OPTIMAL RENTING/SELLING
STRATERGIES IN OLIGOPOLY
DURABLE GOODS MARKETS

by

Golovko Sergiy

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Approved by ____

Head of the KSE Defense Committee, Professor Roy Gardner

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Date ____________________________________

Kyiv School of Economics

Abstract

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This paper studies a simultaneous-move three-period model in which firms choose the durability of their goods, whether rent or sell and how much to produce. We show that a firm's profitability tends to improve if it lowers durability of its output. Then we show that the previously known results regarding commitment to the selling strategy are robust with respect to time if the firms make their renting/selling decisions at the beginning of each period. However, if the firms make their renting/selling decisions at the pre-play stage of the game, they are less likely to commit to the selling strategy, choosing instead some mix of renting/selling strategies.

If the firms are infinitely lived, they should be more patient to sustain trigger strategies when the good is less durable. Moreover, analyzing the model under different specifications of cooperation, we show that the firms should be more patient when they cooperate both in choosing renting/selling strategies and in choosing quantities than when they cooperate only in choosing renting/selling strategies.

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Chapter 1

INTRODUCTION

In this paper, we study the behavior of two oligopolists producing durable goods, such as cars, houses, refrigerators, clothes etc. Durable goods constitute a substantial part of overall consumption in modern economies. For instance, durable goods consumption in Ukraine is equal to around 30% of overall consumption. Not surprisingly, there has been a lot of research interest in analyzing durable goods markets.

In the real world we observe coexistence of firms that sell durable goods and firms that provide renting services for consumers, even so there is no clear evidence in literature that would explain this fact. Markets for cars and real estate markets are examples of markets where selling firms coexist with renting ones naturally. However, it is not difficult to show that firms make a higher profit from renting, Coase (1972), Bullow (1982).

On the one hand, monopolies can easily switch to renting strategies if there are no government restrictions. On the other hand, in some other market structures, such as oligopolistic ones, due to the competitive nature of the market it is not always possible to implement the renting strategy, Poddar (2004). Thus, in this thesis, a major focus is laid on explaining why renting/selling types of firms coexist in the market.

Bullow (1982) and Garella (1999) discuss robustness of the results obtained from the two-period monopolistic model if the number of periods is to increase to three periods. While they have established that their results are robust with respect to the number of periods, there is no evidence that the conclusion about the impossibility of implementing the renting strategy in the oligopoly market remains valid if the number of periods is to increase to three periods. So we study a natural extension of the two-period model to a three-period model.

Even if the renting strategy is better for a monopolistic firm, the monopoly can easily make up for the disutility associated with the selling strategy by decreasing the durability of the good, or by discriminating consumers either in price or in quality, or by requiring an appropriate cost for maintenance and repairs. As a result, there is a need in verifying whether an oligopoly can benefit from implementing one of this market tools. Specifically, in this thesis, we consider durability of the good as a variable of choice.

Under some circumstances, oligopoly firms prefer selling vs renting and there is a reason for them to cooperate and choose the renting strategy to get higher life-time profits. Since an oligopoly firm can use part of their monopolistic power to reduce the durability of their product, dynamic considerations come in play. So we also consider a simultaneous-move infinitely repeated model of competitive interaction among oligopolistic firms.

The remainder of this paper is structured as follows. In Chapter 2 a review of related literature is provided. In Chapter 3 we discuss the main results from two-period model. In Chapter 4, the three-period model is constructed, examined and compared with the two-period model. Also in this chapter we discuss the limitations of three-period model. In Chapter 5, we consider durable goods dynamic models for more than three periods. Our conclusions are presented in Chapter 6.

Chapter 2

LITERATURE REVIEW

The existing literature on the durable goods market theory devoted to comparing selling to renting strategies has two distinct directions. The first one is dedicated to the analysis of different possible market structures such as monopoly, oligopoly, socially concerned firms and mergers markets. The second direction is concerned with the analysis of different tools used by firms having monopolistic power such as decrease in durability, quality differentiation, choice of location and pricing commitment and reasons of firms to choose one or another strategy.

Market Structure Analysis

In their seminal papers Coase (1972) and Bullow (1982) show that renting is more profitable for a durable goods monopolistic firm than selling. The intuition behind this is that monopolists produce their products until marginal cost equals price. Durable goods produced today are also in use tomorrow and demand in the next period will be lower than in the previous one. This means that rational consumers expecting a fall in demand in the subsequent periods are unwilling to pay for the good too much in the current period. As a result, today's prices tend to decrease and monopoly behaves more or less likely to competitive firms.

Bullow (1982, 1986) uses a two-period model of the durable good monopoly market. Below, in Chapter 4, we consider extended version of their model to the case of an oligopolisctic market. However, it is worth mentioning that the main reason why this model is so popular (Malueg and Solow 1987, Suslow(1986), Goering(1993, 2005, 2008) and many others) is its simplicity. In fact, the model has some drawbacks and limitations. For instance, Goering(1993) adds uncertainty to consideration and shows that with a "small" level of uncertainty, the socially optimal durability is attained under a renting monopoly, not a selling one.

However, since in real world pure monopoly markets are rare, there have been several attempts to analyze other market structures. For example, Goering (2008) examines socially concerned firms using an extended version of Bullow (1982)'s two-period model. He shows that socially concerned firms prefer the renting strategy to the selling one because the renting strategy provides the socially optimal durability that coincides with the previous finding for the monopolistic case.

Oligopoly is another example of more realistic real world's market structure. Saggi and Kettas (2000), Poddar (2004), Sagasta and Saracho (2008) examine durable good oligopolies. Saggi and Kettas (2000) study an asymmetric duopoly case when both firms are renting and selling in each period. They show that the renting to selling ratio highly depends on the cost of production. More precisely increase in cost of production of firm leads to increase in renting to selling ratio for specific firm. Poddar (2004) using oligopoly's analog of two-period model show that in the case of duopoly, firms will be better off by renting than selling. In a contrast, since action "rent" turns out to be dominated by action "sell" (selling, selling) strategy profile is a unique Nash equilibria. Therefore, rational firms will sell, unless they cooperate. Sagata and Saracho (2008) consider a case when there are more than two firms in the market. They show that renting firms has "more" incentive to merging than selling ones.

In this paper we develop three-period model that is the extension version of two-period Poddar (2004)'s model. It was shown that under oligopoly, market structure (selling, selling) is the unique Nash equilibria strategy profile. We will show that under assumption that firms make their renting/selling strategies only in the first period, according to the three-period model with two-period durability of the good firms prefer to use some mix of selling/renting strategy while renting in some periods and selling in other ones. This finding partly explains coexisting of renting and selling firms in real world market.

Monopolistic Tools Analyze

Even so, in general firms are better off by renting than selling there are several reasons why firms prefer selling behavior instead of renting one. First of all, for some durable goods such as intermediate durable goods, some kind of clothes renting behavior is impractical and so impossible, Bullow (1982); other ones can be restricted to rent due to antitrust law, Bullow (1986).

The second reason is decreasing durability. Selling monopoly that produces durable good in general will prefer producing less durable goods, even in the case if increase in good's durability is costless, Bullow (1982), Basu (1987) with shorter durability will be better off by selling than by renting. In contrast, renting monopoly is better off by producing goods with higher durability, Malueg and Solow (1987). However, renting monopoly produces their goods with lower durability that is socially optimal, Goering (2005).

The other reason that is closely related to decreasing durability is discrimination in quality. If monopoly produces durable goods with the same quality level, the number of high-valuation consumers will decrease over time, and as a result in the future monopolist should provide low-demand consumers with cheaper goods. It means that rational high-valuation consumer predicts future decreasing in prices will be unwilling to pay too much today; that finally causes reduction in prices. In order to overcome this problem, monopoly can provide high-demand and low-demand consumers with different packages of quality of product and prices, Kumar (2002), Inderst (2008).

