WEALTH CONCENTRATION IN THE TOP RANGES:

AN AGENT-BASED MODEL

by

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A thesis submitted in partial fulfillment of
the requirements for the degree of

MA in Financial Economics

Kyiv School of Economics

2012

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Abstract

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A simple random growth process describing wealth accumulation by a stochastic differential equation allows the agents to invest in (a) assets exposed to stock market fluctuations and (b) assets giving stable returns. The model sheds light on the origin of Pareto distribution of wealth, described by a single parameter. This Pareto coefficient serves as measure of wealth concentration among Forbes billionaires. Its high negative correlation with the equity index S&P 500 confirms that in periods of stock market booms, wealth concentration in the upper tails becomes more acute, while the opposite in case of financial crashes. The model is further enriched allowing agents to trade under bargaining power of the wealthier, so modeling the preferential attachment. In this agent-based model (ABM) the role of government resumes to taxation and redistribution. The numerical simulations suggest that bigger size of government results in wealth more equally spread out. Limited trade between agents has the same effect. However, the bargaining power of the wealthier makes wealth concentrates. The simulations results are confirmed empirically in a cross-country analysis, where the number of super wealthy individuals serves as proxy for level of wealth concentration when controlled for size of nation's wealth and population. Business and labor regulations in favor of the poorer, as well as reducing corruption seem to be efficient means to reduce wealth concentration in the top ranges.

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ACKNOWLEDGMENTS

I am grateful to KSE for endowing me with the necessary intellectual resources and skills to make reading this work not a waste of time. Thanks to KSE, I was given the opportunity to step in the area of economics, particularly finance, at another level. And it's not only about that, it's about new horizons opened, new targets set, and new visions born. And it's about new people and good friends which I am content to have met at KSE.

Probably, this experience wouldn't be possible without the free tuition offered by the Swedish Ministry of Foreign Affairs and the Swedish International Development Agency, and of course my family's funds. All due credits to my donors.

Not last, I am deeply thankful and humble to one special person for supporting me during these two years of KSE and giving me the equilibrium that can be easily lost in this rush. None of the sacrifices and efforts was in vain.

GLOSSARY

ABM. Agent-Based Model.

DDV. Discrete Dependent Variable.

IRR. Incidence Rate Ratio.

MLE. Maximum Likelihood Estimation.

NB. Negative Binomial.

OLG. Overlapping Generations.

SDE. Stochastic Differential Equation.

ZINB. Zero-Inflated Negative Binomial.

Chapter 1

INTRODUCTION

The number of billionaires represented 0.9% of world population in 2010 and they owned 36.1% of global wealth (Global Wealth Report 2011). The society is not a "social pyramid" where the proportion of rich to poor is increasing gradually from the lower niche to the top elite. It is rather a kind of "social arrow" with a very fat base where the majority live and struggle to pay bills and a very narrow top with a few plutocrats. That arrangement is valid for every society – it's something "in the nature of man" (Pareto 1897) and is generally cogent for all the countries around the world. In the top range of wealthy people this "inequality" follows power law distributions proved empirically long ago by Pareto (1897) and Zipf (1949), then more recently by Quadrini *et al.* (1997), Piketty (2001, 2003), Yakovenko (2005), Klass *et al.*(2006), Wolff *et al.* (2008). In simplified terms, a power law describes phenomena when extremely big events are rare while small ones are widespread. The frequency of an event (wealth) changes as a power of an underlying characteristic (size of wealth). Different from the "bell shaped" Gaussian distribution, the power law distribution would have long tails (for a formal discussion and basic notions used in the text, see Appendix A). The presence of power law was later found in many areas and it is considered by certain researchers as being very universal. For instance, Davis (1941) put it: "…no one however, has yet exhibited a stable social order, ancient or modern, which has not followed the Pareto pattern at least approximately".

The ones staying in the end of the long tail of wealth distribution are individuals with extraordinary wealth. And because they have a disproportionate share of economic influence, being owners of big corporations and constituting a large part of tax base, and because they possess, directly or indirectly, a disproportionate share of political influence – the super-rich individuals represent an important group in every society and one cannot understand thoroughly the forces of economic and political changes while does not understand how and where wealth is concentrated. Thus, the level of wealth concentration is of high importance and materiality, because the more the wealth is concentrated at the top, the more the mass struggle to keep control of their lives and their government.

The aim of this work is (a) to show what the mechanics of wealth concentration are and how wealth is distributed among individuals at the high wealth ranges, and (b) to test empirically the significance of factors that condition wealth concentration. To detect the forces behind wealth concentration, a bottom-up approach is taken. The model starts with a stochastic differential equation (SDE) that describes the wealth accumulation process relying on the assumption of market efficiency. The equilibrium solution of the Fokker-Planck equation to the initial SDE suggests the density distribution function for wealth – an approach pioneered by Champernowne (1953). To catch the interaction between agents as a part of wealth accumulation process, the model is enriched with a trade mechanism of how wealth is exchanged between agents and how bargaining power (induced by corruption) enforces the preferential attachment effect and makes wealth concentrate. The theoretical model gives grounds to the inclusion of explanatory variables (size of government, corruption, ease of trade and regulation of business and labor market) in the specification model tested empirically. The novelty of the agent based model constitutes of (a) allowing wealth to be accumulated from three different sources (stocks, fixed-income assets and trade), while (b) trade entails the effect of preferential attachment which is amplified by bargaining power of the wealthier individual, but constrained by business and labor regulations; (c) the government is also present in the model by both taxation and redistribution.

The empirical study presumes a cross-country analysis to test the hypotheses aroused from the theoretical part. The focus is on the number of super-rich individuals used as a proxy of wealth concentration when controlled for population and size of the economy. The cross-country study will give insights about the causes behind the discrepancies of the number of billionaires across countries. A part of the story, of course, is to be explained by discrepancies in macro environments, size of economies and population. That was empirically demonstrated in the works of Neumayer (2004) and Torgler *et al.*(2009). Another part is explained by wealth concentration. Otherwise stated, having more wealthy people in a country than in another is not solely explained by differences in wealth of the whole nation and/or size of population. It also depends of how wealth is distributed – more concentration, more billionaires. Hypothetically, the ease of trade within agents of the same country as well as ease of trade between countries has to exert a positive impact on the frequency of extraordinary fortunes around the world. Another factor that skews the distribution of wealth is the bargaining power of wealthier agents, which is amplified by corruption. This is expected to be a key ingredient that favors accumulation of wealth, especially for already wealthy people following the "cumulative advantage" (preferential attachment) principle (Gibrat 1931, Simon 1955, Souma and Nirei 2005). The bargaining power, however, can be constrained by regulations on business and labor market, making trade between agents (firms, individuals, etc.) a fair game, avoiding ripping off poorer agents in favor of wealthier ones. It is also argued that a minimalist state with small intervention into the economy through taxes and their redistribution is a positive pre-condition to uncommon wealth concentration. Supposedly, bigger size of government makes wealth more equally distributed.

Before further discussion, it should be distinguished that wealth and income, even if they have distributions which qualitatively tend to move together (wealth being more concentrated) and may serve as comparable indicators for financial position of an agent, they represent different things. The main difference between those two concepts is that income is by its nature a flow, while wealth is a stock. It is also well understood that one depend on another, as today stock depends on the current flow and future flow will depend on the available stock. There are plenty other reasons why in certain situations those should be differentiated as they may relate different insights. As mentioned earlier, this paper will revolve on the wealth concept.

Doing an empirical study on people's wealth is not a trivial task; at least, it is more complicated compared to a study on people's income. The reason is availability of data. The information about an individual's wealth can't be easy obtained and each case of extraordinary fortune may necessitate a separate treatment in order to estimate it. A high net worth individual is not prone to disclose his wealth, and actually he is not supposed to. Wealth is not always a subject of taxation and unlike income is not integrally monitored by the government for purpose of taxation. In conclusion, estimating the actual value of the assets and liabilities an individual owns is a tedious challenge and there are no systematic mechanisms (as in the case of income earned) that may give an exact number of the net worth a given individual possess. However, this burdensome and seemingly intractable job of assessing individuals' extraordinary fortunes has started to be done recently in a more meticulous manner by institutions like Credit Suisse, Wealth-X and of course for many years by Forbes magazine. Those represent the principal sources of data on number of wealthy individuals and their net worth. The other data regarding the independent variables come from open sources as KOF, Heritage Foundation, Transparency International, Fraser Institute and Penn World Tables.

Next section presents the literature review. In the third section the theoretical model is developed and the hypotheses are exposed. Within the fourth chapter the specification model is discussed along with the description of data used. In the fifth chapter the empirical results are analyzed and main empirical limitations are explained. The final section comes with the conclusions.

Chapter 2

LITERATURE REVIEW

Even if the extraordinary wealthy individuals attract much attention from the mass-media and the public, the phenomenon of wealth concentration expressed in accumulation of exceptional fortunes is still not enough researched and only a few papers attempted to explain the wealth concentration focusing on the individual level. There are plenty of papers explaining the distribution of income, as the access to the data is more facile, but less papers explaining the wealth distribution, or further the phenomenon of wealth concentration.

One of the most widely accepted approaches to explain wealth accumulation is to employ neoclassical growth models or overlapping generations (OLG) models which provide good frameworks to study the cross-sectional wealth distribution in equilibrium, grounded on optimal inter-temporal choices of consumption and saving by agents. The topic of wealth accumulation in context of growth models was opened by Bewley (1986) then was further developed by Aiyagari (1994), Chatterjee (1994) and Huggett (1996), later to become a workhorse into explaining the equilibrium cross-sectional wealth distribution. In context of OLG models there are various developments regarding intergenerational transfers (Kotlikoff and Jagadeesh 1999), consumption time preferences and precautionary savings (Caballero 1990) to explain wealth accumulation, but less wealth inequality. In fact, both OLG and growth models attempt to explain wealth distribution for the entire range of wealth, both high-wealth range and low-wealth range, in a single model. It is a complex task, because wealth accumulation at the low levels is primarily determined by consumption and production, while at higher levels wealth accumulation is determined by returns to capital (Anglin 2005). In these circumstances, these "one size fits all" models struggle to fit empirically the distributions observed in reality, especially when it comes to fat tailed wealth distribution in the top ranges.

The growth model of Quadrini *et. al.*(1997) embeds the infinite horizon and life-cycle settings, but the models "still fall short of accounting for the high concentration of wealth observed in the US data". In the periods following, the original Bewley-Aiyagari framework has been extended in many different ways, incorporating more realistic settings in order to explain better the fat tails in the empirical distribution of wealth. The work of Cagetti *et. al.* (2005) provides a thorough summary for these papers giving the essential empirical facts and criticizes the performance of different models.

An alternative to the orthodox models was certainly needed in order to shed more light on the mechanics of wealth concentration. Overlooked, the alternatives were already there, and first have been proposed by Champernowne (1953) who designated an "exogenous" model, not driven internally by optimal consumption-saving behavior, but by stochastic terms – allowing wealth today to depend on yesterday's wealth and the chance of today, an idea equivalent to market efficiency assumption. The Champernowne stochastic difference equations (with Markov transition matrix) generated Pareto distributions of wealth. Using different approaches, other authors as Simon (1955) developing a stream model, Wold-Whittle (1957) with the birth-and-death model arrived in a complex manner to the same heavy upper tails for the distribution of wealth observed in reality. More recently, models with stochastic drivers of wealth exchange and accumulation were developed by Angle (1986, 2006). Despite their exogenous character, these models explain well the empirical power laws in the top ranges of wealth.

Heavy upper tails were ubiquitously observed in data for top ranges of wealth in works of Atkinson *et al.*(1989), Piketty (2001), Piketty *et al.*(2003), Piketty *et al.*(2006), Saez *et al.*(2005), Dell (2006) and Wolff *et al.*(2008) studying the distribution of wealth over the last century in the US, Japan, UK, Germany, France and Canada. Again, the power law was found to govern the distributions of wealth in the US and Japan from 1960 to 1999 as documented by Souma *et al.*(2005), in Italy from 1977 to 2002 by Clementi *et al.*(2005), in Norway studied by Dagsvik *et al.* (1999) and Ireland studied by Nola (2006).

