investigates whether the minimum wage policy is expansionary or contractionary during zero-lower bound times with help of modified New-Keynesian DSGE model. Not very much attention is devoted to minimum wage increase in the DSGE literature because in most of the countries the share of workers affected by the minimum wage increase is relatively small, unlike Ukraine, where it is suspected that a large part of the economy is affected by the minimum wage increase through "fiscal channel".

Since for the purposes of checking the robustness of the obtained results the model is modified in several directions discussed in the introduction, it is worth mentioning the literature on which these modifications are based.

Labor market frictions are added into the present model in form of the quadratic labor adjustment costs, the approach used in Lechthaler et. al (2013). As has been shown in Lechthaler et. al (2013), the presence of quadratic labor adjustment costs produces similar results in terms of output persistence as the models with more complicated labor market setups.

The open economy aspects in the present study are modeled in the manner similar to Justiniano and Preston (2010). Justiniano and Preston (2010) developed the small scale model of the small open economy featuring imports, exports and access to foreign financial markets. Schmitt-Grohe and Uribe (2003) show that debt elastic risk premium ensures stationarity in the open economy model. I follow this approach in the present study.

Since the resulting model will be calibrated and estimated for Ukraine, it is also worth mentioning the relevant articles used for the calibration process. Since, to the best of my knowledge, there are no estimates for the elasticity of substitution between low and high skilled labor for Ukraine, I rely on the estimate by Behar(2010) for developing countries. The subset of parameters, which cannot be calibrated, will be estimated by using Bayesian estimation technique. The approach for choosing priors is to use the same priors, which were used in Shintani (2016) for the price adjustment cost parameter and Iacovello et al (2015) for the rest of the estimated parameters.

Chapter 3

THE MODEL

In this section I describe the model and the two model extensions: open economy extension and the non-zero labor adjustment costs extension.

The present model is a modified and extended version of the textbook New-Keynesian Dynamic stochastic general equilibrium model (see Gali, 2008). The time of the model is discrete and the time horizon is infinite.

There are two types of labor according to skill level: low-skilled labor and high-skilled labor. Low-skilled labor is hired at the minimum wage level while the high-skilled labor is hired at the wage above the minimum wage level. High-skilled labor is hired at two labor markets: formal labor market and informal labor market. If a firm hires a worker at formal labor market it fully pays tax on the workers true wage. If a firm hires a worker at the informal labor market it pays tax on the minimum wage regardless of the true wage paid to the worker.

Each period the representative firm faces the probability of being audited by the tax authority in which case it pays the fine for each worker hired with underreporting. Similar framework is used for instance by Orsi (2014) for modeling informal economic activity.

The nominal minimum wage grows at stochastic growth rate. The average growth rate of the nominal minimum wage equals to average inflation which ensures that the *real* minimum wage has a well-defined steady state level.

Households

Representative household supplies two types of labor: low-skilled labor and high-skilled labor. Low-skilled labor is assumed to be supplied under legally established minimum wage. It is assumed that at the minimum wage level supply of low-skilled labor exceeds demand and, consequently, the amount of employed low-skilled labor is fully demand constrained. That is, household will always chose to supply as much low-skilled labor as is demanded under the minimum wage.

High-skilled labor can be supplied formally or with wage underreporting. For the high-skilled labor supplied with underreporting household derives additional disutility. The difference in after tax wages creates an incentive for household to supply both formal and informal high-skilled labor. Functional form of the utility function is close to Orsi (2014).

Household consumes aggregate consumption index and trade domestic private one-period bonds. It maximizes expected discounted lifetime utility:

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\left(\frac{c_t^{1-\sigma}}{1-\sigma}-\Gamma^h\frac{(n_t^f+n_t^i)^{1+\varphi}}{1+\varphi}-\Gamma^i\frac{(n_t^i)^{1+\varphi^i}}{1+\varphi^i}\right)\right. \quad (1)$$

where c_t- consumption index; n_t^f- high-skilled labor supplied formally; n_t^i- high-skilled labor supplied with underreporting; σ - relative risk aversion coefficient; φ - inverse Frisch elasticity of high-skilled labor supply; φ^I- inverse Frisch elasticity of labor supplied with underreporting; Γ^h, Γ^i- preference parameters controlling for relative disutility from different types of labor; β- time discount factor for utility.

Each period budget constraint of a household is given by:

$$P_t c_t + B_t = W_t^{min} n_t^u + W_t^f n_t^f + W_t^i n_t^i + R_{t-1} B_{t-1} + D_t \quad (2)$$

where P_t- price of consumption index; R_t- nominal interest rate; B_t- nominal bonds purchases; W_t^{min} - nominal minimum wage (wage paid for low-skilled labor); W_t^f - nominal wage for formal high-skilled labor; W_t^i - nominal wage for underreporting high-skilled labor; n_t^u- low-skilled labor hours worked; n_t^f - high-skilled formal labor hours worked; n_t^i - high-skilled semi-formal labor hours worked; D_t - other transfers to households (profits from owning the firms).

First-order conditions with respect to c_t, n_t^f, n_t^i, B_t:

$$c_t^{-\sigma} = P_t \lambda_t \quad (3)$$

$$\Gamma^h (n_t^f + n_t^i)^{\varphi} = W_t^f \lambda_t \quad (4)$$

$$\Gamma^h (n_t^f + n_t^i)^{\varphi} + \Gamma^i (n_t^i)^{\varphi^i} = W_t^i \lambda_t \quad (5)$$

$$\lambda_t = \beta E_t \lambda_{t+1} R_t \quad (6)$$

From household preferences it follows that the discount factor for future nominal payments is:

$$Q_{t,t+1} = E_t \left(\beta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1}{\pi_{t+1}} \right) \quad (7)$$

where π_t – inflation rate.