Monopoly can overcome even more if there is a possibility for resale trading in the market, Kumar (2002). In this case, monopolist will be better off by increasing quality of durable goods over time. As a result, high-valuation consumer will buy products with highest quality that currently available in the market and will resale this product to lower-valuation consumer in the next period of time when good with greater quality becomes available. It does not mean however, that future prices are going to rise when quality increases. Such situation we can observe in the computer market, where the quality of computer increases over time even so, the prices remain almost stable. Kuman (2002) shows that prices can even decrease in future.

Mann (1992) indicates that another possibility is to choose the appropriate cost of repairing. In this situation, even so the quality of goods increase over time, monopoly can be better off by selling than renting if used good are close substitutes to new one. Monopolist can get consumer surplus that correspond to the renting strategy by charging maintenance cost on relatively high level.

One more reason examined in the literature is the choice of location. Garella (2002) shows that under assumption that monopoly can charge delivery prices, the selling monopoly will overcome Coase problem and will get the same profit that under renting strategy. Also he indicates that "monopolist will not necessarily choose a social optimally location".

Maybe the most controversial reason why selling can be more profitable than renting, is commitment to sales strategy. If firms can credibly commit to a chosen price level, high demand consumers will not expect future reduction in prices and will buy in current period rather than postponing their buying decision till future periods, Suslow (1986). However, there is an incentive to deviate in future periods from previously announced strategy, and as a result there are too few circumstances when pricing policy can be credible, Garella (1999).

If monopoly can benefit from such market tools as reduction in durability, increasing delivery prices, increase maintenance cost then the reasonable question arises. Will be oligopoly firms better off by using such tools? We partly fill in this gap in the literature. Mainly we focus on two directions. First, we show that under assumptions of three-period model, firm will get higher profit with two-period product durability than with three-period product durability. Second, we construct trigger strategies for oligopoly firms corresponding to the case when oligopoly firm can control durability of its goods.

Chapter 3

TWO-PERIOD MODEL, MAIN RESULTS

In this chapter we discuss the results that follow from Poddar (2004)'s two-period model. These results will be used in Chapter 5 for analyzing trigger strategies in an infinite-horizon problem. They are also a cornerstone for the three-period model developed below and will be used for comparison purposes.

Consider an industry consisting of two firms. In each of two periods, the firms make a decision. At the first stage, they decide whether to rent or to sell, and at the second stage, they make a decision regarding the quantity of the product to be sold. Note that in the second period there is no difference between renting and selling, because consumers that buy the product at the second period use it only one period.

Assume that there is a continuum of consumers that live for two periods during which firms sell their products. Each of them is an expected utility maximizer, given the selling/renting strategies chosen by the firms. There is no secondhand market, it means that there is no possibility for reselling in the market. Without loss of generality, we assume that there is no discounting (for firms and consumers) in this dynamic model.

As was mentioned, in the second period, there is no difference between renting and selling. Therefore there are four possible scenarios: both firms rent, both firms sell, the first sells and the second rents, and finally the first firm rents and the second sells. Corresponding profits for each firm represented in Table 1 (Poddar 2004). For example, if the first firm rents and the second firm sells the profit of the first firm equals to $\frac{208}{1225}a^2$, and the profit of the second firm — $\frac{284}{1225}a^2$.

Table 1

Total Profits, Two-Period Model

Firm 1	Firm 2	
	Renting	Selling
Renting	$\frac{2}{9}a^2, \frac{2}{9}a^2$	$\frac{208}{1225}a^2, \frac{284}{1225}a^2$
Selling	$\frac{284}{1225}a^2, \frac{208}{1225}a^2$	$\frac{11}{64}a^2, \frac{11}{64}a^2$

Interesting fact is that in a scenario when one firm sells and another one rents the selling firm gets higher profit. The economical reason for this fact is straightforward: since only one firm sells, the prices in the second period expected to be higher than in the case if both sell, as a result consumers are willing to pay in first period higher price that gives some sort of monopolistic power to the selling firm. So, the strategy profile (renting, renting) is not Nash equilibria in this game because each player has an incentive to deviate.

In a contrast, (selling, selling) is a Nash equilibria because, for example, the strategy profile (selling, renting) produces a lower profit for renting firm, and so there is no incentive for any firm to deviate. By the way, the strategy profile (renting, renting) produces a higher profit for each of the firms than (selling,

selling). This situation is known in game theoretical literature as the prisoners' dilemma, and formally can be stated as a proposition:

Proposition 1. (Poddar, 2004) In a duopoly durable good market where firms are allowed to rent or sell; (selling, selling) turns out to be the unique dominant strategy equilibria. Moreover, since the players' payoffs corresponding to the strategy profile (renting, renting) are larger than their payoffs corresponding to the strategy profile (selling, selling), and each player's action "sell" always strictly dominates "rent," the first-period game can be described as a prisoners' dilemma.

C h a p t e r 4

THREE-PERIOD MODEL

In this chapter in a contrast to the previous one, we assume that consumers live for three periods. The basic assumptions about zero production cost, the absence of discount factor, the absence of secondhand market and the rationality of the consumers are the same as in the above two-period model.

The remainder of this chapter is structured as follows. In Section 4.1 the model with two-period durability under two different specifications of choosing renting/selling strategies is provided. Subgame perfect equilibria for the perfect durability case is found out in Section 4.2. Perfect durability case is analyzed in Section 4.3. In Section 4.4 we discuss how two oligopolists choose durability of their goods. Finally, in Section 4.5 we discuss the limitations of the model.

4.1 The Case of Two-Period Durability

The target of this subsection is to consider two different cases of strategic firm's behavior under assumption of two-period durability of a good. The first case is the case when firms make their renting/selling decisions in each period. As a result, similarly to two-period model there are two stages in each period. At the first stage, firms decide whether to rent or to sell, and at the second stage they make a decision regarding the quantity of the product to be sold.

The second case is the case when firms make their renting/selling decisions only in the first period. Here and below we will call the stage on which firms make their renting/selling decisions as a pre-play stage. Analogously to two-period game, in the final (third period) there is no difference between renting and selling.

On the pre-play stage the firms make their choices out of four plans: rent in two first periods, sell in two first periods, sell in one period and rent in another one.

As a result, there are 16 possible strategic scenarios with four strategies for each firm available. For example, one possible scenario is when the first firm rents in the first period and sells in the second period, while the second firm rents in both periods. For simplicity, through this paper, a scenario is represented by four capital letters (R-renting, S-selling), where the two first letters correspond to the first firm's strategy, and the other two to the second firm's strategy. For example, (SR, RR) is a scenario.

In order to find a subgame perfect equilibria for the first case, we use the backward induction technique. First, the third period will be examined, then the second, taking into account the players' choices in the third period, and finally the first period will be studied. We will show that regardless the firm's first period decision, firm sells in the second period. Even so, considering the first period we will study all 16 scenarios in order to use them for finding Nash equilibria for the second case.

The third period, both firms rent

In the third period, the firms face the linear demand curve $Q = a - P$ reduced by the quantity purchased in period 2. Note that demand in period 3 is not affected by goods consumed in period 1 because they are not in use anymore. Therefore, the firms' interaction can be described by the standard Cournot's duopoly. The price, quantity and profit for each firm are:

$$p^{3R} = \frac{a - q^{2S}}{3};\ q_t^{3R} = \frac{a - q^{2S}}{3};\ \pi_t^{3R} = \frac{\left(a - q^{2S}\right)}{9},$$

where R denotes renting, S – selling, the numerical upper index indicates period, t=1,2 – firms.