The empirics sustain the analytical findings of so called econo-physicists, though their methods are criticized for the lack of well-defined economic foundations. The tools borrowed from statistic physics or thermodynamics remain unfamiliar for the most of economists and usually those papers do not appear in the "mainstream" literature with few exceptions in the later years, after the mixed success of econo-physicists in the field of quantitative finance. A few recent works that represent interest for this study are mentioned below.

Klass *et al.* (2006) found the power law at the very top range of the wealth distribution in US using the Forbes 400 lists for 1988-2003. The average Pareto exponent for the period examined was $\alpha=2.49$. An interesting result is that in 1988 the index was $\alpha=2.60$ and decreased sharply until late '90s to $\alpha=2.10$, then started to increase again (lower alpha means greater inequality). The result confirms the fact that in periods of stock market boom the inequality will increase as "not all investors ride the boom in the same extent". In my paper, it is expected to observe jumps in the exponent after the financial markets crashes in 2000 and 2008 and decreasing pattern in periods of boom. Or simpler, there should be a negative correlation between Pareto coefficients describing the level of wealth concentration for different years and, say, S&P500 equity index describing stock market movements. Yakovenko *et al.* (2003) studied the probability distributions of money, income, and wealth in society using statistic physics and also showed that wealth is described by exponential and power law functions.

An impressive collection of papers – "Econophysics of income and wealth distributions" were published under the editorship of Chakrabarti *et al.* (2005). Among those, there are several papers, more empirical oriented – as the one of Yakovenko *et al.* (2005) where it is proved that the upper class (1-3% of population) is characterized by the Pareto power law distribution ("super-thermal"). The "super-thermal" tail parameters are easily affected by fluctuations in the stock market. In the context of my paper, it is expected that Pareto exponents will increase after year 2000 (the dot-com bubble) and 2008 (the financial crisis), indicating to a more equal distribution of wealth in the very top range of population. The same paper found that the majority of population belongs to the lower class (97-99%) that is characterized by the Boltzmann-Gibbs distribution, which is not easily perturbed by financial markets fluctuations and is more static over time. The same insights were found by Clementi *et al.* (2005) with evidence from US, UK and Germany. Namely, the power law in the very top was found and parameters of Pareto functional form changes over time, mostly due to stock market fluctuations.

Stock market fluctuations explain very well the short-run fluctuations of net worth owned by super-rich individuals, as a considerable fraction of their wealth are financial assets. The other fraction, however, consists of non-financial assets and forces behind their unequal distribution are quite different. One would look at the macroeconomic environment (conditions of doing business and regulations, size of government, corruption, ease of trade within country and across countries as few factors to mention) in order to understand the differences in shapes of wealth distribution for different countries. There are papers attempting to explain wealth distributions by some of above mentioned "fundamentals" using agent-based models. Ispolatov *et al.* (1998) constructs various specifications for an asset exchange model and finds that under greedy multiplicative exchange between agents, the wealth tends to be distributed following power-law distribution. Different than Ispolatov *et al.* (1998), my agent-based model considers a random exchange between agents under bargaining power of the wealthier, thus entailing the preferential attachment concept or so called "rich get richer effect".

The cross-country model which is based on the variables suggested by the theoretical part is similar in some extent to two papers: Neumayer (2004) and Torgler *et al.* (2009) – presented briefly below. The first paper that investigated the variation in the frequency of exceptional fortunes occurrence among countries at the international level is the work of Neumayer (2004), where a cross-sectional analysis was performed. The explained variable is the number of billionaires in each country (based on Forbes lists). The study confirmed the property rights protection, as the core of economic freedom, being associated with the incidence of unusual big wealth. Another conclusion was that it is easier to accumulate exceptional net worth in richer countries than in poorer ones. The hypothesis that more competitive economies tend to rule out the opportunities of super-great net worth buildups was not confirmed. One difference between the work by Neumayer (2004) and this study is that I use a different vector of explanatory variables as a slightly different estimation model is constructed, in my case more parsimonious – based on the theoretical model. Additional to the number of billionaires (provided by Forbes) as the only dependent variable in both works of Neumayer (2004) and Torgler *et al.* (2009), in this work I employ the number of wealthy people that surpass lower thresholds of net worth. In consequence, the problem of "a lot of zeros" in the dependent variable is solved by simply using a lower threshold, say \$500 mm or \$30 mm[1]. The excess of zeros first represents a lack of variance in the dependent variable and the econometric model may have issues in predict them. On top of that, both authors erroneously misused the econometric tools to estimate their models – an argument made clearer in Chapter 3.

Additional to cross-sectional analysis it would be interesting to elucidate the questions related to dynamics of the number of wealthy people worldwide, for example to explain why the number of billionaires decreased in 2009 and then started rebounding afterwards, so that in 2010 existed more billionaires than at the previous peak in 2007. A panel study seems to be the solution. But at this moment, things may get complicated as there are minor time variations in the explanatory variables, namely the economic freedom (corruption, government size) and globalization. They change very gradually in time. Globalization in essence is a gradual process, and that comes natural. The minor changes up and down in the globalization index during 7 years of 1996-2003 are not material for a country's economic environment. And more, it would conceptually incorrect to use, say, the level of corruption or other persisting distortionary forces in one year to explain the wealth accumulated in the same year. Most probably, a part of wealth was inherited and the newly accumulated wealth was possible because of the favorable conditions that persist since decades ago, as for example globalization. It's a delicate issue and the possible methods to solve it, as for example an average over years or lags, raise even more questions (how many lags, weighted or simple average, averaging make variables even less variable etc.). Despite these arguments Torgler *et al.* (2009) tackled a panel study for period of 1996-2003. The results were mostly similar to Neumayer (2004), while it was found that besides other factors, globalization also influences positively the

[1] As of 2011, the billionaires reside only in 55 countries out of 137, while individuals with wealth of more than \$30M are present in 85 out 85 countries with data available for.

number of billionaires in a country. The main difference between work of Torgler *et. al.* (2009) and my study is the approach to explain the evolution of world's wealth in time. I do consider that varying number of extraordinary wealthy people is closely related to financial market fluctuations – an aspect completely ignored by Torgler *et. al.* (2009). Even though a bigger data set can be used, covering the period of 1996-2010 (14 years), rather than only 1996-2003 (7 years) as of Torgler *et. al.* (2009), the panel study doesn't make too much sense for reasons given above. When it comes to explain the time evolution of wealth, the emphasis is on stock markets. It is the financial wealth that fluctuates more; the non-financial assets are less affected by stock markets variation. The link between varying number of billionaires and fluctuating stock markets is confirmed by the results of Clementi *et al.* (2005) and Yakovenko *et al.* (2005).

There are other empirical studies that answer questions related to the topic and brought considerable insights in the area. One is the work of Wolff *et al.* (2008) that looked at the geographic distribution of wealth and income. It explicated that "globally, wealth is more concentrated than income both on an individual and national basis". Siegfried *et al.* (1991, 1994, and 1995) focus on single country studies exploring what segments of an economy gave better soil for exceptional wealth to be accumulated and whether those segments are characterized by a competition free of any distortions. Siegfried *et al.* (1991, 1994, and 1995) used evidence from US, UK and Australia. The main finding was that over two thirds of fortunes were made in competitive segments of the market, while market power is responsible only for less than a third of the great fortunes. This result confirms that economic freedom (competitive markets) is a factor that influences positively the occurrence of great fortunes. In my work the economic freedom is described by corruption variable, business and labor regulation variable and also the size of government.

If to summarize briefly what this work proposes different than existing literature is: (a) an agent-based model that allows agents to invest in two kind of assets (fixed return and stocks), designs trade under the bargaining power of the wealthier (so modeling the preferential attachment) and the model introduces government by taxation and redistribution; (b) a study of the variation in wealth concentration levels relative to stock market fluctuations; (c) testing the results of the numerical simulations by a cross-country regression, taking super wealthy individuals for different thresholds of wealth as an indicator of wealth concentration when controlled for nation's wealth and population (an approach to measure concentration never taken before in the existing literature).

Chapter 3

THEORETICAL MODEL

3.1 A simple model of wealth accumulation

To model the real-world system a bottom-up approach is tackled, so that the system is modeled as a collection of autonomous interacting agents. To keep the equations tractable the appropriate simplifications are made, but at the same time the work follows the known principle that "everything should be made as simple as possible, but not simpler". To catch all the determinants and mechanisms underlying the process of wealth accumulation in the real world is rather an impossible exercise, Champerpowne (1953) has commented this as follows: "the forces determining the distribution of wealth in any community are so varied and complex and interact and fluctuate so continuously, that any theoretical model must either be unrealistically simplified or hopelessly complicated".

Starting from the very beginning, let's say that wealth (W), as a stock, is accumulated in discrete time following the equation:

$$W_t = A_t(.)\rho W_{t-1} + \eta_t \gamma W_{t-1} \quad (1)$$

,where η_t is an i.i.d. random variable that catches external shocks as fluctuations in stock markets or other sources of spontaneous growth to which the fraction γ of wealth is exposed. The element A_t can be whatever and individual does with a fraction ρ of his wealth to enlarge it. This element can entail within itself very complex processes which depend upon the scope of arguments, but has to keep a discretionary character, something under full control of the investor. There is no idle wealth, so that fractions ρ and γ sum up to one ($\rho + \gamma = 1$)[2]. This sort of stochastic difference equation does not always ensure the existence of a closed form solution and it is not always tractable in case A_t is too sophisticated. To make calculations malleable into deriving an explicit solution, one may find it easier to consider the continuous time approach and solve a differential equation to describe the change of wealth. In general form the equation may look as follows:

$$\frac{dW_t}{dt} = a(\rho W_t, t) + \sigma(\gamma W_t, t)\eta_t \qquad (2)$$

,where $a(W_t, t)$ is to describe the growth of wealth resulted from whatever an individual does to grow its wealth (A_t can entail the trade with other individuals, investing in risk-free assets, etc., but not investments sensible to stock market fluctuations) , while $\sigma(W_t, t)$ is to describe how spontaneous shocks affects his investments in stock markets. Assume η_t is a Gaussian random variable that permits the transformation[3] of (2), such that (2) can be written as an Itô stochastic differential equation with multiplicative noise:

$$dW_t = a(\rho W_t, t)dt + \sigma(\gamma W_t, t)dB_t \qquad (3)$$

,where B_t is a white noise Brownian motion (Wiener process) called also the driving process of the equation (3) which combined with $\sigma(W_t, t)$ forms a multiplicative process. The idea to develop a model of wealth accumulation

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[2] In general case: $\rho + \gamma \leq 1$, i.e. investor may have idle wealth, not engaged in investments. Formally there is no difference; the argument is still the same. At the end of the day, idle wealth also have a return, be it null or negative.

[3] In order to have a Wiener process the following has to be respected: $B_t - B_s = dB_t \sim N(0, t - s = dt)$

through a dynamic differential equation was previously undertaken in the works of Atkinson *et al.*(1978, 1979)[4], Bouchaud *et al.*(2000)[5]. To mention is that the pioneer to explain the exact form of wealth distribution through a stochastic process through Markov chains was Champernowne (1953). But, what I propose at this stage is different, in the sense that instead of putting emphasis on a well-defined mechanism of wealth accumulation, I consider a random growth process, where $a(W) = g(\rho W)$ and $\sigma(W) = r(\gamma W)$. In discrete model terms where $W_{t+dt} = \varphi_{t+dt} W_t$, that is equivalent to saying that $\varphi_{t+dt} = 1 + g\rho dt + r\gamma dB_{t+dt}$, so wealth accumulates as following:

$$W_{t+dt} = (1 + g\rho dt + r\gamma dB_{t+dt})W_t \qquad (4)$$

In simple words, the rate of growth is composed from a deterministic part g yielded by whatever an agent does to enlarge the wealth besides investing in assets exposed to market shocks (investing in bonds or other fixed-income instruments) and a stochastic part $r\eta_t$ yielded by investments in stock markets which are volatile.

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4 The accumulation model for wealth, K, is governed by a differential equation: $\frac{dK}{dt} = \theta K + \sigma$, where θ is the "internal rate of accumulation" and σ is "dominated by savings out of earnings".