From the above equations the supply for informal labor can be written as:

$$\Gamma^i (n_t^i)^{\varphi^i} = (W_t^i - W_t^f)\lambda_t \tag{8}$$

That is, the amount of informal labor supplied is positively related to the wage differential between informal and formal wage.

Firms

Production of final consumption index is split into three stages: production of intermediate good using labor input, producing differentiated final goods from the intermediate good by brand naming, aggregating differentiated final goods into final output.

Final good index

The continuum of differentiated final goods is aggregated into final output via CES aggregator:

$$Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{9}$$

Where Y_t - final output; $Y_t(i)$ - final good produced by i-th producer; ϵ - elasticity of substitution between different final goods.

Demand for each particular final good $Y_t(i)$ is given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \tag{10}$$

Intermediate good production

Representative intermediate good producer hires all types of labor and produces homogeneous intermediate good. Low-skilled and high-skilled labor are hired on the respective markets. Markets for both formal and underreporting high-skilled labor are perfectly competitive and low-skilled labor is hired under the minimum wage. Intermediate good is produced according to technology:

$$Y_t^I = A_t L_t \tag{11}$$

where Y_t^I- intermediate output; L_t - labor input; A_t - total factor productivity.

Labor input is assumed to be CES aggregate of low-skilled and high-skilled labor:

$$L_t = (b(L_t^u)^{\frac{\epsilon_L - 1}{\epsilon_L}} + (1-b)(L_t^f + L_t^i)^{\frac{\epsilon_L - 1}{\epsilon_L}})^{\frac{\epsilon_L}{\epsilon_L - 1}} \tag{12}$$

where L_t^u- low-skilled labor; L_t^f - formal labor; L_t^i - underreporting labor; ϵ_L - elasticity of substitution between low-skilled and high-skilled labor; b - share parameter.

Total factor productivity is exogenous AR(1) process:

$$\frac{A_t}{A_{ss}} = (\frac{A_{t-1}}{A_{ss}})^{\rho_A} e^{\epsilon_t^A}, \epsilon_t^A \sim N(0, \sigma_A) \tag{13}$$

where A_t – total factor productivity at time t; A_{ss} – steady-state level of total factor productivity.

Intermediate good producer pays tax for each type of labor it employs. For underreporting labor firm pays tax only on the reported part of wage (which equals to the minimum wage). Each period intermediate good producer faces probability of being audited by tax authority. In case of being audited firm

pays fine equal to s minimum wage levels for each underreporting worker. Intermediate good producer minimizes expected costs given production level:

$$Cost_t = (1+\tau)W_t^{min}L_t^u + (1+\tau)W_t^f L_t^f + W_t^i L_t^i + \tau W_t^{min}L_t^i + psW_t^{min}L_t^i \quad (14)$$

where τ - tax rate; p- probability of being audited.

First order conditions with respect to L_t^u, L_t^f, L_t^i:

$$W_t^{min}(1+\tau) = \mu_t A_t b\left(\frac{L_t^u}{L_t}\right)^{-\frac{1}{\epsilon}} \quad (15)$$

$$W_t^f(1+\tau) = \mu_t A_t(1-b)\left(\frac{L_t^f + L_t^i}{L_t}\right)^{-\frac{1}{\epsilon}} \quad (16)$$

$$W_t^i + W_t^{min}(\tau + ps) = \mu_t A_t(1-b)\left(\frac{L_t^f + L_t^i}{L_t}\right)^{-\frac{1}{\epsilon}} \quad (17)$$

where $\mu_t = MC_t$- price of intermediate good.

Since formal and underreporting labor are assumed to be perfect substitutes, in equilibrium expected cost of formal labor equals to expected cost of informal labor:

$$W_t^f(1+\tau) = W_t^i + W_t^{min}(\tau + ps) \quad (18)$$

Final goods production

Presence of positive trend inflation makes government to review the nominal wage level often while presence of nominal rigidities makes nominal wage changes to affect the real variables.

The nominal rigidity in form of costly price adjustment under positive trend inflation is incorporated in the behavior of the final good producers.

There is a continuum of monopolistically competitive final goods producers. Each final good producer buys homogeneous intermediate good and converts it into differentiated final good according to technology:

$$Y_t(i) = Y_t^I(i) \tag{19}$$

where $\mathrm{Y_t^I}$ - amount of intermediate good bought by i-th firm.

Each period it faces real quadratic price adjustment costs a la Rotemberg (1982) per each unit of production. I depart from textbook NK DSGE model of sticky prices a la Calvo. The reason is that under positive trend inflation the region of determinacy of the standard NK DSGE model is smaller under Calvo type price stickiness (see Ascari and Rossi, 2012).

Each final good producer maximizes expected discounted stream of future profits subject to i-th final good demand constraint::

$$E_0 \sum_{t=0}^{\infty} Q_{0,t} \Bigg(P_t(i) Y_t(i) - MC_t Y_t^I(i) - P_t Y_t(i) \frac{\Phi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi_{ss} \right)^2 \Bigg) \tag{20}$$

where $\mathrm{MC_t}$ - nominal marginal costs of the firm (equal to the price of intermediate good); Φ - price adjustment cost parameter.

First-order condition with respect to $P_t(i) = P_t$:

$$Y_t(\epsilon - 1 - \epsilon\frac{MC_t}{P_t} - \epsilon\frac{\Phi}{2}\left(\pi_t - \pi_{ss}\right)^2 + \Phi(\pi_t - \pi_{ss})) = Q_{t,t+1}E_tY_{t+1}\Phi\pi_{t+1}^2(\pi_{t+1} - \pi_{ss}) \quad (21)$$

Government

Government finances its purchases of final output index by collected taxes:

$$P_tG_t = \tau(W_t^{min}L_t^u + W_t^fL_t^f + W_t^{min}L_t^i) + psW_t^{min}L_t^i \quad (22)$$

where G_t – government purchases of final good.