The second period, both firms rent

Similarly to the third period, the demand curve is affected only by the number of items sold in the first period. When the firms make decisions in the first period, they do not care about the third period at all because their decisions do not affect the demand in the next period. The price, quantity and total profit during two last periods for each firm can be described by those corresponding to the Cournot's equilibria strategy profile:

$$p^{2R} = \frac{a - q^{1S}}{3};\ q_t^{\ 2R} = \frac{a - q^{1S}}{3};\ \pi_t^{\ 2R} = \frac{(a - q^{1S})^2}{9} + \frac{a^2}{9}.$$

The second period, both firms sell

In this case the choice, made in the second period, affects the demand curve in the third period. As a result, the second period's demand curve should be modified in order to reflect the fact that the marginal consumer is indifferent between buying good in the second period, and waiting till the third period and then renting good for one period. This condition can be written as $2\left(a - q^{2S} - q^{1S}\right) - p^{2S} = \left(a - q^{2S} - q^{1S}\right) - p^{3R}$, where $q^{iS} = q_1^{\ iS} + q_2^{\ iS}, i = \overline{1,2}$.

Substituting the value of p^{3R} into the last expression, the demand curve for the second period can be written as $p^{2S} = \frac{4}{3}\left(a - q^{2S}\right) - q^{1S}$. The two-period (for the second and third periods) profit maximization problem for each firm is as follows:

$$\pi_t^{2S} = \max_{q_t^{2S}} \left(\frac{4}{3}\left(a - q^{2S}\right) - q^{1S} \right) q_t^{2S} + \frac{1}{9}\left(a - q^{2S}\right)^2, t = \overline{1,2}.$$

The first order conditions for this maximization problem are:

$$\frac{4}{3}\left(a - q^{2S}\right) - q^{1S} - q_t^{2S} + \frac{2}{9}\left(q^{2S} - a\right) = 0, t = \overline{1,2}.$$

Solving the last system simultaneously, the prices, quantities and total profits during last two periods for each firm are:

$$q_t^{2S} = \frac{5}{16}a - \frac{9}{32}q^{1S};\ p_t^{2S} = \frac{1}{2}a - \frac{1}{4}q^{1S};\ \pi_t^{2S} = \frac{11}{64}a^2 - \frac{11}{64}aq^{1S} + \frac{27}{256}\left(q^{1S}\right)^2, t = \overline{1,2}.$$

The second period, one firm sells and the other rents

Consider marginal consumer, she is indifferent between buying the good in the second period or waiting till the third period and then renting the good for one period. This condition implies:

$$2\left(a - q^{2S} - q^{2R} - q^{1S}\right) - p^{2S} = \left(a - q^{2S} - q^{2R} - q^{1S}\right) - p^{3R},$$

or after substituting in the last equality the expression for p^{3R} we get $p^{2S} = \frac{4}{3}\left(a - q^{2S}\right) - q^{2R} - q^{1S}$. In addition, equality $p^{2S} = p^{2R} + p^{3R}$ should hold to allow the selling and renting firms coexist in the market. Otherwise, if $p^{2S} > p^{2R} + p^{3R}$ nobody will buy, consumer will be better off by renting a good in the second and third periods; if $p^{2S} < p^{2R} + p^{3R}$ nobody will rent in the second period, instead consumers will buy the good in the second period and will use it for two periods. Combining the market clearing condition with the condition for the marginal consumer allow us to get the expression for the renting price: $p^{2R} = a - q^{2S} - q^{2R} - q^{1S}$.

Then the two periods profit maximization problem for the selling and renting firms are:

$$\pi^{2S} = \max_{q^{2S}} \left(\frac{4}{3}\left(a - q^{2S}\right) - q^{2R} - q^{1S} \right) q^{2S} + \frac{1}{9}\left(a - q^{2S}\right)^2,$$

$$\pi^{2R} = \max_{q^{2R}} \left(a - q^{2S} - q^{2R} - q^{1S}\right) q^{2R} + \frac{1}{9}\left(a - q^{2S}\right)^2.$$

The first order conditions for the profit maximization problem are:

$$\frac{4}{3}a - \frac{8}{3}q^{2S} - q^{2R} - q^{1S} + \frac{2}{9}\left(q^{2S} - a\right) = 0,$$

$$a - q^{2S} - 2q^{2R} - q^{1S} = 0.$$

Solving the above system of the two equations, the prices, quantities and total profits during last two periods for each firm are:

$$q^{2S} = \frac{11}{35}a - \frac{9}{35}q^{1S};\ p^{2S} = \frac{4}{7}a - \frac{2}{7}q^{1S};\ \pi^{2S} = \frac{284}{1225}a^2 - \frac{242}{1225}aq^{1S} + \frac{99}{1225}\left(q^{1S}\right)^2,$$

$$q^{2R} = \frac{12}{35}a - \frac{13}{35}q^{1S};\ p^{2R} = \frac{12}{35}a - \frac{13}{35}q^{1S};\ \pi^{2R} = \frac{208}{1225}a^2 - \frac{264}{1225}aq^{1S} + \frac{178}{1225}\left(q^{1S}\right)^2.$$

The second period analysis revised

The total profits that correspond to each of four considered above scenarios in the second period are represented in the Table 2:

Table 2

Total Profits, Three-Period Model, Second Period

Firm 1	Firm 2	
	Renting	Selling
Renting	π^{RR}, π^{RR}	π^{RS}, π^{SR}
Selling	π^{SR}, π^{RS}	π^{SS}, π^{SS}

$\pi^{RR} = \frac{(a - q^{1S})^2}{9} + \frac{a^2}{9}$ - is total profit that corresponds to situation when both firms rent;

$\pi^{SS} = \frac{11}{64}a^2 - \frac{11}{64}aq^{1S} + \frac{27}{256}(q^{1S})^2$ - is total profit that corresponds to situation when both firms sell;

$\pi^{SR} = \frac{284}{1225}a^2 - \frac{242}{1225}aq^{1S} + \frac{99}{1225}(q^{1S})^2$ - is total profit of selling firm that corresponds to situation when one firm sells and another one rents;

$\pi^{RS} = \frac{208}{1225}a^2 - \frac{264}{1225}aq^{1S} + \frac{178}{1225}(q^{1S})^2$ - is total profit of renting firm that corresponds to situation when one firm sells and another one rents.

In fact, if firms in period one makes a rational decision (choose the output that corresponds to positive profit) it can be shown that firms will sell in the second period. Moreover, the second period renting/selling strategic interaction can be described as a prisoner dilemma in a similar fashion we do for two-period model.

Proposition 2. In a duopoly durable good market where firms are allowed to rent or sell and two-stage interaction process is described by three-period model; for each subgame that follows after first period (selling, selling) turns out to be the unique dominant strategy equilibria unless firms are not profit maximizers in the first period. Moreover, since the players' payoffs corresponding to the strategy profile (renting, renting) are larger than their payoffs corresponding to the strategy profile (selling, selling), and each player's action "sell" always strictly dominates "rent," the second period subgame can be described as a prisoners' dilemma.

Proof. Let us show that player's action "sell" strictly dominates "rent" that will automatically mean that (selling, selling) strategy is a unique Nash equilibria. For this we need to show that $\pi^{SR} > \pi^{RR}$ and $\pi^{SS} > \pi^{RS}$, under assumption of profit maximizing firms in the first period $a > q^{1S}$:

$$\pi^{SR} - \pi^{RR} = 0.0096a^2 + 0.0247aq^{1s} - 0.0303(q^{1s})^2 \geq 0.004(q^{1s})^2 \geq 0,$$

$$\pi^{SS} - \pi^{RS} = 0.0021a^2 + 0.0436aq^{1s} - 0.0398(q^{1s})^2 \geq 0.0059(q^{1s})^2 \geq 0.$$

To end the proof of the proposition we need to show that $\pi^{RR} > \pi^{SS}$:

$$\pi^{RR} - \pi^{SS} = 0.0503a^2 - 0.0503aq^{1s} + 0.0056(q^{1s})^2 \geq 0.0056(q^{1s})^2 \geq 0.$$

Q.E.D.