5 In a simple economy the wealth is exchanged between N individuals at a constant and equal for everyone rate J (in spirit of a Markov process) and a stochastic element characterize the growth of wealth for each individual i due to investment in real estate, stock markets or other volatile investments: $\frac{dW_i}{dt} = J(\overline{W} - W_i) + \eta_i(t)W_i$. One can easily distinguish $a(.)$ element as a function of wealth, trading rate J and average wealth $\overline{W}$.

One fundamental reason at the base of the random part from the growth process is the assumption of market efficiency[6]. Otherwise stated, an investor can't systematically beat the market and can't always hedge against all risks such that his returns are deterministic and not affected by market fluctuations. Let's assume that the process in (3) governs the wealth accumulation only for fortunes that exceed a given threshold $W_{\min}$. The reason of this assumption is twofold: (a) this process characterizes in a less extent wealth accumulation for low-wealth individuals where consumption can't be neglected[7] and (b) power-law distributions of wealth are found only in the upper-tail: Yakovenko *et al.* (2005), Klass *et al.*(2006), Newman (2006). Formally, that would be equivalent to writing that for $W_{t+dt} > W_{\min}$:

$$W_{t+dt} = a(\rho W_t)dt + \sigma(\gamma W_t)B_t = g\rho W dt + r\gamma W dB_t \tag{5}$$

Let's denote by $f(w,t)$ the distribution of wealth at time t. Given the initial condition $f(w,0)$, the time evolution of the density function can be described by the Fokker-Plank equation:

$$\frac{\partial f(w,t)}{\partial t} = -\frac{\partial[a(w,t)f(w,t)]}{\partial w} + \frac{1}{2}\frac{\partial[\sigma^2(w,t)f(w,t)]}{\partial w \partial w} \tag{6}$$

6 The hypothesis of market efficiency was long debated starting with the influential articles of Fama (1970) and Jensen (1968) but never came to a consensus. An argument about market efficiency is out of the scope of this work. For a broad summary over the debate about efficient market hypothesis and its critics, one may check the work of Malkiel (2003).

7 Consumption can't be neglected when considering wealth accumulation of poor people as consumption affects considerably the level of wealth. In contrast, for wealthy people the relative fraction of wealth dedicated to consumption is very small and can be neglected. The processes that govern wealth accumulation for these two different classes are not the same.

Using the Fokker-Planck equation, it is easily to compute the steady state (equilibrium) solution, which means $\frac{\partial f(w,t)}{\partial t}$. If the long-run distribution of wealth is $f(w)$, then (6) becomes:

$$\frac{\partial f(w)}{\partial t} = -\frac{\partial[g\rho w f(w)]}{\partial w} + \frac{1}{2}\frac{\partial[r^2\gamma^2 w^2 f(w)]}{\partial w \partial w} = 0 \tag{7}$$

If to consider a Pareto solution, as one explicitly derived by Bouchaud *et al.*(2000) and others:

$$f^*(w) = K\frac{1}{w^{1+\alpha}} \tag{8}$$

,and insert it in the Fokker-Planck equation from (7) the following is true:

$$-\frac{\partial[g\rho w K w^{-(1+\alpha)}]}{\partial w} + \frac{1}{2}\frac{\partial[r^2\gamma^2 w^2 K w^{-(1+\alpha)}]}{\partial w \partial w} = \square K w \square^{\dagger}(-(1+\alpha))\,)\,[g\rho\alpha \tag{9}$$

If the trivial solution $\alpha = 0$ is disregarded, then the only solution is:

$$\alpha = 1 \tag{10}$$

This coefficient α is so called the Pareto index of the probability density function (Type I). In general form, Pareto distribution (Type I) can be written as follows:

$$f_W(w) = P(W = w) = \alpha w_{\min w^{-(1+\alpha)}} = K w^{-(1+\alpha)} \tag{11}$$

The coefficient α represents the Pareto exponent (index), which in this context quantifies the wealth inequality (concentration), as α decreases, wealth becomes more unequally distributed and more concentrated, i.e. larger share held by few individuals (see Appendix A). The distribution is said to be scale free, which means that the distribution remains the same whatever scale we consider, i.e. shape of distribution is the same for any K in (11).

The first striking conclusion that follows after (11) is that the wealth accumulation under current setup will generate fat tails characterized by Pareto distribution. That confirms the findings of Mezard *et al.*(2000), Scafetta *et al.*(2004), Yakovenko *et al.*(2000), Yakovenko *et al.*(2005), Yarlagadda *et al.*(2005), Chakraborti *et al.*(2005), Mimkes *et al.*(2005), Lux (2005) and Braun (2006) though derived in different manners and different contexts. The second conclusion after relation (10) is that in periods of stock market booms ($r \gg 0$), the concentration of wealth will become more acute. At the same time, after stock market crashes ($r \ll 0$), wealth in the upper tail becomes more equally distributed. It would be of interest to check whether those two conclusions are confirmed by real data.

The easiest way to confirm empirically the Pareto distribution is to check the so-called Zipf plots for the individuals of the upper tail analyzed. In this paper those are Forbes billionaires. After binning them in rankings based on their net worth, the logs of wealth are plotted against logs of rankings. In case the linear relation between logs is valid, then it is possible to describe the distribution of wealth by a single parameter, which is the Pareto coefficient. This argument comes from (11), which implies:

$$W_r = Cr^{(-\beta)} \tag{12}$$

,where W_r – wealth of individual ranked **r**, $|\beta| = \frac{1}{\alpha}$, α – Pareto coefficient, C – constant. Relation (12) transformed in log-log form supports the approach behind Zipf plots[8]:

$$log(W_r) = \log(C) - \beta * log(r) \tag{13}$$

The Zipf plots for each year of 1996-2012 are represented in Fig.1. The linear relation between logs is quite evident, which means the wealth of billionaires can be well described by Pareto distribution[9] and measuring the concentration by Pareto parameter is the correct approach.

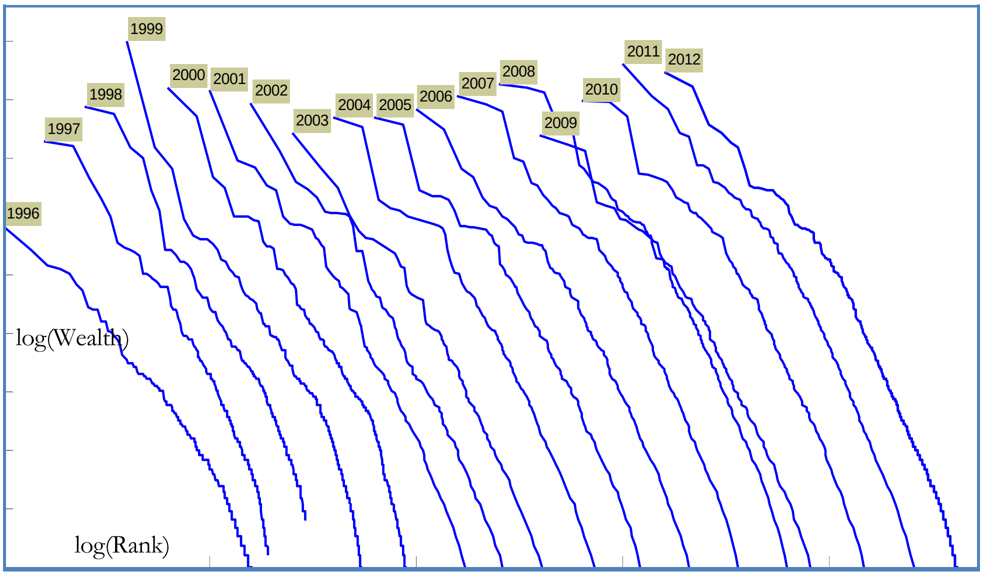

Figure 1. Zipf plots on Forbes billionaires (1996-2012)

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[8] This approach to test whether wealth is distributed according to Pareto distribution follows straight from the properties of this distribution. For a formal explanation and more details the reader can check the work of Adamic (2000) and Newman (2006).

[9] In a more rigorous way the samples of data were tested for many other different distributions using Kolmogorov-Smirnov test, Anderson-Darling test and Chi-squared test. All those rank Pareto as one of the best fits. Other more sophisticated distributions fit sometimes the data better. However, Pareto Type I distribution remains a good balance between quality of fit and parameters required to describe it.

In conclusion, Pareto coefficient may serve as a very exact measure of concentration. Using this parameter the Lorenz curve can be rigorously described and Gini coefficient exactly computed. Gini coefficient would be computed from Pareto coefficient as[10]:

$$G = \frac{1}{2\alpha - 1} \tag{14}$$

Thus, $\alpha = \infty$ implies perfectly equal distribution of wealth ($G = 0$) and the case $\alpha = 1$ corresponds to complete inequality ($G = 1$). For a better understanding Fig. 2 represents the relation between Pareto coefficient and Gini coefficient.

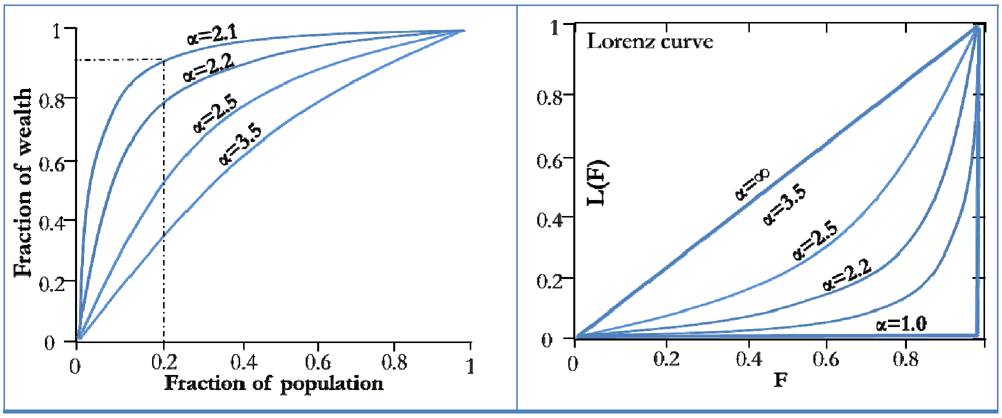

Figure 2. Relation between Pareto coefficient and Gini coefficient.

[10] $G = 1 - 2\int_0^1 [[1] - L(F)]\,dF$, where $(F) = 1 - (1 - F)^{1-\frac{1}{\alpha}}$, $F(w)$ – is the CDF of Pareto distribution: $F(w) = 1 - w^{\alpha}$. The formula to compute the Gini coefficient from the Lorenz curve (Lorenz 1905) as a measure of concentration was suggested by Gini (1936).

A simple and reliable method to estimate the Pareto index is the ML estimator developed by Newman (2006)[11]:

$$\alpha = 1 + n\left[\sum_{i}^{n} \frac{\ln w_i}{w_{\min}}\right]^{-1} \quad (15)$$

The expected standard error for the estimated α is given by:

$$\sigma = \sqrt{n}\left[\sum_{i}^{n} \frac{\ln w_i}{w_{\min}}\right]^{-1} = \frac{\alpha - 1}{\sqrt{n}} \quad (16)$$

After Pareto coefficients were obtained (for each year of 1996-2012 based on information on world billionaires offered by Forbes), it is a useful exercise to plot the time evolution of Pareto indexes compared to S&P500 time evolution which is a good proxy for stock market evolution (Fig. 3). The result confirms the analytical findings derived previously, namely that in period following the stock market crashes (as the dotcom bubble, then financial crisis of 2008) the wealth becomes less concentrated. In periods of financial booms as housing bubble, the concentration in the upper tails becomes more acute. The negative correlation between Pareto coefficients and S&P500 and S&P500 adjusted for inflation is evident. The correlation coefficients were computed to be equal to 95% and respectively 70% for 2000-2011 and negative 42% and respectively 57% for 1996-2011. The correlation coefficients given in Table 1 and Fig. 4 also suggest that total wealth held by billionaires is highly correlated with the stock market, which mean wealth is sensitive to stock market shocks. This naturally comes as a consequence of that billionaires' wealth partly constitutes of financial assets, exposed to stock market fluctuations.

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[11] Pareto coefficient can be also estimated by OLS from equation (13), but OLS method is not that exact compared as ML method. OLS estimated Pareto coefficients are upward biased. Newman (2006) proves the superiority of ML estimates over OLS.