Central Bank

Central bank follows Taylor(1999) type reaction function:

$$\frac{R_t}{R_{ss}} = ((\frac{\pi_t}{\pi_{ss}})^{\rho_\pi}(\frac{Y_t}{Y_{ss}})^{\rho_y})^{1-\rho_R}(\frac{R_{t-1}}{R_{ss}})^{\rho_R}e^{\epsilon_t^R}, \epsilon_t^R \sim N(0, \sigma_R) \quad (23)$$

where R_{ss}, Y_{ss} and π_{ss} are steady state nominal interest rate, output and inflation.

Minimum wage

It is assumed that nominal minimum wage follows:

$$\frac{W_t^{min}}{W_{t-1}^{min}} = \pi_{ss}e^{\epsilon_t^W}, \epsilon_t^W \sim N(0, \sigma_W) \quad (24)$$

Resource constraint

Resource constraint in this economy is:

$$Y_t = C_t + G_t + Y_t \frac{\Phi}{2}(\frac{P_t}{P_{t-1}} - \pi_{ss})^2 \quad (25)$$

C_t – aggregate consumption.

Other market clearing conditions are:

$$C_t = c_t \quad (26)$$

$$L_t^u = n_t^u \quad (27)$$

$$L_t^f = n_t^f \quad (28)$$

$$L_t^i = n_t^i \quad (29)$$

The full set of non-linear equations in terms of stationary variables and the set of log-linearized equations are available in Appendix A1.

Extension: Open economy framework

Since Ukraine is an open economy it is useful to investigate whether the main findings of the present study hold in the open economy context. Next, I extend the model to include open economy aspects: imports, exports and international risk sharing. The open economy extension is based on Gali (2008), Justiniano and Preston (2010).

First, in the open economy context household has access to both domestic and foreign financial assets. Consequently, its one period budget constraint is:

$$P_t c_t + B_t + e_t B_t^f = W_t^{min} n_t^u + W_t^f n_t^f + W_t^i n_t^i + R_{t-1} B_{t-1} + e_t F i_{t-1} R_{t-1}^f B_{t-1}^f + D_t \quad (30)$$

where B_t^f – foreign assets denominated in foreign currency; e_t – nominal exchange rate; R_t^f – nominal interest rate paid on foreign assets; Fi_t - country specific debt-elastic risk premium (see Schmitt-Grohe and Uribe, 2003).

The first-order condition with respect to B_t^f:

$$\lambda_t e_t = \beta E_t \lambda_{t+1} e_{t+1} F i_t R_t^f \quad (31)$$

Combining the previous equation with the first order condition for domestic assets gives the modified uncovered interest rate parity condition:

$$R_t = F i_t R_t^f \frac{E_t e_{t+1}}{e_t} = F i_t R_t^f E_t \left(\frac{q_{t+1} \pi_{t+1}}{q_t \pi_{t+1}^f} \right) \quad (32)$$

where $q_t = \frac{e_t P_t^f}{P_t}$ – real exchange rate and P_t^f – foreign prices denominated in foreign currency; π_t – domestic CPI inflation; π_t^f – foreign price inflation, $E_t e_{t+1}$ – time $t$expected value of the next period nominal exchange rate.

Second, the consumption good bought by households and by the government is now assumed to be a CES aggregate of domestic good and imported good:

$$C_t + G_t = \left(\omega^{\frac{1}{\epsilon_c}} (C_t^d)^{\frac{\epsilon_c - 1}{\epsilon_c}} + (1-\omega)^{\frac{1}{\epsilon_c}} (\mathrm{C_t^m})^{\frac{\epsilon_c - 1}{\epsilon_c}} \right)^{\frac{\epsilon_c}{\epsilon_c - 1}} \tag{33}$$

where C_t^d – consumption of domestically produced goods by domestic agents; $\mathrm{C_t^m}$ – consumption of imported goods by domestic agents; ϵ_c – elasticity of substitution between domestic and imported consumption; ω – the degree of preferences towards the domestic goods consumption.

Given prices of domestic and imported goods P_t^d and P_t^m as well as the price of the resulting consumption index P_t, cost minimizing allocation of producing fixed amount of the consumption index yields the following demand equations for the domestic consumption and imported consumption:

$$C_t^d = \omega \left(\frac{P_t^d}{P_t} \right)^{-\epsilon_c} C_t^T \tag{34}$$

$$C_t^m = (1-\omega) \left(\frac{P_t^m}{P_t} \right)^{-\epsilon_c} C_t^T \tag{35}$$

Third, the foreign demand for export is given by:

$$Y_t^E = \left(\frac{P_t^d}{e_t P_t^f}\right)^{-\epsilon_E} Y_t^f \tag{36}$$

where Y_t^f – world output; ϵ_E – elasticity of export demand.

The resource constraint in the open economy now includes export:

$$Y_t = C_t^d + Y_t^E + Y_t \frac{\Phi}{2}\left(\frac{P_t}{P_{t-1}} - \pi_{ss}\right)^2 \tag{37}$$

The log-linear form of open economy extensions is presented in the Appendix A2.

Extension: Labor adjustment costs

Labor market frictions are introduced in the form of quadratic labor adjustment costs. Now intermediate good producer maximizes discounted stream of nominal profits and bares costs of labor adjustment proportional to output produced:

$$E_0 \sum_{t=0}^{\infty} Q_{0,t}\left(P_t^I Y_t^I - W_t L_t - P_t \frac{\Phi_L}{2}\left(\frac{L_t}{L_{t-1}} - 1\right)^2 Y_t^I\right) \tag{38}$$

where W_t – cost of CES labor service aggregate; L_t – amount of labor service; Φ_L – labor adjustment costs parameter.

Fist order condition of the firm and the log linearized first order condition are presented in Appendix A3.

Chapter 4

CALIBRATION AND ESTIMATION

In this section I describe calibration and estimation of the model for Ukraine. Most of the parameters are calibrated directly, while the rest is estimated using Bayesian estimation technique.