The first period

There are four subgames that follow after pre-play stage of the game: RR – when both firms choose to rent in the first period, SR and RS – when one firm chooses to rent and other to sell in the first period, and SS – when both firms sell in first period. For case when firms make their renting/selling decisions in each period only SS second period scenario should be considered. However, we consider all four second period scenarios that will be used for analyzing the case when firms make their renting/selling choices on pre-play stage of the game.

The first period, RR subgame

In this case, decisions made in the first period do not affect decisions to be made in the next two periods. As a result, in the first period Cournot's equilibria is achieved with quantities $q_t^{1R} = \frac{a}{3}$, prices $p_t^{1R} = \frac{a}{3}$ and profit $\frac{1}{9}a^2$. For each scenario, the total profit is equal to $\frac{1}{9}a^2$ plus the profit obtained in the first period plus the profit obtained in the two other periods (derived in previous section) taking into account that $q^{1S} = 0$. The results are summarized in the Table 3:

Table 3

Total Profits, Three-Period Model, RR Subgame

Firm 1	Firm2	
	Renting	Selling
Renting	$0.3333a^2, 0.3333a^2$	$0.2809a^2, 0.3429a^2$
Selling	$0.3429a^2, 0.2809a^2$	$0.2830a^2, 0.2830a^2$

The first period, SS subgame

As before, the marginal consumer should be studied to identify the inverse demand curve in the period under consideration. There are different conditions for the marginal consumer depending on the strategies the firms choose in the second period. Let us consider all possible cases:

a) Scenario (SR, SR). The marginal consumer is indifferent between buying the good in the first period and waiting till the second period and then renting the good. It implies $2\left(a-q^{1S}\right)-p^{1S}=\left(a-q^{1S}\right)-p^{2R}$. Substituting the value of p^{2R} into the last equality the inverse demand curve is: $p^{1S}=\frac{4}{3}\left(a-q^{1S}\right)$.

b) The scenarios (SS, SR), (SR, SS) and (SS, SS). The marginal consumer is indifferent between buying the good in the first period and then renting the good for one period, and waiting till the second period and then buying the good. It implies $2\left(a-q^{1S}\right)-p^{1S}+\left(a-q^{1S}\right)-p^{3R}=2\left(a-q^{1S}\right)-p^{2S}$. Substituting the value of p^{3R} and p^{2S} into the last equality gives the inverse demand curves: $p^{1S}=\frac{11}{8}a-\frac{23}{16}q^{1S}$ for the (SS, SS) scenario, and $p^{1S}=\frac{47}{35}a-\frac{48}{35}q^{1S}$ for both (SS, SR) and (SR, SS) scenarios.

For each of the scenarios, the three-period maximization problem can be written as:

$$\pi_1^{1S}=\max_{q_1^{1S}} p^{1S}q_1^{1S}+\pi_1^{2j},$$

$$\pi_2^{1S}=\max_{q_2^{1S}} p^{1S}q_2^{1S}+\pi_2^{2j},\ j=S,R.$$

The first-order conditions for the maximization problem for each scenario give us two equations with two unknowns: q_1^{1S} and q_2^{1S}. Solving these two equations simultaneously, we compute the quantities to be produced by each of the firms in the first period. The corresponding profits are presented in the table below:

Table 4

Total Profits, Three-Period Model, SS Subgame

Firm 1	Firm2	
	Renting	Selling
Renting	$0.2830a^2, 0.2830a^2$	$0.2557a^2, 0.2818a^2$
Selling	$0.2818a^2, 0.2574a^2$	$0.2562a^2, 0.2562a^2$

SR and RS Subgames

By symmetry, we assume that the first firm sells and the second firm rents (in the other words, the SR subgame is considered). As in the SS subgame case, different conditions for the marginal consumer depending on which strategies the firms choose in the second period are to be specified. Moreover, in order to identify not only the inverse demand curve for the selling firm, but also the demand faced by the renting firm, some market clearing conditions should be analyzed. Let us derive each firm's inverse demand curve for the possible scenarios that are present in the SR subgame:

a) The scenario (SR, RR). The marginal consumer is indifferent between buying the good in the first period and waiting till the second period and then renting the good. It implies: $2\left(a-q^{1S}-q^{1R}\right)-p^{1S}=\left(a-q^{1S}-q^{1R}\right)-p^{2R}$. Substituting the value of p^{2R} into the last equality the inverse demand curve gives us: $p^{1S} = \frac{4}{3}\left(a - q^{1S}\right) - q^{1R}$. The marketing clearing condition give as the inverse demand curve for the renting firm: $p^{1R} = a - q^{1S} - q^{1R}$.

b) The scenarios (SS, RR), (SR, RS) and (SS, RS). Marginal consumer is indifferent between buying the good in the first period and then renting good during one period, and waiting till the second period and then buying the good. It implies $2(a - q^{1S} - q^{1R}) - p^{1S} + \left(a - q^{1S} - q^{1R}\right) - p^{3R} = 2\left(a - q^{1S} - q^{1R}\right) - p^{2S}$.

Substituting the value of p^{3R} and p^{2S} into the last equality gives the inverse demand curves: $p^{1S} = \frac{61}{48}a - \frac{43}{32}q^{1S} - q^{1R}$ for the (SS, RS) scenario, and $p^{1S} = \frac{47}{35}a - \frac{48}{35}q^{1S} - q^{1R}$ for both (SS, RR) and (SR, RS) scenarios.

The marketing clearing condition for each scenario is $p^{1S} = p^{1R} + p^{2R}$, which leads to the same inverse demand curve for the renting firm: $p^{1R} = a - q^{1S} - q^{1R}$.

As a result, for each scenario, the three-period maximization problem can be written as:

$$\pi^{1S} = \max_{q^{1S}} p^{1S}q^{1S} + \pi_1{}^{2j},$$

$$\pi^{1R} = \max_{q^{1R}} p^{1R}q^{1R} + \pi_2{}^{2j},\ j = S, R.$$

The first-order conditions for the maximization problem for each scenario are given by two equations that can be easily solved. The corresponding profits are:

Table 5

Total Profits, Three-Period Model, SR Subgame

Firm 1	Firm2	
	Renting	Selling
Renting	$0.3429a^2, 0.2809a^2$	$0.2964a^2, 0.2918a^2$
Selling	$0.3559a^2, 0.2359a^2$	$0.2856a^2, 0.2509a^2$

The whole game, firms make their decisions in each period

To summarize first period selling/renting strategic interaction, three-period firms' payoffs are represented in the table below:

Table 6

Total Profits, Three-Period Model, Whole Game

Firm 1	Firm2	
	Renting	Selling
Renting	$0.2830a^2, 0.2830a^2$	$0.2509a^2, 0.2856a^2$
Selling	$0.2856a^2, 0.2509a^2$	$0.2562a^2, 0.2562a^2$

It is easy to verify that the strategy profile (selling, selling) is the unique Nash equilibria for the first-period game, taking into account the players' actions in the second period. We single out with subgame perfect equilibria that can be formally stated in a form of proposition:

Proposition 3. In a duopoly durable good market where the firms are allowed to rent or sell, live for three periods, produce three-period durable goods and make their renting/selling decisions in each period, the strategy profile (selling, selling) in the first period and (selling, selling) in the second period turns out to be the unique subgame perfect equilibria.