Table 1. Correlations between estimated Pareto coefficients, S&P500 and wealth.

1996-2012 \ 2000-2012	S&P500	Wealth	S&P500 (1995 $)	Wealth (1995$)	Pareto α (ML)	Gini coefficient
S&P500		0.83	0.85	0.88	**-0.95**	0.95
Wealth	0.76		0.42	0.98	**-0.89**	0.90
S&P500 (1995 $)	0.89	0.39		0.55	**-0.71**	0.70
Wealth (1995$)	0.80	0.98	0.47		**-0.90**	0.92
Pareto α (ML)	**-0.42**	**-0.13**	**-0.57**	**-0.17**		-1.00
Gini coefficient	0.37	0.07	0.54	0.11	-1.00	

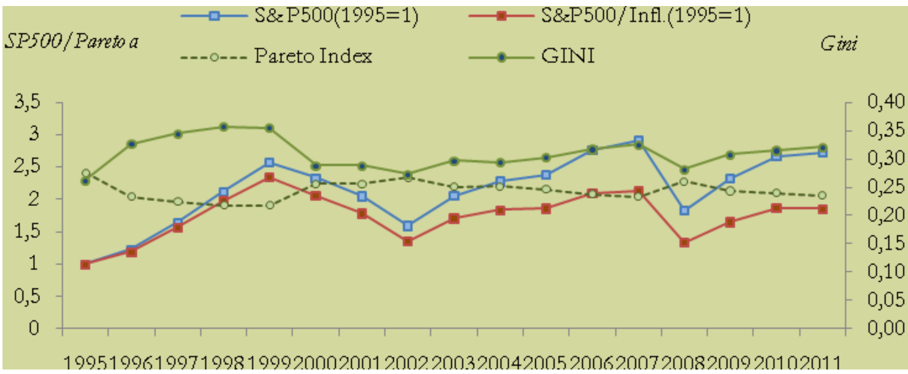


Figure 3. Time evolution of Pareto coefficient (α), Gini coefficient (based on Pareto α), S&P500 and total wealth held by billionaires.

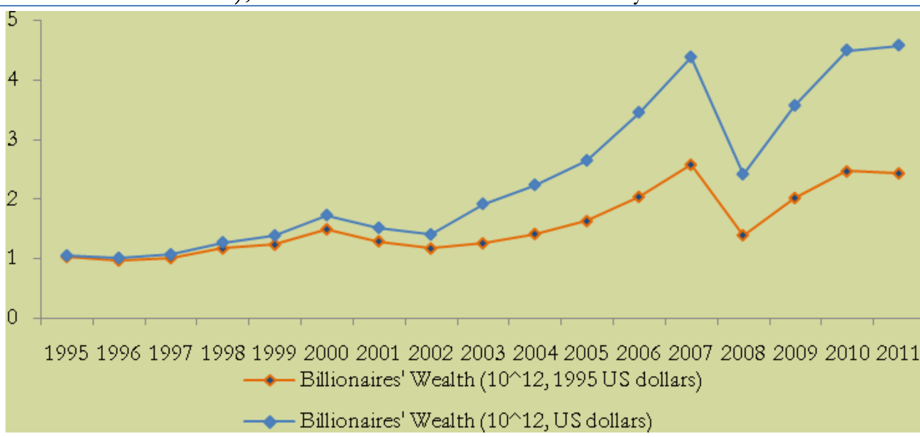


Figure 4. Time evolution of total wealth held by billionaires in nominal terms and in constant 1995 US dollars.

3.2 An agent-based model with wealth exchange

To complicate the model and answer additional questions, a more complex form for the equation (5) is considered. Namely, additionally to investing in assets that give a deterministic growth for wealth, the part $A(W_t, t)$ will entail a mechanism of trade between individuals. For that, let consider an economy with *N* individuals. Each of them owns a net-worth equal to $W_i, 2 \leq i \leq N$. The transfer of wealth through exchange between individuals is allowed. A transition matrix E describes the wealth transfer rates between them:

$$E = \begin{bmatrix} e_{11} & \cdots & e_{1j} \\ \vdots & \ddots & \vdots \\ e_{i1} & \cdots & e_{ij} \end{bmatrix}, \quad \sum_j e_{ij} \leq 1 \;, \forall e_{ij} \geq 0$$

,where e_{ij} represents the transfer rate of wealth from individual j to individual i if the trade occurs. If assume rational agents, then the wealthier individual i ($W_i > W_j$) never risks more than the maximum the poorer can trade (W_j). Thus, the maximum stake in a trade is the wealth of the poorer agent, which is $\min[\{W\}_i, W_j\}$. Of course, in real-life an agent very rarely dedicate his whole wealth for trading, and normally only a fraction of it is involved in transactions. Let consider this fraction to be equal to θW_j, where $\theta \in [0,1]$. Thus, the part of wealth evolution in time, driven only by trade, can be described by the following equation:

$$\frac{dW_i}{dt} = \sum_{j \neq i} [(e]_{ij} W_j - e_{ji} W_i) = \sum_{j \neq i} w_{j \to i} - \sum_{j \neq i} w_{i \to j} = \sum_{j \neq i} \Delta w_{i \to j} \tag{14}$$

This approach was suggested by Mezard *et al.*(2000), but it is now extended by considering individual transfer rates, which are not necessarily deterministic and equal for all agents as it was assumed by Mezard *et al.*(2000). The transfer rates may change in time and the only natural restriction is that they can't be greater than one, i.e. an individual can't afford to trade more than he owns. Given these settings it would be convenient to assume that $\Delta w_{j\to i}$ follows a Gaussian distribution with probability density function:

$$f_w(w_{j\to i}) = \qquad e^{\left[\frac{w_{j\to i} - \bar{w}_{j\to i}}{2s^2}\right]} \tag{15}$$

The mean (expected) wealth $\bar{w}_{j\to i}$ ($\bar{w}_{\downarrow}(i \to j)$) that is to be exchanged between two individuals is equal to:

$$\bar{w}_{j\to i} = \bar{w}_{i\to j} = \frac{\left(\left|W_i - W_j\right|\right)}{W_i + W_j}(1 \pm \omega)s \tag{16}$$

,where ω catches the inequality of bargaining power between individuals and $s = \theta W_j$ represents the deviation the transferred wealth may have. A coefficient ω with a positive sign in the formula, makes the trade favorable for the individual i (with more wealth, i.e. $W_i > W_j$) and the less is ω the less bargaining power agent i. In case $\omega = 0$ the trade is equally profitable for both individuals and the success is a question of chance. This construction allows considering the argument of preferential attachment, also called "the rich get richer" effect introduced by Yule (1925) and developed by Simon (1955). More recently, the "the rich get richer" effect was elaborately studied and proved empirically by Angle (1986) within the framework of a surplus theory, introducing a unidirectional model of wealth exchange that prescribes only a fraction of wealth passing from one agent to another, where the direction of the flow is determined by the relative difference between the agents' wealth. In this context, when two agents with similar wealth ($W_i \approx W_j$) trade, the gain from the transaction is equally likely for both of them and the bargaining power has no effect (as $\overline{w}_{j\to i} = \overline{w}_{i\to j} \approx \mathbf{0}$). Instead, when agents have different statuses (induced by different levels of wealth $W_i > W_j$), the wealthier agent gain a larger profit from the trade. In this case, the bargaining power plays its role and multiplies the "the rich get richer" effect. That serves as the ground of introducing regulation and corruption into discussion and testing them in the empirical model. Of course corruption skews the bargaining power in a favor of the wealthier people. With that being said, corruption or other tools amplifying the bargaining power will have a positive effect on extraordinary wealth accumulation and its concentration.

After introducing trade into the model the equation (5) changes to:

$$dW_t^i = \sum_{j\neq i} [(w_{j\to i}] - w_{i\to j}) + a(\rho W_t^i)dt + \sigma(\gamma W_t^i, t)dB_t \tag{17}$$

Following the setting from the initial model the stochastic differential equation becomes:

$$dW_t^i = \sum_{j\neq i} [(w_{j\to i}] - w_{i\to j}) + g\rho W dt + r\gamma W dB_t \tag{18}$$

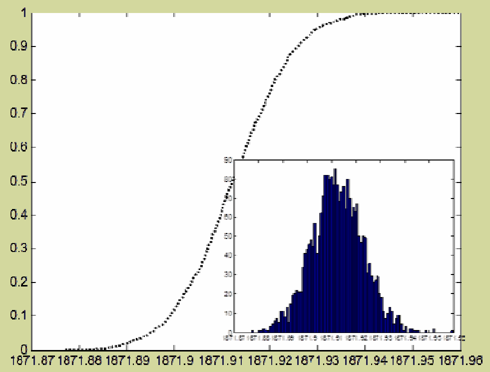
If to complicate it further and include the government in this game, then the equation entails an additional parameter (τ) for taxes:

$$dW_t^i = (1-\tau I_c)\sum_{j\neq i}[(w_{j\to i}] - w_{i\to j}) + (1-\tau)g\rho W dt + r\gamma W dB_t + \bar{T}$$

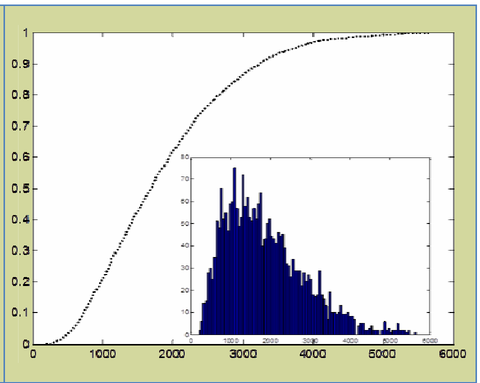
(19)

,where $\bar{T} = \frac{1}{N}\sum_i [\sum_j \left(\tau(w_{j\to i}] - w_{i\to j}) + \tau g\rho W\right)$; I_c =1 if $[(w]_{j\to i} - w_{i\to j}) > 0$ and $I_c = 0$ otherwise, i.e. there is no taxation for negative inflows. The government imposes a unique for everyone tax τ on every positive wealth inflow without any difference between corporate and income tax. The government also redistributes the taxes collected to every taxpayer in an equal manner, without any discrimination. This redistribution term $\bar{T}$ attempts to cover existent in reality public goods and social policies financed by taxpayers. Of course that is a simplified role of government. One may argue that a progressive tax system is more appropriate and equal distribution to rich and poor is not that realistic. Indeed, but departing closer to reality from the baseline assumed in the model, makes the argument even stronger – taxes and redistribution make wealth more equally distributed. That point is made clear in the results of the simulations performed. The analytical solution for the Fokker-Planck equation following from the above SDE is complex and deriving it explicitly can be a tedious challenge. Alternatively, the form of the distribution that eventually can be suggested after computer simulations for the SDE.

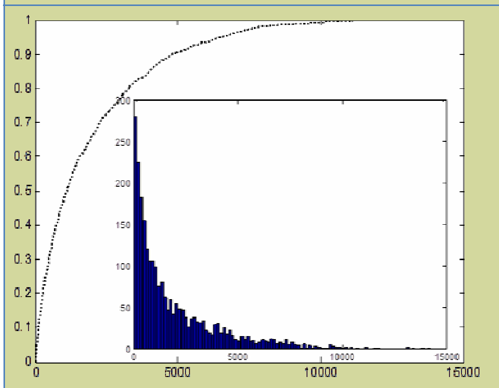
For that an economy with N agents is considered. The agents have equal endowments of wealth (W_0 units) and meet each other in T sessions of trading to exchange the wealth. Their wealth inflow may come from three sources: (a) trading, (b) investment in stock markets that are volatile and (c) fixed-income instruments that give a fixed rate of return as described by equation (19). Through simulations different cases were analyzed. For the sake of illustration, the simulations starts with N =2000, W_0 =1000, T =5000, g =0.01, r =0.005, half of wealth is exposed to shocks (γ =0.5) and another half is invested in "safe" assets (ρ =0.5); individuals do not trade (θ =0) and there is no bargaining power (ω =0). That serves as a baseline case to start from. It is easy to notice that in this situation (Fig. 4A) wealth is almost equally distributed and the distribution has a bell shape. However, after letting agents to trade, with a minimal fraction of wealth (θ =0.1%) each session, the skewness appears and there is already a disproportionate distribution of wealth (Fig. 4B). Following this hint and increasing the fraction of wealth involved in exchange (θ =1.5%), the distribution gets an exponential shape (Fig. 4C). By setting coefficient ω equal to 1%, the bargaining power of the wealthier is introduced in this game, which means that in every trade the wealthier agent capitalize on his bargaining power and increases his expected return by 1%. That leads to more concentrated wealth and the gap between the poor and rich widens even more (Fig. 4D). Bargaining power of the wealthier can be easier exercised in an unregulated market, where business and labor regulations don't protect the poorer and/or don't limit the wealthier. To make the point clearer, in Fig. 5E ω is set equal to 5%, which pictures a very corrupt society where a few individuals own huge fortunes, while others are very poor. Of course, if government is present the picture looks different. Imposing a tax rate of τ =20% has several effects: (a) wealth becomes more dispersed, (b) the gap between poor and rich narrows and (c) there are less individuals excessively poor (Fig. 5E). Keeping on this track and increasing tax rate to τ =40% (Fig. 5G), then to τ =60% (Fig. 5H) make the effects more visible.