I set the coefficient of relative risk aversion σ to 1 which corresponds to logarithmic utility function. Average quarterly inflation π_{ss} is calibrated to 1.0298 given the average CPI inflation for 2016:Q1-2018:Q2. Utility discount factor is set to $\beta = \frac{\pi_{ss}}{R_{ss}}$ = 0.9989 where R_{ss} is the average quarterly return on 3-6 months deposits over period 2010:Q1-2018:Q2. Minimum wage to average wage ratio is calibrated as the average ratio for the period 2006:Q1-2017:Q4 to 0.33. Probability of being audited p is set to 0.006 given the ratio of the number of firms which Ukrainian fiscal authority plans to inspect per quarter to the total number of firms in Ukraine. Surcharge over minimum wage in case of detecting underreporting s is set to 10 according to the Ukrainian Labor Code, which prescribes a special fine in case of underreporting detection to be equal 30 amounts of monthly minimum wages adjusted for quarterly frequency. Tax rate τ is set 0.4 as sum of currently relevant income tax rate and social security tax rate. Steady state share of workers reporting the minimum wage is set to 0.273 as share of workers who earned less than 4000 UAH in wage distribution of Ukraine in the year 2017.

The rest of the parameters are estimated using Bayesian estimation technique. Prior distributions of the parameters are chosen in line with the existing literature. Prior distribution for inverse Frisch elasticity of labor supply φ is set to be Gamma with mean 1 and standard deviation 0.1. Prior distribution of inverse elasticity of informal labor supply with respect to wage differential

φ^I is set to Gamma with mean 1 and standard deviation 0.5. Prior distribution for elasticity of substitution between different monopolistically competitive final goods ϵ is Gamma with mean 6 and standard deviation 1.Prior for elasticity of substitution between unskilled and skilled labor ϵ^L is Gamma distributed with mean 2 and standard deviation 1. Prior for price adjustment costs parameter Φ is set to Gamma with mean 20 and standard deviation 10. Prior distributions for autocorrelation parameters ρ_A and ρ_R are set to Beta with mean 0.85 and standard deviation 0.1. Prior distributions for Taylor rule parameters ρ_π and ρ_y are set to Normal with means 1.7 and 0.1 and standard deviations 0.1 and 0.05 respectively. For the steady state share of informal labor in those reporting the minimum wage the prior distribution is chosen to be Beta with mean 0.69 and standard deviation 0.1. Prior distributions for the standard deviations of shocks ϵ_t^R, ϵ_t^A and ϵ_t^W are set to Inverse Gamma with mean 0.1 and standard deviation 0.05.

In the open economy modification export-to-GDP ratio, imports to GDP ratio and consumption to GDP ratio are calibrated to respective average ratios over the period 2016:Q1-2018:Q2 and set to 0.3348, 0.4753 and 0.9687 respectively. The elasticity of substitution between foreign and domestic consumption and the elasticity of export demand are set to 1 (see Gali, 2007 textbook).

The estimation is performed on three time series: GDP, inflation and minimum wage. The period of the data used for estimation is 2010:Q2 – 2018:Q2. Then the estimated parameter values are set to the mean values of the posterior distributions. The foreign debt to GDP ratio is set to 0.926 as the World Bank projection for year 2019. Posterior means of the estimated parameters are presented in Table 1.

Table1. Posterior means of estimated parameters

Parameter	Description	Posterior mean
φ	Inverse Frisch elasticity of labor supply	0.9989
φ^I	Elasticity of informal labor supply with respect to wage differential	0.8704
ϵ	Elasticity of substitution between final goods	5.4502
ϵ^L	elasticity of substitution between unskilled and skilled labor	2.2627
Φ	Price adjustment costs parameter	25.0521
ρ_A	TFP persistence	0.6491
ρ_R	Interest rate persistence	0.4912
ρ_π	Monetary policy response to inflation	1.6978
ρ_y	Monetary policy response to output gap	0.1211
$Shinf$	Steady state informal labor ratio	0.6809
ϵ_t^R	Monetary shock standard deviation	0.2166
ϵ_t^A	TFP shock standard deviation	0.0662
ϵ_t^W	Minimum wage shock standard deviation	0.1488

Chapter 5

DISCUSSION OF RESULTS

In this chapter I discuss the response of the model economy to the minimum wage shock.

Figure 3 shows impulse responses of the model economy to the minimum wage shock under different degrees of wage underreporting.

As we can see, the positive minimum wage shock brings increase in inflation and drop in output. When the minimum wage goes up costs of both low-skilled labor and underreporting labor increase and, consequently, marginal costs of the firms increase, which induces the firms to raise prices for their products. Under the high degree of wage underreporting the positive response of inflation and the negative response of output to the minimum wage increase are smaller in magnitude. When the degree of wage underreporting is high, the increase in the labor costs of the firms related to the minimum wage hike is smaller. This happens because if large share of those workers for whom the minimum wage is reported are indeed the underreporting workers, the true cost of labor for them will increase only by $\tau \Delta W_t^{min}$ and not by $(1+\tau)\Delta W_t^{min}$. Since labor costs don't increase as much, the decrease in employment is smaller and the drop in consumption is smaller. Moreover, when the degree of underreporting workers is high, there is an increase in tax revenues due to the minimum wage increase, and, consequently, there is an increase in government spending, since it is assumed that government runs balanced budget. Consequently, in case of high degree of underreporting, aggregate demand is supported by the increase in government spending and output drops less, comparing to the case of low degree of wage underreporting.