The whole game, firms make their decisions in the first period

In this case, there are four strategies for each player. For example, the renting/selling (RS) strategy corresponds to the case when firm rents in the first period and sells in the second period. Above we considered all possible scenarios for this game. Let us summarize firms' profits in the table:

Table 7

Total Profits, Three-Period Model, Pre-Play Stage

Firm1	Firm2			
	RR	RS	SR	SS
RR	$0.3333a^2, 0.3333a^2$	$0.2809a^2, 0.3429a^2$	$0.2809a^2, 0.3429a^2$	$0.2359a^2, 0.3559a^2$
RS	$0.3429a^2, 0.2809a^2$	$0.2830a^2, 0.2830a^2$	$0.2918a^2, 0.2964a^2$	$0.2509a^2, 0.2856a^2$
SR	$0.3429a^2, 0.2809a^2$	$0.2964a^2, 0.2918a^2$	$0.2830a^2, 0.2830a^2$	$0.2557a^2, 0.2818a^2$
SS	$0.3559a^2, 0.2359a^2$	$0.2856a^2, 0.2509a^2$	$0.2818a^2, 0.2557a^2$	$0.2562a^2, 0.2562a^2$

From Table 7 we can easily find out all Nash equilibria. Our findings can be formally stated as a proposition:

Proposition 4. In a duopoly durable good market where firms are allowed to rent or sell, live for three periods, produce goods with two-period durability and make their renting/selling decisions in the first period; strategies profiles (SS, SS), (SR, RS) and (RS, SR) are Nash equilibria. Moreover, strategy (renting/renting) is strictly dominated by other three strategies (selling/selling), (selling/renting) and (renting/selling).

So, we find out strict evidence that firms will commit to selling strategy in the case when firms make their renting/selling decisions in each period. However, under another specification, when firms make their renting/selling decisions only in the first period there are also the possibility of choosing renting strategy in one period and selling strategy in another one. It is worth to mention that firms more likely to sustain equilibria (SR, RS) and (RS, SR) than equilibria (SS, SS) because of two main reasons. First of all, firms get higher profit in equilibria (SR, RS) and (RS, SR). Second, even if in a process of reaching one of two equilibria (SR, RS) or (RS, SR) firms misunderstand each other and both choose the same strategy (renting/selling) or (selling/renting) they pick up with higher profit that under (SS,SS) equilibria ($0.2830a^2 > 0.2562a^2$).

4.2 The Case of Three-Period Durability

In this section we discuss the results of three-period model for the case when durability of good is three periods (perfect durability). Since after first period firms observe permanent decrease in demand for good by quantity sold in first period, the subgame that follows after first period is two-period game with demand function $Q = \tilde{a} - P = a - q_1^S - P$. As a result, total profits during last two periods are:

Table 8

Total Profits, Three-Period Model, Three-Period Durability, Second Period

Firm 1 .	Firm 2	
	Renting	Selling
Renting	$\frac{2}{9}\left(a-q_1^S\right)^2, \frac{2}{9}\left(a-q_1^S\right)^2$	$\frac{208}{1225}\left(a-q_1^S\right)^2, \frac{284}{1225}\left(a-q_1^S\right)^2$
Selling	$\frac{284}{1225}\left(a-q_1^S\right)^2, \frac{208}{1225}\left(a-q_1^S\right)^2$	$\frac{11}{64}\left(a-q_1^S\right)^2, \frac{11}{64}\left(a-q_1^S\right)^2$

We can make the same conclusions that we do for two-period model. The main result is that in subgame that follows after first period strategy profile (selling, selling) is a unique Nash equilibria. The corresponding for this strategy profile quantities and prices in the second and the third periods are (Poddar 2004):

$$q_1^{2S} = q_2^{2S} = \frac{5}{16}\left(a-q^{1S}\right), p^{2S} = \frac{1}{2}\left(a-q^{1S}\right);$$

$$q_1^{3R} = q_2^{3R} = \frac{1}{8}\left(a-q^{1S}\right), p^{3R} = \frac{1}{8}\left(a-q^{1S}\right).$$

In this section we restrict our analysis only to the case when firms make their renting/selling decisions in each period and are not considering case when firm make their renting/selling decisions on pre-play stage because of its complexity and our belief that it less likely to give some additional insight into the problems discussed in this paper.

There are four subgames that follow after pre-play stage (when firms choose their renting/selling strategies): RR, SR, RS and SS and taking into account that in the second period firms choose to sell four corresponding scenarios are possible: (RS, RS), (SS, RS), (RS, SS) and (SS, SS).

Scenario (RS, RS)

Scenario (RS, RS) produce the same results in terms of total profit as for the case when durability is two periods. The reason for this is straightforward. Since in the first period both firms rent, the second and third periods' demand functions are not affected by first period's decision; but in the second period there is no difference between selling a good with two or three period durability because people will live only the remaining two periods. As a result $\pi^{RR} = 0.2830a^2$.

Scenario (SS, SS)

The marginal consumer is indifferent between buying the good in the first period, and waiting till the second period and then buying the good for two periods. It implies: $3\left(a - q^{1S}\right) - p^{1S} = 2\left(a - q^{1S}\right) - p^{2S}$ or after substituting in the last equality the expression for p^{2S} we get $p^{1S} = \frac{3}{2}\left(a - q^{1S}\right)$. The profit maximization problem that corresponds to this scenario can be written as:

$$\pi_t^{SS} = \max_{q_t^S} \frac{3}{2}\left(a - q_1^{1S} - q_2^{1S}\right) q_t^S + \frac{11}{64}\left(a - q_1^{1S} - q_2^{1S}\right)^2, t = \overline{1,2}\,.$$

The first order conditions for this maximization problem are:

$$\frac{37}{32}\mathrm{a} - \frac{37}{32}\mathrm{q}_2^{1S} - \frac{85}{32}\mathrm{q}_1^{1S} = 0,$$

$$\frac{37}{32}\mathrm{a} - \frac{37}{32}\mathrm{q}_1^{1S} - \frac{85}{32}\mathrm{q}_2^{1S} = 0.$$

Solving the last system simultaneously, the prices, quantities and total profits for each firm are:

$$q_1^{1S} = q_2^{1S} = 0.3033a,\, p^{1S} = 0.5902a, \pi^{SS} = 0.2056a^2 .$$

Scenario (SS, RS) and (RS, SS)

The marginal consumer is indifferent between buying the good in the first period, and waiting till the second period and then buying the good for two periods. This condition implies: $3\left(a - q^{1S} - q^{1R}\right) - p^{1S} = 2\left(a - q^{1S} - q^{1R}\right) - p^{2S}$ or after substituting in the last equality the expression for p^{2S} we get $p^{1S} = \frac{3}{2}\left(a - q^{1S}\right) - q^{1R}$. Additionally, from market clearing condition $p^{1S} = p^{2S} + p^{1R}$ we get inverse demand function for renting firm: $p^{1R} = a - q^{1S} - q^{1R}$. The profit maximization problem that corresponds to this scenario can be written as:

$$\pi^{SR} = \max\left(\frac{3}{2}\left(a - q^{1S}\right) - q^{1R}\right)q^{1S} + \frac{11}{64}\left(a - q^{1S}\right)^2,$$
$$\pi^{RS} = \left(a - q^{1S} - q^{1R}\right)q^{1R} + \frac{11}{64}\left(a - q^{1S}\right)^2.$$

The first order conditions for this maximization problem are:

$$\frac{37}{32}\mathrm{a} - \frac{85}{32}\mathrm{q}^{1S} - q^{1R} = 0,$$

$$\mathrm{a} - \mathrm{q}^{1S} - 2\mathrm{q}^{1R} = 0.$$

Solving the last system simultaneously, the prices, quantities and total profits for each firm are:

$$q^{1S} = 0.3043a,\ p^{1S} = 0.3478a, \pi^{SR} = 0.2949a^2, \pi^{RS} = 0.2042a^2.$$

The whole game

Table 9 summarizes three-period game:

Table 9

Total Profits, Three-Period Model, Three-Period Durability, Whole Game

Firm 1	Firm2	
	Renting	Selling
Renting	$0.2830a^2, 0.2830a^2$	$0.2042a^2, 0.2942a^2$
Selling	$0.2942a^2, 0.2042a^2$	$0.2056a^2, 0.2056a^2$

It is easy to verify that the strategy profile (selling, selling) is the unique Nash equilibria for the first-period game, taking into account the players' actions in the second period. The situation is quite familiar for us from two-period model, player's action "sell" strictly dominates player action "rent" even so the strategy profile (renting, renting) produces a higher profit for each of the firms than (selling, selling). This finding can be formally stated in a form of proposition:

Proposition 5. In a duopoly durable good market where the firms are allowed to rent or sell, live for three periods, produce two-period goods and make their renting/selling decisions in each period, the strategy profile (selling, selling) in the first period and (selling, selling) in the second period turns out to be the unique subgame perfect equilibria.