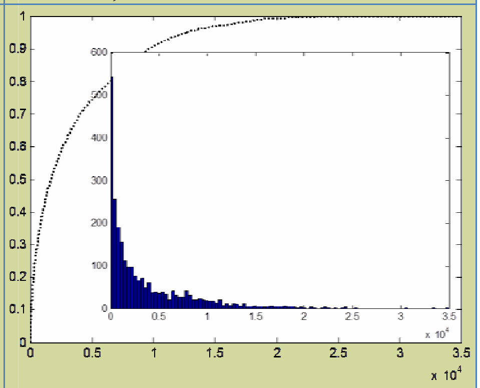
A. γ =50%, ρ =50%, θ =0, ω =0, τ =0



B. γ =49.995%, ρ =49.995%, θ =0.1%, ω =0, τ =0

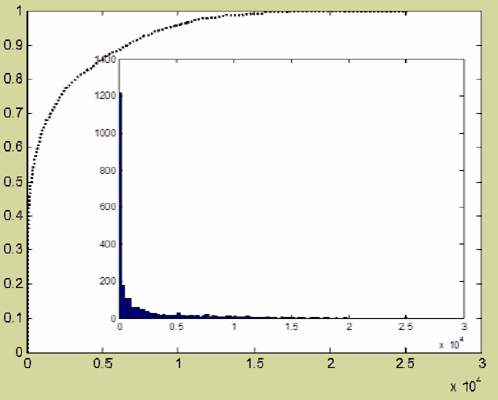


C. γ =49.25%, ρ =49.25%, θ =1.5%, ω =0, τ =0

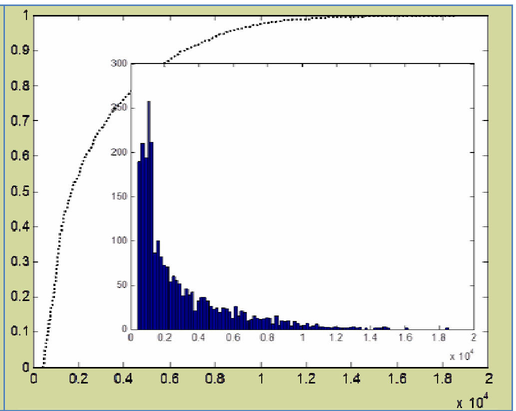


D. γ =49.25%, ρ =49.25%, θ =1.5%, ω =1%, τ =0

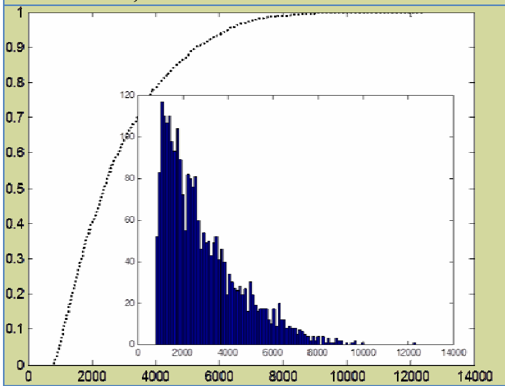
Figure 5. The empirical CDFs and histograms of wealth distribution for different parameters (γ, ρ, θ, ω) of the ABM, keeping $\tau = 0$ (no taxation).



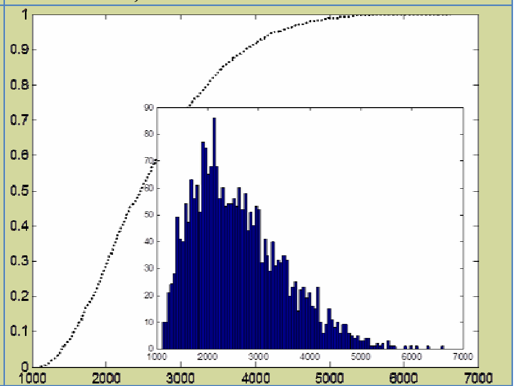
E. γ =49.25%, ρ =49.25%, θ =1.5%, ω =5%, τ =0



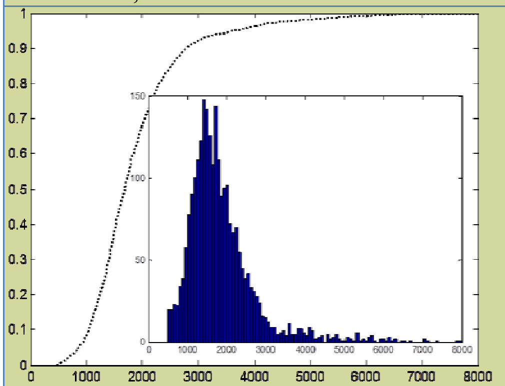
F. γ =49.25%, ρ =49.25%, θ =1.5%, ω =1%, τ =20%



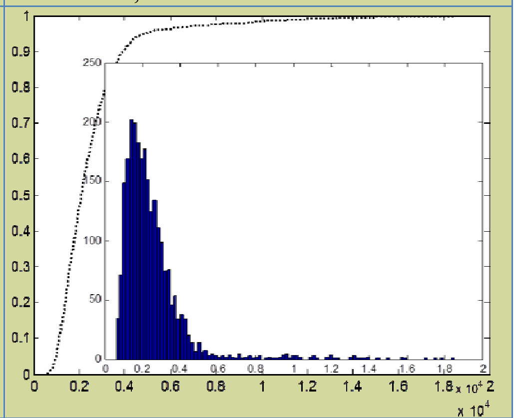
G. γ =49.25%, ρ =49.25%, θ =1.5%, ω =1%, τ =40%



H. γ =49.25%, ρ =49.25%, θ =1.5%, ω =1%, τ =60%



I. γ =49.25%, ρ =49.25%, θ =1.5%, ω =1%, τ =60%, 20%agents evade taxation



J. γ =49.25%, ρ =49.25%, θ =1.5%, ω =1%, τ =60%; 10% agents evade taxation

Figure 6. The empirical CDFs and histograms of wealth distribution for different parameters (γ, ρ, θ, ω, τ <>0) of the ABM.

The conclusion is that size of government narrows the gap between rich and poor and makes the wealth be more evenly distributed. However, even in conditions of big taxes, corruption can make wealth concentrate by tax evasion. To illustrate the effect two additional situations were simulated, when the same 20% of population do not pay taxes (Fig. 5I), and even worse when the same 10% evade taxation (Fig. 5J). The conclusion is rather natural – corruption pays off if one is a big fish, but corruption harms the poorer agents. Their tax contribution is redistributed to those not paying taxes, (a) make them wealthier and (b) put them in a more favorable position when meet to trade. The pictures in Fig. 5I and Fig. 5J clearly suggest this point.

The findings of this section are summarized in Table 2. Those serves as hypotheses to be tested by the empirical model described in details in the following section.

Table 2. The effect of ABM suggested factors on wealth concentration.

FACTOR	EFFECT ON WEALTH
SIZE OF GOVERNMENT	
- Larger/Smaller	Less/More Concentration
BARGAINING POWER	
- More/Less Corruption	More/Less Concentration
REGULATION (Business, Labor)	
- More/Less Regulation	Less/More Concentration
EASE OF TRADE	
- More/Less Freedom of Trade	More/Less Concentration

One more thing to notice is that, the ABM developed in this work doesn't intend to cover other aspects that may affect wealth accumulation and distribution. As a few to mention are the growth in population, birth and death processes, bequests, optimal choice of financial portfolio, differences in risk profiles and preferences, etc. With that being said, the ABM described above doesn't have a steady state and for simulations ran until infinity, extreme concentrations are not excluded, i.e. one person own all the wealth. Of course, the model can be complicated further to represent better the real-world system and avoid extreme results, not observed in reality. But, that can be a really complex task, at some extent impossible. However, being aware of existent limitations and doing simulations on the ABM in proper ways, allow drawing the major conclusions without excess of complications and assumptions.

Chapter 4

METHODOLOGY AND DATA

The main task of the empirical model is to confirm or to infirm the hypotheses about factors behind wealth concentration as suggested by the ABM developed in Sections 3.2. Ideally, in the regressions analysis the dependent variable should be the level of wealth concentration (expressed by the Pareto coefficient) for different countries which should be computed for each country from a reliable sample of wealthy individuals as it has been done in Section 3.1 for world billionaires of 2011. That regression would have the following form:

$$CON_{(IND)i} = \beta_0 + \beta_2 GOVSIZE_i + \beta_3 CORR_i + \beta_4 REG_i + \beta_5 ECGLOB_i + \varepsilon_i \tag{20}$$

The variables on the left hand side are the explanatory variables suggested in the ABM of the previous section, namely the size of government ($GOVSIZE$), corruption ($CORR$), business and labor regulation (REG) and the freedom of trade ($ECGLOB$). The methods those are quantified are described in the next paragraph. The dependent variable CON_{IND} is namely the index to describe the level of wealth concentration in the top ranges for each country. However, there is no available data at the individual level for each country to compute the concentration index and make this approach feasible. Alternatively, the level of wealth concentration can be obtained by taking the number of super wealthy individuals in a country per nation's wealth and population. In other words, having the number of individuals with extraordinary net worth on the left hand side of equation and controlling for country's GDP and population on the right hand side is a method to obtain good measure of wealth concentration in that country. With that being said, the regression should have the following form:

$$NR_{(W \geq \overline{W})i} = \beta_0 + \beta_2 GOVSIZE_i + \beta_3 CORR_i + \beta_4 REG_i + \beta_5 ECGLOB_i + \beta_6 GDP_i + \beta_7 POP_i + \varepsilon_i \tag{21}$$

The dependent variable $NR_{(W \geq \overline{W})}$, as said before, is the number of super wealthy individuals with net worth W surpassing a threshold $\overline{W}$, which in this study takes values of \$1 Billion, \$500 million, \$100 million, \$50 million and \$30 million[12]. The data come from three different sources (Forbes, Credit Suisse and Wealth-X) for the same year of 2011. This tandem wouldn't be possible prior to 2011, as that year was the first time when Credit Suisse and Wealth-X have offered publicly this specific information on wealthy individuals around the world. The reason of taking number of rich people for different levels of wealth is that certain countries are poorer than others and exceptional fortunes are measured at different scales. For instance, billionaires are present only in 55 countries out of 137 (Forbes 2011) having GDP greater than one billion dollars (PPP, as of 2011). Obviously, the threshold should be taken lower in order to have more information about poorer countries, more than only "no billionaires in country X". However, it should be understood that differences in scales do not affect the consistency of the method to measure concentration in the upper tails for reason that wealth in the upper tails follow Pareto power law distribution which is said to be scale invariant (concept introduced in Section 3.1 and explained in more details in Appendix A).

12 The data on billionaires (\$ 1B<) comes from the Forbes lists. The data on UHNWI's (\$ 30M<) is offered by Wealth-X in the World Ultra Wealth Report 2011. The data on number of individuals having Net Worth surpassing the remained thresholds is taken from the Global Wealth Report Data-book 2011 offered by the Research Institute of Credit Suisse.