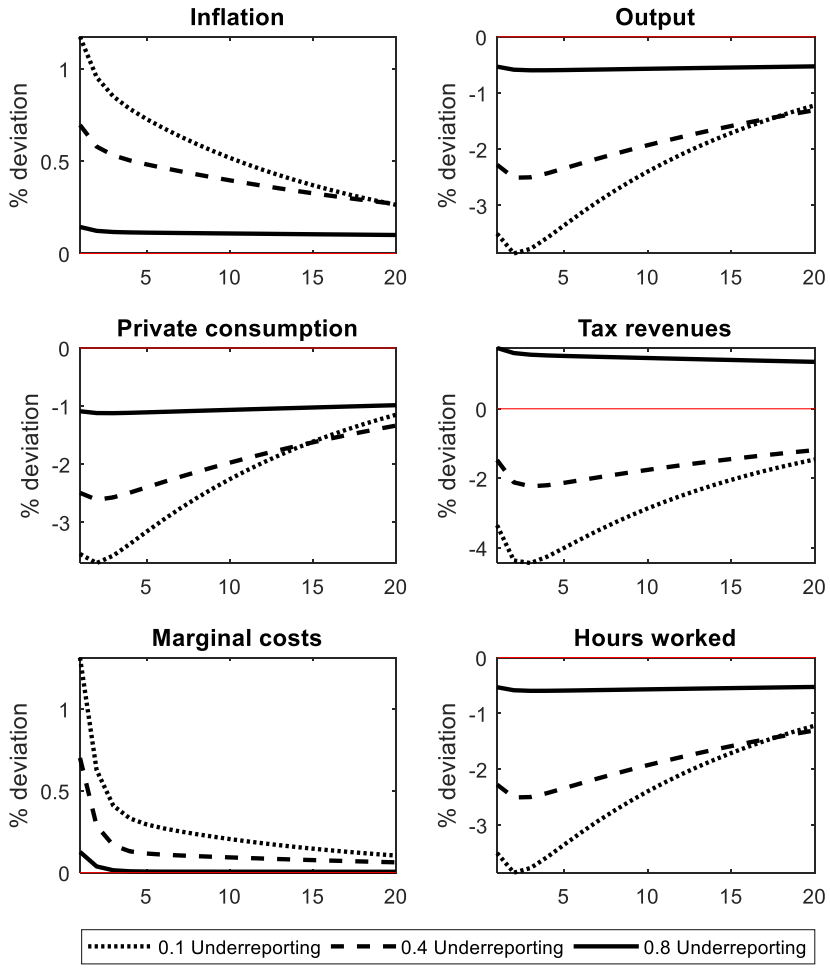

Figure3. Impulse responses to the minimum wage shock depending on the degree of underreporting

Figure 4 shows the initial response of inflation, output, marginal costs and tax revenues to the minimum wage shock. The strength of response of inflation, output and marginal costs linearly declines in share of wage underreporting. Also with high enough degree of wage underreporting tax revenues go up with the minimum wage increase.

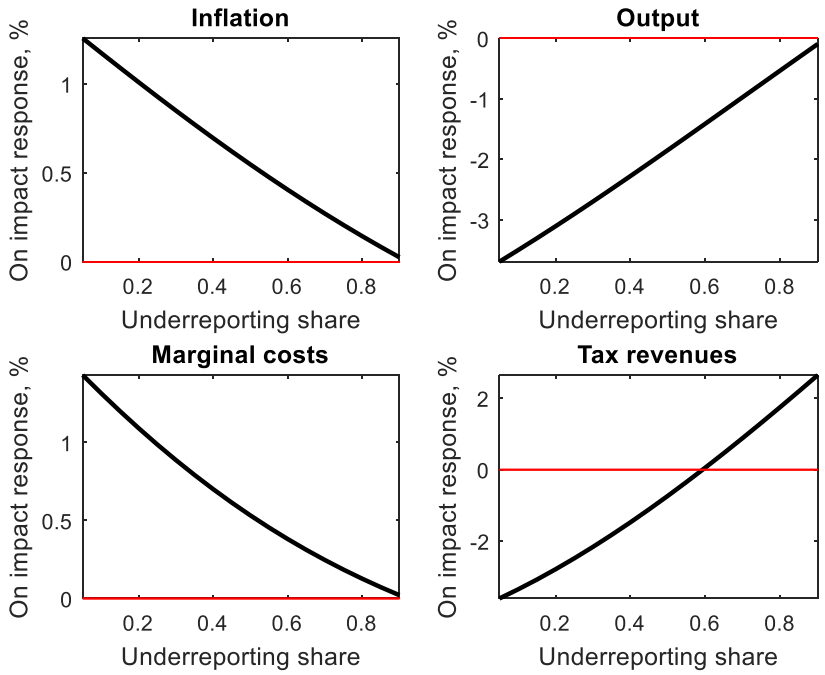

Figure 4. On impact response to the minimum wage shock depending on the degree of underreporting

Next I look at the response of the small open economy model to the minimum wage shock under different degree s of wage underreporting. As we can see on Figure 5, the result holds in the open economy context – the higher is the degree of wage underreporting the less responsive is the economy to the minimum wage shock. Quantitatively, however, the impact of the minimum wage increase in the small open economy differs from the one obtained in the closed economy. In the small open economy agents can substitute imported consumption for domestic consumption. Private consumption doesn't drop as much in an open economy as it does in a closed economy. Since labor supply is inversely related to the level of consumption, households are ready to supply less labor at any given level of wage in an open economy comparing to the closed economy. Consequently, the equilibrium marginal costs are affected more strongly in an open economy

context. Inflation is less affected in the open economy context since consumption basket also includes imported goods.