4.3 The Case of Asymmetric Durability

In this section we consider the game with asymmetric durability of the goods. One firm produces the good with two-period durability and another one with three-period durability. Without loss of generality, assume that first firm produces two-period durable good and second firm – three-period durable good. As in Section 3.3 we are studying only case when firms make their renting/selling strategies in each period.

In fact, the asymmetry arises only if two firms sell in the first period. If both firms rent in the first period the model can be viewed as three-period model with two-period durability of the good with corresponding firms' profits $\pi_1^{1R} = \pi_2^{1R} = 0.2830a^2$. The reason for this is the following. After first period consumers will consider three and two period durable goods as identical because these goods can be useful for them only during last two periods of their life. For the same reason the situation when in the first period the first firm rents and the second sells can be viewed as a model with three-period durability ($\pi_1^{1R} = 0.2042a^2, \pi_2^{1S} = 0.2942a^2$), and the situation when in the first period the first firm sells and the second rents can be viewed as a model with two-period durability ($\pi_1^{1S} = 0.2856a^2, \pi_2^{1S} = 0.2509a^2$).

Let us consider the situation when both firms sell in the first period. In the second period firms observe temporal decrease in the demand for good by quantity q_1^{1S} sold by the first firm in the first period, and the permanent decrease by quantity q_2^{1S} sold by the second firm. Since there is no difference in the second period between two and three period durability of the good, the subgame that follows after the first period can be viewed as a subgame of a model with two-period durability (with substituting a by $a - q_2^{1S}$).

It means that Proposition 2 holds and firms sell in the second period. The prices, quantities and total profits during last two periods for each firm are:

$$q_t^{2S} = \frac{5}{16}\tilde{a} - \frac{9}{32}q_1^{1S};\ p_t^{2S} = \frac{1}{2}\tilde{a} - \frac{1}{4}q_1^{1S};\ \pi_t^{2S} = \frac{11}{64}\tilde{a}^2 - \frac{11}{64}\tilde{a}q_1^{1S} + \frac{27}{256}\left(q_1^{1S}\right)^2, t = \overline{1,2},$$

where $\tilde{a} = a - q_2^{1S}$.

Clearly, in the first period firms will set-up different prices for their products. Thus, the marginal consumer is indifferent between buying two-period durable good in the first period and then renting the good for one period, and waiting till the second period and then buying the good. This condition implies: $3\left(a - q^{1S}\right) - p_1^{1S} + p^{3R} = 2\left(a - q^{1S}\right) - p^{2S}$ or after substituting in the last equality the expression for p^{2S} and p^{3R} we get $p_1^{1S} = \frac{11}{8}a - \frac{23}{16}q_1^{S} - \frac{11}{8}q_2^{S}$.

In the same time, the marginal consumer is indifferent between buying three-period durable good in the first period and waiting till the second period and then buying the good. It implies $3\left(a - q^{1S}\right) - p_2^{1S} = 2\left(a - q^{1S}\right) - p^{2S}$ or after substituting in the last equality the expression for p^{2S} we get $p_2^{1S} = \frac{3}{2}a - \frac{5}{4}q_1^{S} - \frac{3}{2}q_2^{S}$.

As a result, the three-period maximization problem can be written as:

$$\pi_t^{1S} = \max_{q_t^{1S}} p_t^{1S}q_t^{1S} + \pi_t^{2S}, \overline{t = 1,2}.$$

First-order conditions for the maximization problem are given by two equations that can be easily solved. The corresponding total profits for scenario (SS, SS) are: $\pi_1^{1S} = 0.2121a^2, \pi_2^{1S} = 0.2549a^2$.

The total profits for all scenarios are represented in Table 10:

Table 10

Total Profits, Three-Period Model, Asymmetric Durability

Firm 1	Firm2	
	Renting	Selling
Renting	$0.2830a^2, 0.2830a^2$	$0.2042a^2, 0.2942a^2$
Selling	$0.2856a^2, 0.2509a^2$	$0.2121a^2, 0.2549a^2$

As in the cases with two and three period durability the strategy profile (selling, selling) is a unique Nash equilibria. Another finding is that under equilibria firms with higher durability of a good get higher profit. Intuitively, firm that produces their goods with three-period durability has a strategic advantage in the first period by charging the prices on the higher level than prices for two-period durable goods. Our findings can be formally stated in a form of proposition:

Proposition 6. In a duopoly durable good market with asymmetric durability of a good, where the firms are allowed to rent or sell, live for three periods, and make their renting/selling decisions in each period, the strategy profile (selling, selling) in first period and (selling, selling) in second period turns out to be the unique subgame perfect equilibria. Moreover, in equilibria firm that produces their goods with higher durability gets higher profit.

4.4 Choice of Durability

Assume that additionally to choosing renting/selling strategies and quantities to be sold, firms choose the durability of their goods. We assume that each firm has a possibility to choose between three and two period durability. Firms choose the durability of their goods before choosing whether to rent or to sell and how much to produce in each period. However, since in the second and third periods there is no difference between selling the two and three period durable good, there is only reason to distinguish between choices of the durability made by firms in the first period.

As a result, the choice of the durability can be thought of as a pre-play stage. There are four possible scenarios after pre-play stage: both firms produce two-period durable good (discussed in Section 3.2), both firms produce three period durable good (discussed in Section 3.3) or one firm produce two period durable good while another produces three period durable good (discussed in Section 3.4). In each of this scenario firms prefer selling strategy for renting one in each of three periods. Table 11 summarizes the results of three-period model with the choice of durability:

Table 11

Total Profits, Three-Period Model, Choice of Durability

Firm 1	Firm2	
	Two period	Three perid
Two period	$0.2562a^2, 0.2562a^2$	$0.2121a^2, 0.2549a^2$
Three period	$0.2549a^2, 0.2121a^2$	$0.2042a^2, 0.2042a^2$

Firm's action "produce two-period durable good" strictly dominates action "produce three-period durable good". We single out with subgame perfect equilibria that can be formally stated in a form of proposition:

Proposition 7. In a duopoly durable good market where the firms are allowed to rent or sell, live for three periods, can choose the durability of good and make their renting/selling decisions in each period, the strategy profile where both firms choose two-period durability of good and selling strategy in each period is the unique subgame perfect equilibria.

4.5 Limitations

We show that oligopolists will commit to selling strategy if they can choose their renting/selling strategies in each period. It coincides with previously received results for two-period model (Poddar 2004). The same conclusions hold for two and three period durability cases as well as asymmetric durability case. However there is an incentive to commit to renting strategy in some periods if firms choose their renting/selling strategies in the first period only. Another finding is that in the presence of the choice of durability, firms will prefer to produce their goods with lower durability. However, there are some limitations of our model.

First, we restrict our analysis to three periods only. The reasonable question arises. What would happen if we consider strategic behavior of oligopolists when number of periods is more than three periods? Following the intuition behind our findings, especially Proposition 2, the permanent or temporary decrease in demand for goods does not affect strategical firm's renting/selling behavior. So, the results of three-period model can be used for analyzing subgame that follows after first period of four-period model, which in the same way can be used for analyzing five-period model.