Quantifying the explanatory variables from (21) is also not a trivial task. Besides that reliable data are needed, all the variables should be measured in a standardized way, identical for each country so that a regression analysis would be correct. Heritage Foundation, Transparency International, Fraser Institute and ETH Zurich offer publicly all the necessary data and meet the conditions above mentioned. Using data from these sources the explanatory variables were constructed as indexes ranging from 1 to 100.

Bargaining power of the wealthier is reflected in two explanatory variables. One is the variable to measure corruption in a country ($CORR$) computed as the average between Corruption Perception Index (by Transparency International) normalized to range 1-100 and the average of two other sub-indexes for Property Rights Freedom and Freedom from Corruption offered by Heritage Foundation. After the composite index was normalized to range 1-100, higher values mean lower levels of corruption – more freedom from corruption, thus lower level of bargaining power of the wealthier. Figure B1 in Appendix B gives the structure of the index.

The other variable to catch the bargaining power of the wealthier is the regulation variable REG, quantifying the degree of business and labor market regulations (see Fig. B2 in Appendix B for more details). A higher index value indicates to a less regulated economy. It is believed that less regulated labor markets make the employees easier to be ripped off by the employer, which is considered to be the one with more bargaining power – able to capitalize on his position. Of course, more collective bargaining power of the workers, larger hiring and firing costs as well as raised minimum wage diminish from the bargaining power of the employer, which is classified in this trade as the "wealthier" one in the context of the ABM model. The part with business regulations has another connotation and rather talks about ease of doing business in a country, so that more freedom for business in a country means more chances for extraordinary fortunes to be made. Overall, business and labor market regulation result in a more evenly distributed wealth among agents.

The size of government ($GOVSIZE$) variable intends to catch both, taxation and redistribution as argued in the ABM. Thus, this index was computed as an average between size of government sub-index elaborated by Lawson *et al.* (2011) of Fraser Institute and "limited government" sub-index by Heritage Foundation, normalized to 1-100 range, so that a higher value of the index indicates to a smaller government size. A detailed illustration of its composition is given in Fig. B3 in Appendix B. As argued in context of the ABM, a larger size of government make the wealth more equally spread out.

The explanatory variable $ECGLOB$ catches to ease of trade between agents of different countries. In fact, $ECGLOB$ is the "Economic Globalization" sub-index of the Globalization Index elaborated by Dreher (2006) and later updated by Dreher *et al.* (2008). This index is available in time series for each country since 1970 made publicly available by Swiss Federal Institute of Technology. Important is to use this index after averaged for the past decade or even for a longer period. Again, a higher value of the index indicates to a more open economy. The argument behind this is that the extraordinary fortunes were accumulated during decades or even longer, especially if bequeathed. So, it would be unfair to rank similarly two countries with the same index today, when one globalized during last decade, while another globalized two decades ago. In this work, the index was averaged for the last 15 years, due to availability of data for certain countries.

After all, the specification model has similarities with one model developed by Neumayer (2004), being now constructed in a more parsimonious way as it is based on hints given by the ABM (Section 3.2). His ad-hoc model was however estimated in a wrong way. Neumayer (2004) added several dummies for the OECD countries and US, the latter taking into account US special character (approx. a third of billionaires were Americans). That was technically a necessary thing, because the inclusion of dummies was the way of catching the extreme values and outliers while performing a Tobit left-censored regression. The choice of Tobit is questionable, because having count data and a lot of zeros shouldn't lead to consideration of Tobit model censoring at zero, but rather of Poisson distribution or negative Binomial distribution having different log-likelihoods.

Table 3. Summary statistics of cross-country regression variables.

Variable	Comments	Obs.	Mean	Std. Dev.	Min	Max
M30	*Individuals with NW > $ 30M*	87	2,089	6,694	35	57,860
M50	*Individuals with NW > $ 50M*	87	1,025	3,878	15	35,522
M100	*Individuals with NW > $ 100M*	53	550	1,651	26	11,968
M500	*Individuals with NW > $ 500M*	91	52	290	0	2,756
BLN	*Individuals with NW > $ 1B*	159	8	35	0	412
GOV	*Size of Government Index*	142	64	16	20	92
ECGLOB	*Economic Globalization Index*	159	60	16	27	93
REG	*Regulation Index*	159	41	22	16	94
CORR	*Corruption Index*	139	41	21	44	89
GDP_trln	*GDP ($ trln.)*	159	0.46	1.52	0.00	14.62
POP_bln	*Population (bln.)*	159	0.04	0.15	0.00	1.34

The two control variables added to the model are the population size (*POP_mln*) and the GDP (*GDP_bln*). The argument behind doing that is that a country with larger population will offer a more propitious medium for great fortunes to be accumulated than smaller countries do. Controlling for population permits the discussion of wealth concentration (wealthy individuals for a given population), thus number of wealthy individuals becomes a measure of wealth concentration. The GDP controls for the size of the economy. Obviously, the bigger is the economy, the higher the odds for extreme levels of net worth to be attained. The simulations on the ABM proved that the more wealth is available in a system, the wider can be the gap between poor and rich. The descriptive statistics of the variables used in the regression analysis are given in Table 3.

Before choosing the estimation method is should be figured out what distribution for the dependent variable is more appropriate. The histograms of the dependent variables in Fig. 7 clearly show that OLS is not an option. Since, the dependent variable is countable (discrete, hence after DDV – Discrete Dependent Variable) with a few large values and a lot of small values (including zeros) which seems to have a log link, the Poisson estimation would be the first candidate (with a null parameter alpha measuring the degree of overdispersion).

After a detailed analysis of data, it was figured out that the observed variance is way larger than the mean violating the restrictive condition $E[Y] = \lambda$ and $Var[Y] = \lambda$. In such circumstances the DDV can't be considered a Poisson variable. The problem of over-dispersion ($E[Y] < Var[Y]$) is solved using the negative binomial distribution instead. It is possible to estimate the extra parameter alpha and test it against null value. If alpha is significantly different than zero, then the negative binomial distribution is superior to Poisson, which is a particular case of negative binomial when alpha is equal to zero. If the regressions give an alpha statistically different than zero, then the negative binomial is superior to the Poisson. The coefficients are estimated by MLE.

To be more precise about the dependent variables, they follow closely two variations (general and type II) of the Pareto distribution (Appendix C). These kinds of distributions are well enough approximated by the negative binomial (Pólya distribution) with small occurrence coefficient.

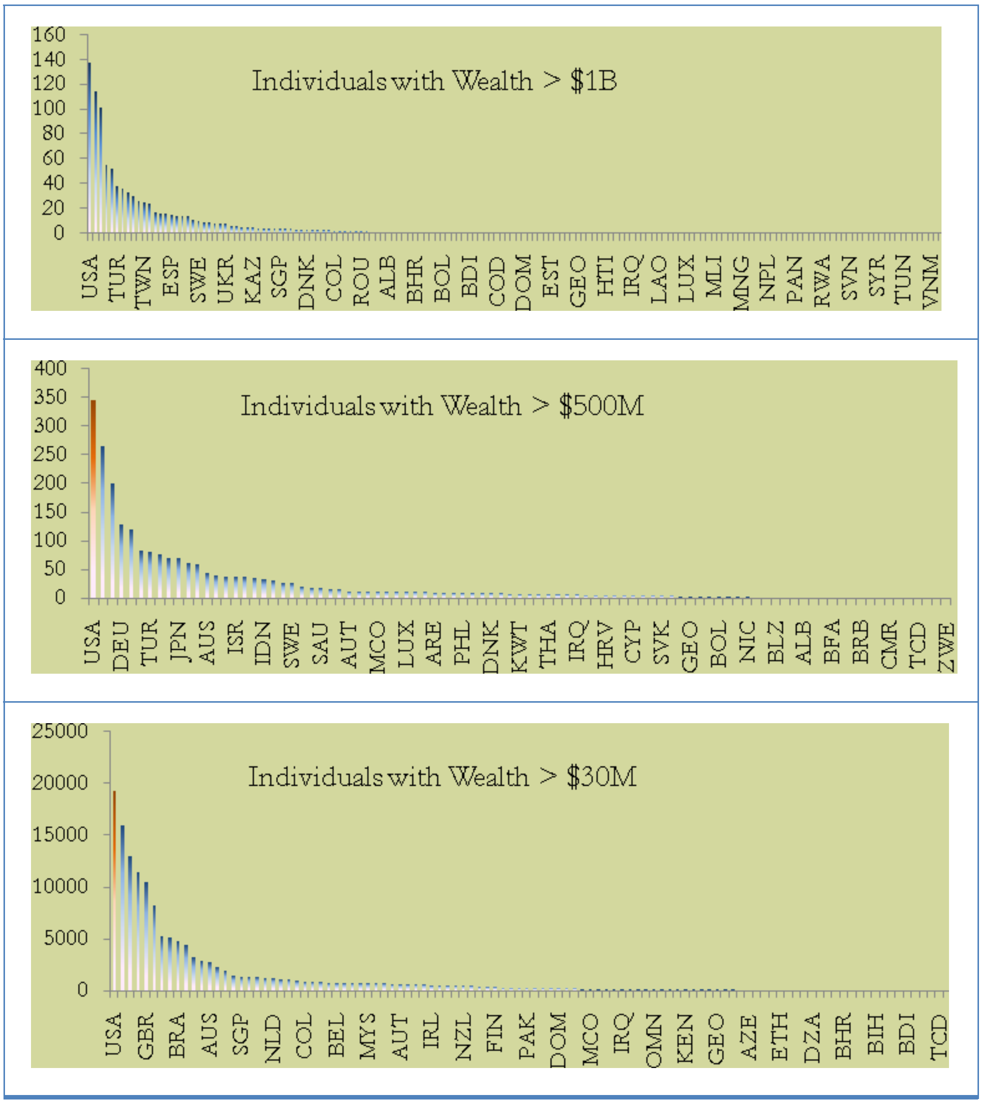


Figure 7. Pareto charts of the dependent variables (<1B, 500M<, 30M<)

However, the excess of zeros in the dependent variable (*BL*) representing the number of billionaires give some difficulties and may require a special treatment. The standard model may suffer because of the misspecification. Having 102 (out of 159) countries with no billionaires leads to consider another type of model that will explain the zeros better than the standard negative binomial does. The zero-inflated negative binomial (ZINB) seems to be the solution. This kind of regression is for modeling count variables with excessive zeros, usually for over-dispersed (discussed above) count dependent variables. The theory suggests that the excess zeros in DDV are generated by a distinct process of the count values, so that the excess zeros can be modeled independently. The zero-inflated model consists of two parts: the binary model, which is a logit to model the excess zero outcome and the count model, which in this case is a negative binomial, to model the count process. The hypothesis is that the absence of billionaires is mainly determined by small size of economy, so the GDP variable would be sufficient to predict the zeros in the dependent variable.

All the models mentioned above are estimated. The main ones to be discussed are presented in the next section.

Chapter 5

EMPIRICAL RESULTS

The estimation results confirmed the hypotheses enunciated ex-ante thus confirming the appropriateness and usefulness of the ABM in explaining the mechanics of wealth concentration.

Table 4 offers the incidence rate ratios (IRR) and p-value of each variable. These ratios are not the original coefficients that the NB or ZINB regressions give, but modified coefficients with scope of an easier interpretation. The IRR has a multiplicative effect on the dependent variable and tells how the number of wealthy individuals changes due to a unit increase in the independent variable other things equal. First three equations were estimated by negative binomial regressions which model the count variable (wealthy individuals) better than Poisson method given that the dependent variable is over-dispersed. That is sustained by significant different than zero coefficients measuring the degree of overdispersion, which as discussed before should be null in order to consider Poisson regressions as correct and superior to negative binomial.

Regressions [4] and [5] are also negative binomial, but they are inferior to zero inflated negative binomial regressions [6] and respectively [7] because of the excess of zeros in the dependent variables, which are additionally predicted by a logit equation. The Vuong closeness test for model selection sustain this argument, especially for regression [7] superior to [5], something that was expected given 102 countries out of 137 with no billionaires. Also, due to excess of zero in the dependent variable, in regressions [4]-[6] it makes sense to drop the constant from the specification model. For those equations, when constant is left in the model, its coefficient is close to be null and is statistically insignificant.

Table 4. Estimation results for NB and ZINB regressions.