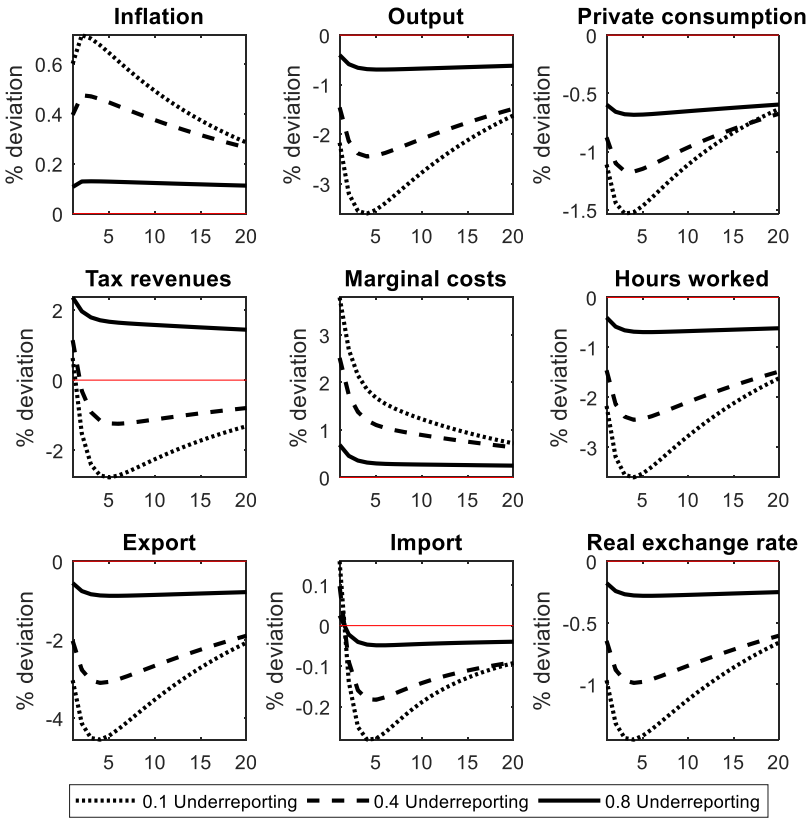

Figure 5. Impulse responses to the minimum wage shock in the small open economy model

Since Central Bank reacts to increase in inflation with higher domestic nominal interest rate, the domestic currency appreciates (real exchange rate drops). Stronger domestic currency and domestic inflation lead to decrease in export.

Finally I look at the effect of non-zero labor adjustment costs. If it is costly to adjust the amount of labor input in production, the marginal costs of

production increase more sharply with the minimum wage shock as you can see on figure 6.

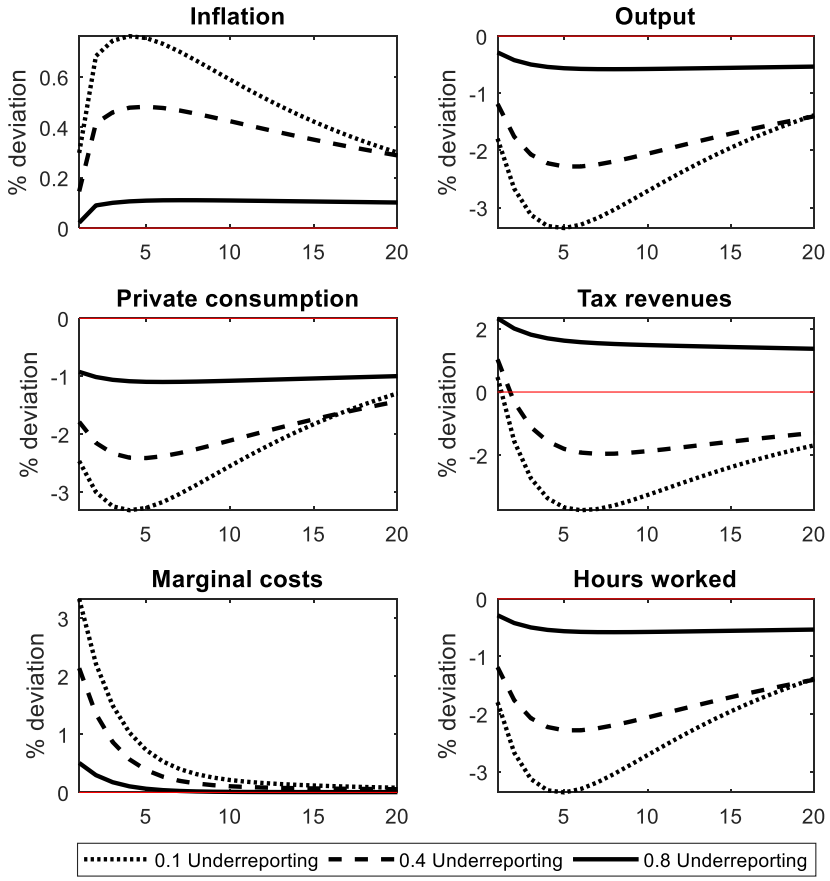

Figure 6. Impulse responses to the minimum wage shock with non-zero labor adjustment costs ($\Phi_L = 5$)

Also, the response of inflation to the minimum wage shock is hump-shaped in case of non-zero labor adjustment costs with peak at around 5 quorters. But the main result still holds – under higher underreporting an economy is less responsive to the minimum wage increase.

Chapter 6

CONCLUSIONS

Minimum wage increase affects the economy not only through labor and goods market channels but also through the "fiscal channel" since when minimum wage goes up the taxes paid by the minimum wage workers also go up. When wage underreporting is present both true minimum wage workers and the underreporting workers are affected by the minimum wage increase, but affected differently.

The response of the economy to the minimum wage shock depends on the share of underreporting workers in the observed share of the minimum wage workers. The larger is the share of underreporting workers, the less responsive is the economy to the minimum wage shock.

Since employment is less affected when the degree of wage underreporting is high and the increase in tax revenue per underreporting worker equals the increase in tax revenues per true minimum wage workers, government can achieve higher tax revenues as a result of the minimum wage increase.

The result holds in the open economy framework and in the model where changing the amount of labor input is costly.

An important policy implication is that in an economy with high degree of wage underreporting, the government can be less cautious about the negative effects of the minimum wage increase and even use the minimum wage increase as a tax collecting tool.

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APPENDIX A1

Full set of model equations in terms of stationary variables.