Knowing that in the following periods it will be optimal to commit to selling strategy firms can choose between two options: either sell in current period, or switch to renting strategy. If firm switches to renting strategy it gives an incentive for opponent to sell their goods for higher prices than renting one and get extra profit in current period. Also in real world it may be costly to switch from one strategy to another one from period to period. In this case pre-play stage plays its important role and firms may commit to renting strategy in a similar fashion we showed in Section 3.2.

Second, we restrict our analysis to linear demand functions. We do this for two main reasons. On the one hand, we use it for comparison purpose with two-period model. On the other hand, even in the case of a linear demand functions our calculation is quite complicated and we did not find easy way to make some generalization for the case of other demand functions.

Third, we assume that there is zero cost of production and there is no asymmetry in a cost of production among firms. While this assumption is sufficient for our analysis of strategic firm's behavior (Poddar, 2004), the assumption of different cost of production two and three-period durable good may be crucial for the choice of durability analysis.

Since we show that oligopolists prefer to produce goods with lower durability they should be interested in finding ways to reduce the durability of their goods that may be associated with some additional cost. Intuitively, there is some level of durability after which it is not profitable anymore to reduce the durability of their goods. It happens due to both, reduction of prices and increasing in cost of producing less durable goods. If we assume that firms have the same access to technology that reduces durability of a good, they should both commit to this optimal level of durability.

Finally, the absence of discounting factor restricts our analysis in some sense. We argue that introducing discount factor is unlikely to change our main results. The first reason is that consumers similar to firms value tomorrow consumption less than today one. As a result, following Poddar(2004) the second reason is that firms get lower profit both under renting and selling strategies, and so it should not change the strategic renting/selling interaction of the firms. We provide further discussion of discounting problem in the next chapter.

C h a p t e r 5

REPEATED INTERACTION MODEL

In this section we discuss how firms' strategies are affected when firms will repeatedly interact infinitely many times. The main assumption to be made is that firms can restrict the durability of their goods to two periods. It can be made by several ways: increase quality of new supplied goods, introducing new style or simply decreasing durability of currently supplied goods. Even so this assumption can be violated in the real world, there are such things as a habit, technological process or catastrophe that cause consumers to change durable goods from time to time. Such dynamic processes when one goods are out of the use and others become popular in the market can be with some approximation described by repeated interaction model presented below.

Assume that the durability of the good is limited to two or three periods depending on type of model we are considering. Moreover, we assume that once firm introduces "new" good nobody will buy "old" goods, even so they may be still in the market. As a result, discussed above two-period and three-period models, both with two and three period durability of a good, can be viewed as infinitely interacted model with payoff matrix representing in Table 1, Table 6 and table 9 correspondingly. We assume that each consumer lives for three or two periods depending on model that is basis of our analysis. Therefore, there is no need to introduce discounting factor for consumers. Moreover, this assumption makes our model even more realistic because new generation of people is more likely to prefer new sort of goods, while old people may be opposed to any changes.

In infinitely interaction game, based on three-period model with two-period durability, there are two types of durability: durability because the good is out of the use (two periods) and durability due to introduction new type of good (three periods). Such situation we can observe in the computer or mobile phone markets. Mobile phone can be out of use due to damage of battery, screen or keyboard. In most cases, such damage can be fixed. However, once new type of mobile phone is introduced there is no need to supply spares for old models and such repair is impossible or too expensive in comparison with the price of a new model of the phone.

As previously, we assume that there is no discounting between two consequent periods in which one type of goods is in use. However, there is a discounting factor δ between period when "old" good was in use and period when "new" good is introduced. This assumption does not confine the analysis presented below because of two reasons. First of all, firms should be more patient between two consequent periods when one type of good is in the use because there is more or less stable demand of their goods in each period. However, once "new" good is introduced firms are highly interested in compensating their cost for developing new type of good as soon as possible. Second, even so there is a need to introduce this discounting between periods, the consumer discounting also should be introduced and this factors are very likely to compensate each other. Finally, we are more interested in comparison analysis of how patient firms should be under different assumptions and specification of the model rather than in particular values of discount factors.

There are two possible situations that will be considered. The first situation is when firms sustain renting strategy in each period. The second situation is when additionally to the sustaining renting strategy they produce half of the monopoly optimal output in each period. For each of these two cases critical value δ will be found that describes how patient firms should be so that trigger strategy profile remains credible.

For simplicity of representation of our results we will call each of two (three) periods during which one type of good is in use as a "sub-period" and two (three) periods together as a "period".

Trigger strategies when both firms follow renting strategies in each period

Consider trigger strategy in which each firm cooperates in period t playing renting strategy as long as other firm cooperates in all previous periods and rents their goods. However, if opponent did not cooperate and switch to selling strategy in some sub-period of period t-1, firm will choose selling strategy forever, including all remaining sub-periods of period t-1. Note, that there is no possibility to deviate in last sub-period of particular period because there is no difference between renting and selling. The results for each of three models (two-period model, three-period model with two-period durability and three-period model with three-period durability) are stated as the propositions.

Proposition 8.a In a duopoly durable good market where firms are allowed to rent or sell, consumers live for two periods and durability of the good is two periods, if the discounting factor $\delta \geq 0.16$, then the outcome where the players play their trigger strategies is a subgame perfect equilibria.

Proof. Consider particular period of time t1. Let us calculate the present value of infinite stream of payoffs if both firms play accordingly to the trigger strategies: $\sum_{i=0}^{\infty} \delta^i \pi_1^{1RR} = \frac{1}{1-\delta} \cdot \frac{2}{9} a^2$. If firm deviates in current period by playing selling strategy then firm gets $\pi_1^{1SR} = \frac{284}{1225} a^2$ in current period, but since after this period other firm will play selling strategy forever in the next period firm will get lower profit $\pi_1^{1SS} = \frac{11}{64}a^2$. As a result, present value that corresponds to deviation case is $\pi_1^{1RS} + \sum_{i=1}^{\infty}\delta^i \pi_1^{1SS} = \left(\frac{284}{1225} - \frac{11}{64}\right)a^2 + \frac{1}{1-\delta}\cdot\frac{11}{64}a^2$. From condition that present value will be higher when firm does not deviate than when it deviates we find that $\delta \geq 0.16$. *Q.E.D.*

Proposition 8.b In a duopoly durable good market where firms are allowed to rent or sell, consumers live for three periods and durability of the good is two periods, if the discounting factor $\delta \geq 0.11$, then the outcome where the players play their trigger strategies is a subgame perfect equilibria.

Proof. The present value of infinite stream of payoffs if both firms play accordingly to the trigger strategies: $\sum_{i=0}^{\infty}\delta^i \pi_1^{1RR} = \frac{1}{1-\delta}\cdot\frac{1}{3}a^2$. If firm deviates in the first sub-period by playing selling strategy then firm gets $\pi_1^{1SR} = 0.2856a^2 < 0.3333a^2 = \pi_1^{1RR}$ in current period, so it is clearly no sense to deviate in the first sub-period. If firm deviates in the second sub-period, there is no difference after first period with two-period model, and as a result it gets "cooperative" profit $\frac{1}{3}a^2$ in the first sub-period and $\pi_1^{2SR} = \frac{284}{1225}a^2$ in the second sub-period or totally $\pi^{dev} = 0.3429a^2$. After this period both firms will play selling strategies forever and will get lower profit $\pi^{1SS} = 0.2562a^2$. Present value that corresponds to deviation case is $\pi^{dev} + \sum_{i=1}^{\infty}\delta^i \pi_1^{1SS} = (0.3429 - 0.2562)a^2 + \frac{1}{1-\delta}\cdot 0.2562a^2$. From condition that present value will be higher when firm does not deviate than when it deviates we find that $\delta \geq 0.11$. *Q.E.D.*

The only difference between three-period model with three-period durability and three-period model with two-period durability is that after deviations firms will get lower profit, specifically $\pi_1^{1SS} = 0.2056$. So, we can state the next proposition without proof:

Proposition 8.c In a duopoly durable good market where firms are allowed to rent or sell, consumers live for three periods and durability of the good is three periods, if the discounting factor $\delta \geq 0.07$, then the outcome where the players play their trigger strategies is a subgame perfect equilibria.