	Negative Binomial (NB)					ZINB*	
Variables	**[1]** 30M<	**[2]** 50M<	**[3]** 100M<	**[4]** 500M<	**[5]** 1B<	**[6]** 500M<	**[7]** 1B<
GOV	**1.012** *0.07*	**1.014** *0.06*	**1.011** *0.09*	**1.028** *0.00*	**1.033** *0.00*	**1.029** *0.00*	**1.024** *0.00*
ECGLOB	**1.025** *0.02*	**1.023** *0.00*	**1.011** *0.35*	**1.051** *0.00*	**1.085** *0.00*	**1.041** *0.00*	**1.039** *0.00*
REG	**1.024** *0.00*	**1.027** *0.00*	**1.022** *0.00*	**1.016** *0.05*	**1.023** *0.06*	**1.017** *0.04*	**1.014** *0.07*
CORR	**0.982** *0.27*	**0.970** *0.10*	**0.964** *0.03*	**0.952** *0.03*	**0.932** *0.00*	**0.952** *0.00*	**0.952** *0.00*
GDP_trln	**1.72** *0.12*	**1.611** *0.10*	**1.525** *0.02*	**1.650** *0.09*	**2.723** *0.11*	**1.585** *0.00*	**1.429** *0.00*
POP_bln	**1.877** *0.91*	**1.589** *0.60*	**1.796** *0.75*	**1.476** *0.44*	**1.138** *0.29*	**1.534** *0.38*	**1.654** *0.56*
Constant	**38.35** *0.00*	**43.29** *0.03*	**100.9** *0.03*	No	No	No	No
Logit inflate: GDP_trln	No	No	No	No	No	**-84.88** *0.11*	**-27.95** *0.00*
α	**0.59**	**0.60**	**0.56**	**0.88**	**3.03**	**0.66**	**0.73**
McFadden R^2 Cragg&Uhler R^2	10.40% 82%	10.90% 81%	10.50% 78%	18.30% 78%	16.93% 55%	20.7% 82%	30.3% 71%
Observations	75	74	48	81	137	81	137
Prob>Chi^2 (Overall signif.)	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Prob>Chi^2 (α=0)	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Vuong Test (ZINB vs. NB)	-	-	-	-	-	0.035	0.000

*Zero-Inflated Negative Binomial

** Table offers the incidence rate ratios (IRRs) and p-values below.

Based on IRRs, it is concluded that one percent increase in the government size index, which is one unit increase in the index given its 1-100 range, leads to a 1.1%-3.3% increase in number of wealthy individuals ceteris paribus. That confirms the hypothesis that smaller government size permits unequal wealth distribution. A more impressive change in dependent variable results from a unit increase in economic globalization index, namely 2.3%-8.5% ceteris paribus. Thus, it is more likely to have more wealth concentration in a more open country. The effect of a unit increase in the business and labor regulation leads to a 1.4%-2.7% increase in the number of super-wealthy individuals. This result confirms the hypothesis that less regulated economies have, on average, more accentuated wealth concentration. Corruption, as it was expected, has a positive effect on exceptional wealth accumulation. Put it the other way, one unit increases in freedom from corruption index results in a 1.8%-6.8% decrease in number of super-wealthy individuals ceteris paribus.

Post-estimation analysis was performed and according to Wald tests, all regressions are overall significant. There are no explicit signs of heteroskedasticity, but to be on the safe side, the p-values of the regression coefficients were computed from robust standard errors. There is a degree of multicollinearity in the model, due to correlation between regressors (see Table C1 in Appendix C). The variance inflation factors confirm the issue (see Table C2 in Appendix C). Despite multicollinearity and supposedly inflated standard errors, the coefficients are significant, and that only make stronger their significance in the model. However, all variables were kept in the regression in their initial form, especially that McFadden R-squared isn't that high to consider multicollinearity an acute problem. In some extent upward biased, the likelihood based Cragg and Uhler's R-squared indicates toward satisfactory goodness of fit and a good specification of the model. The correlation coefficients between the regressors and the residuals are minor, except the case of GDP with correlation coefficients ranging from 30% to 50%. To test the exogeneity of GDP, a left censored Tobit regression is used and the GDP is instrumented by two variables: oil and steel production over last 15 years. Those are correlated both with the dependent variables and the GDP variable, but practically uncorrelated with the residuals. The Wald test fails to reject the null hypothesis of exogeneity, for some of the regressions, but for the others it suggests that GDP is statistically significant correlated with the residuals. Overall, the model struggles to predict the extreme values of the dependent variable, leaving much variance unexplained. Usually, the omission of variables from the model is the most plausible and common explanation. But this is not the case here. Taking into account the extreme shape of the distribution of the dependent variable, it is quite clear that explaining it in such extent that the problem disappear, the regression may need some variables with similar distributions, acutely skewed and over-dispersed. Unfortunately, the macroeconomic variables or variables quantifying things as corruption don't have this kind of distribution. In this situation, the omission of variables isn't truly the problem behind large residuals. The issue is the discrepancies between distribution of the dependent and independent variables. In general, the technical issues discussed above are characteristic for negative binomial models estimated by ML and they have another connotation compared to the case of common OLS regressions, where assumptions are much stricter. With that being said, the post-estimation analysis is prone to sustain that regression results are to be trusted, and conclusions drawn upon them are correct and well grounded.

Chapter 6

DISCUSSION AND CONCLUSIONS

The paper studied the phenomenon of wealth concentration in the upper tails of the distribution, so focusing on super-wealthy individuals. A theoretical framework based on agent-based model to explain wealth accumulation mechanics was proposed. The equation describing a random growth process, with assumption of market efficiency, allows wealth to be accumulated from two different sources, one exposed to fluctuations (stock markets) and the other giving stable returns (fixed income instruments). The process gives in limit a Pareto distribution of wealth, which is confirmed empirically based on billionaires net worth distribution. Using the Pareto coefficient as a measure of wealth concentration it is found out that in periods of stock market booms wealth becomes more concentrated, while after crashes is more evenly spread out. The ABM, enriched with a trade mechanism between agents entailing the preferential attachment in favor of the wealthier, suggests that less regulated business environments and labor markets offer a propitious medium for wealth to become concentrated, since the bargaining power of the wealthier is not limited and poorer are not protected. The numerical simulations also show that larger size of the government, which in the ABM has the role of collecting and redistributing taxes, conditions more equal distribution of wealth.

The cross-country regression analysis confirms empirically all the hypotheses hinted by the ABM, thus sustaining the theoretical argument and conclusions. Different than Neumayer (2004) that found the government size being statistically insignificant in his model, the study confirms the result of Mezard *et al.* (2000) which concluded that bigger is government, leads on average to less extraordinary fortunes. The study also confirms the result of Neumayer (2004) and also of Torgler *et al.*(2009), that in a more open country, it is more likely to have more super-wealthy individuals. Additionally, the model sustain the hypotheses that more bargaining power of the wealthier, conditioned by (a) less regulated business and labor markets, and by (b) less freedom from corruption, leads to more concentration of wealth. That confirms "the rich get richer" effect studied by Yule (1925), Simon (1955) and Angle (1986).

In conclusion, enlarging the size of government through taxation and redistributions (public goods, socials programs, etc.) reduces the gap between the poor and rich. However, based on numerical simulations, that seems to have a side effect, namely, less wealth is created and nation as a whole may become poorer. A possible explanation is that wealth taken away from individuals by means of taxation is not invested by government as efficient and profitable as individuals may do. At the end of the day, less return mean less wealth created. That hypothesis is not empirically tested in this paper, but represents an interesting continuation of the topic. Another aspect that still has to be studied is whether progressive tax systems are more efficient in reducing wealth inequalities and whether different tax regimes between individuals and corporations change the argument made in the ABM. So far, the ABM developed in this paper makes no difference between individuals and corporations, nor entails a progressive tax system. A further development may be the subject of another paper.

According to the empirical results, it seems that more regulation on doing business, and more regulation on labor markets is an efficient way to reduce wealth concentration. Usually, such regulations intend to protect the poor and limit the wealthier. Translated in my model, that is equivalent to giving less room for bargaining power of the wealthier. In conclusion, "tools" related to labor markets like raising minimum wages, imposing firing and hiring costs, establishing labor syndicates and "tools" related to business regulation like price controls, licensing restrictions, antimonopoly laws and administrative requirements are effective in reducing wealth inequality within a society.

Without any "side-effects", fighting corruption and leaving less room for discretionary wealth distribution and bargaining power of the wealthier (the ones able to pay bribes in order to "get things done") is a justified effort to reduce wealth concentration in a society. Without rule of law, a country may find itself to be ruled by a few plutocrats, while the mass is left to struggle in mediocrity.

Based on theoretical and empirical results, it seems that the ease of trade between countries, or otherwise called economic globalization, leads to wealth concentration across and within countries. Trade between countries has lots of positive outcomes, and reducing wealth inequality by closing the frontiers is not the solution. The most suitable solution to avoid wealth concentration across countries would be reducing the bargaining power of "bigger" countries over the smaller ones by engaging, for instance, in trade unions. Economic globalization should be a phenomenon with equal benefits for all participating countries.

So far, the ABM model developed in this paper doesn't catch any of the aspects related to life cycle and bequests. The model has a dynamic character and doesn't exclude extreme distributions if ran till infinity. These features may be addressed if model would be complicated and constrained further. And at some point the model can even be used to calibrate empirical distributions.

This study looks only at the top ranges of the distribution. On one hand, this is a justified effort as in the upper tails lie most of nations' wealth, but on the other hand this wealth is held by only little population. Of course this work would be more complete if the whole range of wealth distribution would be studied and another shape of distributions can be found for different segments of it. It would be also interesting to study whether the same factors behind wealth accumulation in the upper tails are valid for lower ranges. If data available, such studies could be done and additional insights on the topic would be learned.

WORKS CITED

Adamic, Lada A. 2000. Zipf, Power-law, Pareto - a ranking tutorial. Information Dynamics Lab.

Aiyagari, Rao S. 1994. Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics* 109 (3): 659–684.

Angle, John. 1986. The surplus theory of social stratification and the size of distribution of personal wealth, *Social Forces* 65 (December): 293-326.

Angle, John. 2006. The Inequality Process as a wealth maximizing process. *Physica A* 367 (July): 388-414.

Anglin, Paul. 2005. Econophysics of wealth concentration: A comment. In *Econophysics of wealth distributions. Econophys-Kolkata I*, 15-23. Springer: New Economic Windows.

Atkinson, Anthony, James Gordon, and Alan Harrison. 1989. Trends in the shares of top wealth-holders in Britain, 1923-1981. *Oxford Bulletin of Economics and Statistics* 51 (3): 315-332.

Atkinson, Anthony and Alan Harrison. 1978. Distribution of personal wealth in Britain. Cambridge: Cambridge University Press.

Atkinson, Anthony and Alan Harrison. 1979. The analysis of trends over time in the distribution of personal wealth in Britain. *Annales de l'INSEE* 33-34: 89-107.

Bewley, Truman. 1986. Stationary Monetary Equilibrium with A Continuum of Independently Fluctuating Consumers. *Contributions to Mathematical Economics: In Honor of Gerard Debreu*: 79–102.

Braun, Dieter. 2006. Non-equilibrium thermodynamics of wealth condensation. *Physica A* 369: 714-722.

Cagetti, Marco and Mariacristina De Nardi. 2005. Wealth Inequality: data and models. Federal Reserve Bank of Chicago Working Paper 1250.

Chakrabarti, Bikas K., Arnab Chaterjee, and Yalagadda Sudhakar. 2005. *Econophysics of wealth distributions. Econophys-Kolkata I*. Springer: New Economic Windows.

Champernowne, David G. 1953. A model of income distribution. *Economic Journal* 83: 318–325.

Clementi, Fabio and Mauro Callegati. 2005. Pareto's Law of Income Distribution: Evidence for Germany, the UK, and the US. In *Econophysics of wealth distributions. Econophys-Kolkata I*. Springer: New Economic Windows: 3-14.

Davis, Harold. 1941. The analysis of economic time series. San Antonio: The Principia Press of Trinity University.

Dell, Fabien. 2006. Top Incomes in Germany throughout the Twentieth Century: 1891-1998. In *Top Incomes over the Twentieth Century*, ed. Athony Atkinson and Thomas Piketty. Oxford: Oxford University Press.