Household's first order conditions:

$$c_t^{-\sigma} = \lambda_t^{\{st\}}$$

$$\Gamma^h (n_t^f + n_t^i)^{\varphi} = w_t^f \lambda_t^{\{st\}}$$

$$(n_t^i)^{\varphi^i} = (w_t^i - w_t^f)\lambda_t^{\{st\}}$$

$$\lambda_t^{\{st\}} = \beta E_t \lambda_{t+1}^{\{st\}} R_t / \pi_{t+1}$$

where $w_t^f = \frac{W_t^f}{P_t}$, $w_t^i = \frac{W_t^i}{P_t}$, $\lambda_t^{\{st\}} = P_t \lambda_t$.

Discount factor for nominal payments:

$$Q_{t,t+1} = E_t \left(\beta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1}{\pi_{t+1}} \right)$$

Intermediate firm technology:

$$Y_t^I = A_t L_t$$

$$L_t = (b(L_t^u)^{\frac{\epsilon_L - 1}{\epsilon_L}} + (1-b)(L_t^f + L_t^i)^{\frac{\epsilon_L - 1}{\epsilon_L}})^{\frac{\epsilon_L}{\epsilon_L - 1}}$$

$$\frac{A_t}{A_{ss}} = (\frac{A_{t-1}}{A_{ss}})^{\rho_A} e^{\epsilon_t^A}, \epsilon_t^A \sim N(0, \sigma_A)$$

Intermediate firm first order conditions:

$$w_t^{min}(1+\tau) = mc_t A_t b (\frac{L_t^u}{L_t})^{-\frac{1}{\epsilon}}$$

$$w_t^f (1+\tau) = mc_t A_t (1-b) (\frac{L_t^f + L_t^i}{L_t})^{-\frac{1}{\epsilon}}$$

$$w_t^i + w_t^{min}(\tau + ps) = w_t^f(1+\tau)$$

where $w_t^{min} = \frac{W_t^{min}}{P_t}$, $\mathrm{mc_t} = \frac{\mathrm{MC_t}}{P_t}$.

Final good producers first order condition:

$$Y_t(\epsilon - 1 - \epsilon mc_t - \epsilon\frac{\Phi}{2}(\pi_t - \pi_{ss})^2 + \Phi(\pi_t - \pi_{ss})) = Q_{t,t+1}E_tY_{t+1}\Phi\pi_{t+1}^2(\pi_{t+1} - \pi_{ss})$$

Intermediate goods market clearing:

$$Y_t^I = Y_t$$

Government:

$$G_t = \tau(w_t^{min}L_t^u + w_t^f L_t^f + w_t^{min}L_t^i) + psW_t^{min}L_t^i$$

Central Bank:

$$\frac{R_t}{R_{ss}} = ((\frac{\pi_t}{\pi_{ss}})^{\rho_\pi}(\frac{Y_t}{Y_{ss}})^{\rho_y})^{1-\rho_R}(\frac{R_{t-1}}{R_{ss}})^{\rho_R}e^{\epsilon_t^R}, \epsilon_t^R \sim N(0, \sigma_R)$$

Minimum wage process:

$$\frac{w_t^{min}}{w_{t-1}^{min}} = \frac{\pi_{ss}}{\pi_t}e^{\epsilon_t^W}$$

Resource constraint:

$$Y_t = C_t + G_t + Y_t\frac{\Phi}{2}(\frac{P_t}{P_{t-1}} - \pi_{ss})^2$$

Labor markets clearing:

$$C_t = c_t$$

$$L_t^u = n_t^u$$

$$L_t^f = n_t^f$$

$$L_t^i = n_t^i$$

Full-set of log-linearized model equations:

$$-\sigma\Delta c_t = \Delta\lambda_t^{\{st\}}$$

$$\varphi\left(\frac{L_{ss}^f}{L_{ss}^f + L_{ss}^i}\Delta L_t^f + \frac{L_{ss}^i}{L_{ss}^f + L_{ss}^i}\Delta L_t^i\right) = \Delta w_t^f + \Delta\lambda_t^{\{st\}}$$

$$\varphi^i\Delta L_t^i = \left(\frac{w_{ss}^i}{w_{ss}^i - w_{ss}^f}\Delta w_t^i - \frac{w_{ss}^f}{w_{ss}^i - w_{ss}^f}\Delta w_t^f\right) + \Delta\lambda_t^{\{st\}}$$

$$\Delta\lambda_t^{\{st\}} = E_t\Delta\lambda_{t+1}^{\{st\}} + E_t\Delta R_t - E_t\Delta\pi_{t+1}$$

$$\Delta\, Y_t = \Delta A_t + \Delta L_t$$

$$\left(\mathrm{b} + (1-\mathrm{b})\left(\frac{\mathrm{L}_{\mathrm{ss}}^f + L_{ss}^i}{L_{ss}^u}\right)^{\frac{\epsilon_L - 1}{\epsilon_L}}\right)\Delta L_t = b\Delta L_t^u +$$

$$(1-b)\left(\frac{\mathrm{L}_{\mathrm{ss}}^f + L_{ss}^i}{L_{ss}^u}\right)^{-\frac{1}{\epsilon_L}}\left(\frac{L_{ss}^f}{L_{ss}^u}\Delta L_t^f + \frac{L_{ss}^i}{L_{ss}^u}\Delta L_t^i\right)$$

$$\Delta A_t = \rho_A\Delta A_{t-1} + \epsilon_t^A$$

$$\Delta w_t^{min} = \Delta mc_t + \Delta A_t - \frac{1}{\epsilon}\Delta L_t^u + \frac{1}{\epsilon}\Delta L_t$$

$$\Delta w_t^f = \Delta mc_t + A_t - \frac{1}{\epsilon}\left(\frac{L_{ss}^f}{L_{ss}^f + L_{ss}^i}\Delta L_t^f + \frac{L_{ss}^i}{L_{ss}^f + L_{ss}^i}\Delta L_t^i\right) + \frac{1}{\epsilon}\Delta L_t$$