Trigger strategies when both firms produce half of monopoly output in each period

Consider trigger strategy in which each firm cooperates in period t playing renting strategy and renting half of monopolistic output in each sub-period of period t, specifically $\frac{q^M}{2} = \frac{1}{4}a$, and get half of monopolistic profit $\frac{\pi^M}{2} = \frac{1}{8}a^2$, as long as other firm cooperates in sub-period t-1. However, if opponent does not cooperate and switch to the selling strategy on the first stage in some sub-period of period t-1, or deviates on the second stage and produces $q^* = \frac{3}{8}a$ amount of good with corresponding profit $\pi^* = \frac{9}{64}a^2 > \frac{1}{8}a^2$, firm will choose selling strategy with Cournot's quantities forever.

Proposition 9.a In a duopoly durable good market where firms are allowed to rent or sell, consumers live for three periods and durability of the good is two periods, if the discounting factor $\delta \geq 0.11$, then the outcome where the players play their trigger strategies is a subgame perfect equilibria.

Proof. The present value of infinite stream of payoffs if both firms play accordingly to the trigger strategies: $\sum_{i=0}^{\infty} 2\delta^i \frac{\pi^M}{2} = \frac{1}{1-\delta} \cdot \frac{1}{4} a^2$. If firm deviates in current period by playing selling strategy then firm gets $\pi_1^{1SR} = \frac{284}{1225} a^2$ in current period. If firm deviates on the second stage in the second sub-period of current period it gets totally $\pi^{dev} = \frac{\pi^M}{2} + \pi^* = \frac{17}{64} a^2 > \frac{284}{1225} a^2$. There is no need to consider deviation on the second stage in the first sub-period because it causes to π^* profit in the first sub-period, but lower than $\frac{\pi^M}{2}$ "punishment" profit in the second sub-period. After this period both firms will play selling strategies forever and will get lower profit $\pi^{1SS} = \frac{11}{64} a^2$. Present value that corresponds to deviation case is $\pi^{dev} + \sum_{i=1}^{\infty} \delta^i \pi^{1SS} = (\frac{17}{64} - \frac{11}{64}) a^2 + \frac{1}{1-\delta} \cdot \frac{11}{64} a^2$. From condition that present value will be higher when firm does not deviate than when it deviates we find that $\delta \geq 0.17$. *Q.E.D.*

Proposition 9.b

In a duopoly durable good market where firms are allowed to rent or sell, consumers live for three periods and durability of the good is two periods, if the discounting factor $\delta \geq 0.11$, then the outcome where the players play their trigger strategies is a subgame perfect equilibria.

Proof. The present value of infinite stream of payoffs if both firms play accordingly to the trigger strategies: $\sum_{i=0}^{\infty} 2\delta^i \frac{\pi^M}{2} = \frac{1}{1-\delta} \cdot \frac{3}{8} a^2$. Accordingly to proof of Proposition 8.b, if firm deviates to selling strategy it gets

$\pi^{dev1} = 0.3429a^2$, but if firms deviates on the second stage in third sub-period (similarly to proof of Proposition 8.b there is no need to consider deviations in the second and first sub-periods) it gets higher profit $\pi^{dev} = \pi^{dev2} = 0.3906a^2$. After this period both firms will play selling strategies forever and will get lower profit $\pi^{SS} = 0.2562a^2$. Present value that corresponds to deviation case is $\pi^{dev} + \sum_{i=1}^{\infty} \delta^i \pi^{1SS} = (0.3906 - 0.2562)a^2 + \frac{1}{1-\delta} \cdot 0.2562a^2$. From condition that present value will be higher when firm does not deviate than when it deviates we find that $\delta \geq 0.12$. *Q.E.D.*

Proposition 9.c is stated without proof because it can be proven by analogy with Proposition 9.b.

Proposition 9.c In a duopoly durable good market where firms are allowed to rent or sell, consumers live for three periods and durability of the good is three periods, if the discounting factor $\delta \geq 0.07$, then the outcome where the players play their trigger strategies is a subgame perfect equilibria.

Analysis of the results

First of all, according to repeated interaction model, the firms should be not so patient to sustain trigger strategies. In all six cases, that are considered, cooperation will be sustainable for $\delta \geq 0.17$. However, we should take into account, that we are requiring from firms to be absolutely patient ($\delta = 1$) during two consequent sub-periods within one period. It means that for correct interpretation, threshold δ-values should be normalized. Thus, got in Proposition 8.a threshold value is equal to $\frac{1+0.17}{2} = 0.59$, while according to the

Proposition 8.b for three-period model with two-period durability threshold value equals $\frac{1+1+0.11}{3}=0.69$.

Second, from Proposition 8.b and Proposition 8.c (as well as Proposition 9.b and Proposition 9.c) we can make a conclusion that firms should be more patient to sustain trigger strategies with shorter durability of their goods. The intuition behind this result is the following. As long as firms cooperate there is no difference between three and two-period durability of the good because renting strategy allows for firm to maintain prices on the same level during three sub-periods. However, when firms do not cooperate, longer durability of the good is associated with lower total profit, and as a result, with higher "punishment" for deviation.

Finally, we find out that firms that cooperate in renting/selling strategies should be less patient than firms that cooperate both in renting/selling strategies and in quantities of good to be produced. The explanation that firms, that cooperate both in renting/selling strategies and in quantities of good to be produced, have more possibilities to deviate (additionally in quantities) is not fully correct. Additional cooperation in quantities is not only associated with higher profit under deviation, but also with higher punishment due to absence of half of monopolistic profit in all subsequent periods after deviation. Our finding indicates that deviation in quantities to be produced is much more profitable than deviation in selling/renting strategies and, as a result, requires firms to be more patient in order to sustain trigger strategies.

C h a p t e r 6

CONCLUSIONS

In this paper we investigate a three-period simultaneous-move model for oligopoly market. We show that in the perfect durability case, the firms will commit to the selling strategy that supports previous findings for two-period models. Moreover, we extend our analysis to goods with a durability of two periods, asymmetric durability cases, and prove that our results are robust with respect to such changes.

We provide the reasons why firms may commit to the renting strategy. The one reason is the existence of a time lag between announcing a strategy and its implementation. We determine Nash equilibria for the case when the firms make their renting/selling decisions each period. Two Nash equilibria are a mixture of renting/selling strategies, with firms preferring the renting strategy in some periods and the selling strategy in the others.

The intuition behind our finding is the following. Selling is also a better choice for oligopoly firms as long as they make their renting/selling decisions each period. However, if the firms make their second period choices in the first period the story is different. There is no sense anymore in switching to the selling strategy each period if the opponent chooses to rent in the first period and sell in the second period. Instead, the firm sells in the first period, rents in the second to support a higher demand in the third period.

The second reason is the repeated nature of the firms' interaction. Under different assumptions and specifications, we consider trigger strategy equilibria. We show that it is easier to sustain the renting strategy when the good is more durable. At the same time, if, additionally to cooperation in choosing the renting strategy, the firms cooperate in quantities they should be more patient to be able to sustain trigger strategies.

However, the nature of this sustainability is different. Difficulties with sustainability for the case of short durability of a good are explained by lower punishment for deviation from renting strategy comparing with long durability case, while cooperation in quantities is less sustainable due to greater incentive to deviate from renting strategy.

A possible direction for further research can be found in the jeans market: It would be interesting to compare the results for monopoly and oligopoly market structures under the assumption that the cost of production increases when the good becomes less durable.

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