Dreher, Axel. 2006. Does Globalization Affect Growth? *Applied Economics* 38 (10) :1091-1110.

Dreher, Axel, Gaston Noel, and Martens Pim. 2008. Measuring Globalization - Gauging its consequences. New York: Springer.

Fama, Eugen. 1970. Efficient Capital Markets: A Review of Theory and Empirical Work. *Journal of Finance* 25 (May): 383-417.

Gini, Corrado. 1936. On the measure of concentration with special reference to income and wealth. In summary of proceedings from the Cowles Commision Research Conference on Economics and Statistics: 73-80.

Gibrat, Robert. 1931. Les Inégalités Économiques. Paris: Sirey.

Jensen, Michael. 1968. The Performance of Mutual Funds in the Period 1945-1964. *Journal of Finance* 23: 389-416.

Klass, Oren S., Biham Ofer, Levy Moshe, Malcai Ofer, and Solomon Sorin. 2006. The Forbes 400 and the Pareto wealth distribution. *Economics Letters* 90 (February): 290-295.

Lawson A. Robert. 2010. Economic Freedom and the Wealth and Well-Being of Nations. Emily Chamlee-Wright: 65-80

Lawson, Robert, Joshua Hall, and James Gwartney. 2011. Economic Freedom Dataset. *Economic Freedom of the World: 2011 Annual Report.* Vancouver: Fraser Institute.

Lorenz, Max O. 1905. Methods of Measuring the Concentration of Wealth. *Publications of the American Statistical Association* 9, (70): 209-219.

Thomas, Lux, Simone Alfarano, and Friedrich Wagner. 2005. Estimation of Agent-Based Models: The Case of an Asymmetric Herding Model. *Computational Economics, Society for Computational Economics*, 26(August): 19-49.

Bouchaud, Jean-Philippe, and Marc Mezard. 2000. Wealth condensation in a simple model of economy. *Physica A: Statistical Mechanics and its Applications* 282 (July): 536-545.

Neumayer, Eric. 2004. The Super-Rich in Global Perspective: A Quantitative Analysis of the Forbes List of Billionaires. *Applied Economics Letters* 11(13): 793-796.

Pareto, Vilfredo. 1897. Cours d'économie politique. Lausanne: Edition Rouge.

Nola, Brian. 2006. Long-Term Trends in Top Income Shares in Ireland. In *Top Incomes over the Twentieth Century*, ed. Atkinson, Athony and Piketty, Thomas. Oxford: Oxford University Press.

Piketty, Thomas. 2001. Income Inequality in France, 1901-1998. *Journal of Political Economy* (111): 1004-1042.

Piketty, Thomas and Emanuel Saez. 2003. Income Inequality in the United States, 1913-1998. *Quarterly Journal of Economics* CXVIII: 1-39.

Piketty, Thomas, Gary Postel-Vinay, and Joachim Rosenthal. 2006. Wealth Concentration in a Developing Economy: Paris and France, 1807-1994. *American Economic Association* (96): 236-256.

Quadrini, Vincenzo and Jose-Victor Rios-Rull. 1997. Understanding the U.S. distribution of wealth. Federal Reserve Bank of Minneapolis Quarterly Review 21 (2): 22–36.

Saez, Emanuel and Marcus Veall. 2005. The Evolution of High Incomes in Northern America: Lessons from Canadian Evidence. *American Economic Review* 95 (June): 831-849.

Siegfried, John J. and Alison Roberts. 1991. How Did the Wealthiest Britons Get So Rich? *Review of Industrial Organization* 6: 19-32.

Siegfried, John J. and David Round. 1994. How Did the Wealthiest Australians Get So Rich? *Review of Income and Wealth* 40: 191-204.

Siegfried, John J. and David Round. 1995. The limited role of power market in generating great fortunes in UK, US, and Australia. *The Journal of Industrial Economics* 3: 46-73

Simon, Herbert A. 1955. On a class of skew distribution functions. *Biometrika* 42 (3-4): 425–440.

Torgler, Benno and Marco Piatti. 2009. Extraordinary Wealth, Globalization, and Corruption. School of Economics and Finance Working Papers Series 247. Queensland: Queensland University of Technology.

Wolff, Edward N., Anthony Shorrocks, James Davies, and Susanna Sandström. 2008. The World Distribution of Household Wealth. WIDER Discussion Paper (March).

Yakovenko, Victor M. and Adrian Dragulescu. 2000. Statistical mechanics of money. *European Physical Journal B* 17 (October): 723-729.

Yakovenko, Victor M. and Adrian Dragulescu. 2003. Statistical Mechanics of Money, Income, and Wealth: A Short Survey. New York: AIP Conference Proceedings 661: 180-183.

Yakovenko, Victor M. and Christian Silva. 2005. Two-class structure of income distribution in the USA: Exponential bulk and power-law tail. In *Econophysics of wealth distributions. Econophys-Kolkata I*, 15-23. Springer: New Economic Windows.

Zipf, George K. 1949. Human behavior and the principle of least effort. Harvard, MA: Addison-Wesley Press.

APPENDIX A

PARETO'S LAW, it is simply the CDF:

$$P[W \geq w] = \left(\frac{w}{w_{\min}}\right)^{(-\alpha)} \quad , \; , \; W, w_{min} > 0 \qquad \text{(A1)}$$

Since $w_{min} = 1$ (threshold of one billion USD): (A2)

$$P[W > w] = w^{(-\alpha)}$$

POWER LAW, it is simply the PDF: (A3)

$$P[W = w] = w^{-(\alpha+1)}$$

α – shape parameter of the Pareto distribution.

ZIPF RANKING

The slope of the „ranked" Zipf plot is β. Expected wealth $\mathbf{E}[W_r]$ of the r[th] ranked wealth (of a billionaire) W_r is: (A4)

$$E[W_r] = Cr^{(-\beta)} \quad , \text{C – constant,} \qquad \text{(A5)}$$

i.e. that there are r variables with expected value greater than or equal to:

$$Cr^{(-\beta)} P\left[W \geq Cr^{(-\beta)}\right] = cr \qquad \text{,c – constant} \qquad \text{(A6)}$$

Changing variables, it gets to : (A7)

$$P[W \geq w] = w^{\left(-\frac{1}{\beta}\right)}$$

PDF from the CDF, is possible by taking derivative w.r.t w:

$$P[W \geq w] = w^{-\left(1+\frac{1}{\beta}\right)} = w^{-\alpha}$$

(A8)

SCALE INVARIANCE („scale free" distribution) of the distribution refers to the fact that distribution keeps its form indeferent to scale. Say, function f has the form:

$$f(r) = Cr^{(-\alpha)} = W_r$$

Scaling r by a constant k :

$$f(kr) = k^{(-\alpha)}Cr^{(-\alpha)} = k^{(-\alpha)}W_r \qquad \text{(A1)}$$

It gives just a proportionate scaling of function, hence the lack of a characteristic length scale. The property gives the liniar form of the relation between logs of $f(kr)$ and r on the Zipf plots.

APPENDIX B

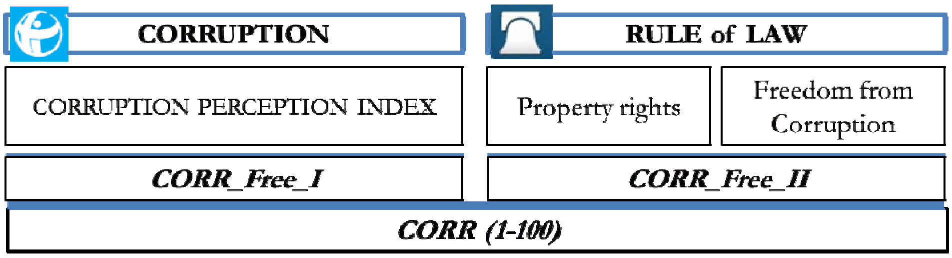


Figure B1. Construction of explanatory variable for corruption (CORR).

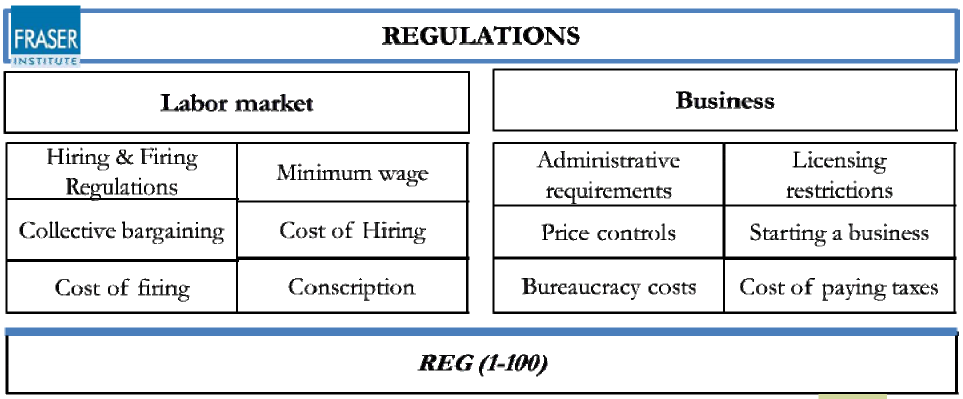


Figure B2. Construction of explanatory variable for regulations (REG).

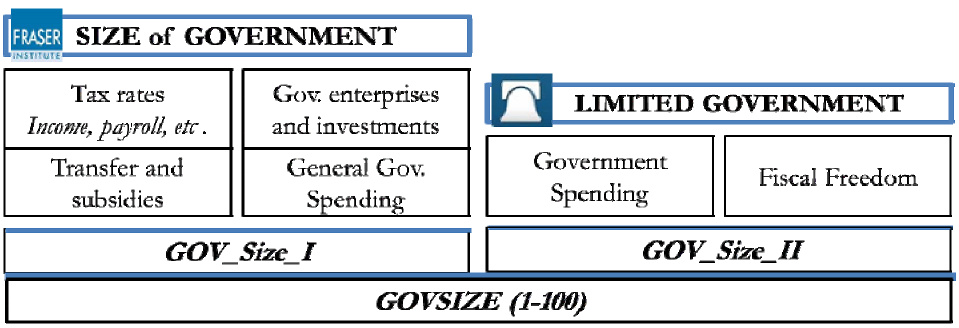


Figure B3. Construction of explanatory variable for size of government ($GOVSIZE$).

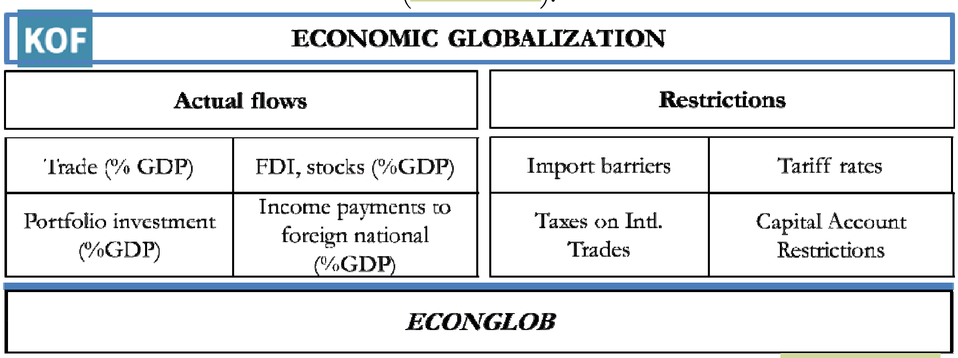


Figure B4. Construction of explanatory variable for ease of trade ($ECONGLOB$).

APPENDIX C

Table C1. Correlation coefficients between explanatory variables.

	GOV	ECGLOB	REG	CORR	GDP	POP
GOV	1					
ECGLOB	-0.3713	1				
REG	-0.428	0.6232	1			
CORR	-0.0595	0.2835	0.4977	1		
GDP	-0.0673	0.3755	0.1779	-0.0028	1	
POP	0.0649	0.2329	-0.0596	-0.1603	0.6542	1

Table C2. Variance inflation factors for regressors.

Variables	**VIF**
GOV	**41.69**
ECGLOB	**18.09**
REG	**17.30**
CORR	**11.46**
GDP	**2.15**
POP	**2.10**
Mean VIF	15.46