$$\frac{w_{ss}^f}{w_{ss}^{min}}(1+\tau)\Delta w_t^f = \frac{w_{ss}^i}{w_{ss}^{min}}\Delta w_t^i + (\tau + ps)\Delta w_t^{min}$$

$$\Delta\pi_t = \frac{(\epsilon - 1)}{\Phi\pi_{ss}^2}\Delta mc_t + \beta E_t\Delta\pi_{t+1}$$

$$\frac{G_{ss}}{Y_{ss}}\Delta G_t = \tau(\frac{w_{ss}^{min}L_{ss}^{u}}{Y_{ss}}(\Delta w_t^{min}+\Delta L_t^{u}) + \frac{w_{ss}^{f}L_{ss}^{f}}{Y_{ss}}\ (\Delta w_t^{f}+\Delta L_t^{f})) +$$

$$(\tau + ps)\ \frac{w_{ss}^{min}L_{ss}^{i}}{Y_{ss}}(\Delta w_t^{min}+\Delta L_t^{i})$$

$$\Delta R_t = (1-\rho_R)\big(\ \rho_\pi \Delta\pi_t + \rho_y \Delta y_t\big) + \rho_R \Delta R_{t-1}\ + \epsilon_t^R$$

$$\Delta w_t^{min} - \Delta w_{t-1}^{min} = \ -\Delta\pi_t + \epsilon_t^W$$

$$\Delta Y_t = \frac{C_{ss}}{Y_{ss}}\Delta C_t + \frac{G_{ss}}{Y_{ss}}\Delta G_t$$

APPENDIX A2

Log-linearized UIP condition:

$$\Delta R_t = \Delta F i_t + \Delta R_t^f + E_t \Delta q_{t+1} - \Delta q_t + E_t \Delta \pi_{t+1} - E_t \Delta \pi_{t+1}^f$$

Log-linear form of CES consumption index:

$$\Delta C_t^T = \omega \Delta C_t^d + (1-\omega) \Delta C_t^m$$

Note: log-linearization is done around steady state in which $\frac{P_t^d}{P_t} = \frac{P_t^m}{P_t}$=1. See Gali (2008) textbook for further detail.

Log-linear form of domestic and imported consumption demand equations:

$$\Delta C_t^d = -\epsilon_c \Delta p_t^d + \Delta C_t^T$$

$$\Delta C_t^m = -\epsilon_c \Delta p_t^m + \Delta C_t^T$$

where $p_t^d = \frac{P_t^d}{P_t}$ and $p_t^m = \frac{P_t^m}{P_t}$ are real prices of domestic and imported goods in terms of consumption index. It is assumed that import is bought at the P_t^f so that $P_t^m = e_t P_t^f$. This yields $p_t^m = q_t$ that is the real price of imports is equal to the real exchange rate.

Log-linearized consumption good market clearing condition:

$$\Delta C_t^T = \frac{C_{ss}}{C_{ss}^T} \Delta C_t + \frac{G_{ss}}{C_{ss}^T} \Delta G_t$$

Export demand can be written as:

$$Y_t^E = \left(\frac{P_t^d}{e_t P_t^f} \right)^{-\epsilon_E} Y_t^f = \left(\frac{p_t^d}{q_t} \right)^{-\epsilon_E} Y_t^f$$

Log-linear export demand:

$$\Delta Y_t^E = -\epsilon_E \Delta p_t^d + \epsilon_E \Delta q_t + \Delta Y_t^f$$

Log-linear resource constraint:

$$\Delta Y_t = \frac{C_{ss}^d}{Y_{ss}} \Delta C_t^d + \frac{Y_{ss}^E}{Y_{ss}} \Delta Y_t^E$$

Also, note that Phillips curve should be now treated in terms of domestic inflation, not the CPI inflation

$$\Delta \pi_t^d = \frac{(\epsilon - 1)}{\Phi \pi_{ss}^2} \Delta mc_t^d + \beta E_t \Delta \pi_{t+1}^d$$

where $\pi_t^d = \frac{P_t^d}{P_{t-1}^d} = \pi_t \frac{p_t^d}{p_{t-1}^d}$ is domestic inflation; $mc_t^d = \frac{MC_t}{P_t^d} = \frac{mc_t}{p_t^d}$ marginal costs in terms of domestic prices. The log-linearized identities are:

$$\Delta \pi_t^d = \Delta \pi_t + \Delta p_t^d - \Delta p_{t-1}^d$$

$$\Delta mc_t^d = \Delta mc_t - \Delta p_t^d$$

APPENDIX A3

First order condition of the firm facing labor adjustment costs:

$$p_t^I A_t - w_t - 1.5\Phi_L A_t \left(\frac{L_t}{L_{t-1}}\right)^2 + 2\Phi_L A_t \left(\frac{L_t}{L_{t-1}}\right) +$$

$$Q_{t+1}A_{t+1}\left(\frac{L_{t+1}}{L_t}\right)^3 \pi_{t+1} - Q_{t+1}A_{t+1}\left(\frac{L_{t+1}}{L_t}\right)^2 \pi_{t+1} = 0$$

Where $p_t^I = P_t^I/P_t$ – real intermediate good price; $w_t = W_t/P_t$ – real cost of labor input.

Note that in this modification of the model the firms decisions regarding labor allocation between different types of labor are separated from the firms decision about the total amount of labor service which is the subject to quadratic adjustment costs.

Log-linearized condition is:

$$p_{ss}^I(\Delta p_t^I + \Delta A_t) - p_{ss}^I \Delta w_t + 0.5\Phi_L \Delta A_t - \Phi_L(\Delta L_t - \Delta L_{t-1}) + \beta\Phi_L(\Delta L_{t+1} - \Delta L_t) = 0$$