DISCOUNT PRICING STRATEGIES FOR DURABLE GOODS MONOPOLY

by

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Abstract

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Coase (1972) revealed a problem that a durable goods monopoly faces. When the high-value consumers have bought the good, the monopoly has an incentive to reduce its price. Knowing this, some of the customers may decide to postpone their purchases, thus reducing the potential profit of the monopolist. The rationing strategy as a solution to this problem were proposed by Denicolo and Garella (1999).

The current study presents a modification of Deniclo and Garella's model. It appears to be a more general one, particulary the rationing strategy is one of the possible cases of this modification. Also the study proposes strategies which allow the monopolist to improve his performance, earning more profit than in rationing and non-rationing cases.

Table of contents

List of figures

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Chapter 1

Introduction

Consumer goods are usually categorized as durables and non-durables. A durable good considered as a good that has a long period of life or more specifically one that brings utility over some period of time.

Many goods are durable: cars, light bulbs, shoes, computers, sell phones ect. Durable goods make a huge part of overall consumption in modern economies. For example, in the United Sates personal consumption expenditures for durable goods (PCEDG) in 2013-08 were about 1,274.4 billions of dollars[1], which is about 11% of overall personal consumption expenditures. In Ukraine, durable goods consumption is approximately 30% of overall consumption.[2]

The importance of product durability and monopoly power was admitted long ago. For example, major U.S. antitrust cases such as the U.S. vs. Aluminum Co. of America (1945) and the U.S. vs. United Shoe Machinery Corp. (1953) and some cases involving IBM (Fisher et al. (1983)) were concentrated on issues of product durability and monopoly power. These issues continue to be a very important part in public policy for durable goods markets.

The results of the following work should not be interpreted only for the case of monopoly. Even though, the producers of durable goods mainly are not monopolists, a big part of them do have some monopoly power. For this purpose, the following analysis can give us useful insights.

Coase (1972) explained why product durability could be a problem for a

[1] U.S. Department of Commerce: Bureau of Economic Analysis
[2] Golovko (2010)

monopoly and how it limits monopoly power. A monopolist selling a durable good encounters the competition with his own future output. If he is not able to commit to a price sequence, he may be willing to lower the price once the consumers with a strong willingness to pay have bought the good, but this leads customers to postpone purchases. As a result, his discounted profit would be lower.

One of the possible ways to solve this problem is by rationing demand. Rationing is considered as the regulated distribution of scarce resources. It means that a seller can commit to a given output in the second period such that if consumers with high willingness to pay wait for the second period, they are not certain of obtaining the product. This provokes them to purchase sooner and lets the monopolist provide the price discrimination over time. This strategy helps the monopoly increase its discount profit and somehow solve this problem.

The two and three-period models of durable goods monopoly proposed by Denicolo and Garella (1999) help us understand some intuition behind the processes that take place. In my thesis I built the two-period discount-model in which the second period price for rationed consumers is different than for those who just postponed the purchase. The intuition here is when consumer knows that there is a chance to be rationed in the first period with the following high price in the second period, she has more incentive to postpone the purchase until the second period and buy the product with discount price. It is worth noting that rationing model presented in the Denicolo's paper appears to be one of the cases of discount model specifically where the price for rationed consumers is the same as for those who just postponed the purchase.
I consider the other two possible cases of discount model. First one is when the price for rationed consumers in the second period is such than they can not afford the product and the second one is when the price for rationed consumers

is affordable for some of the consumers. I find conditions when the discount-model gives more profit than the non-rationing and the rationing model.

The remainder of my thesis is structured as follows. The first, the literature review is provided. Second, we discuss the main results from Denicolo and Garella (1999). Third, the discount model proposed. Conclusions are presented in the final part.

Chapter 2

Literature review

The topic of durable goods is very wide-ranging. Here I will discuss some of the main directions of this topic. However, in my thesis I touch only some aspects of durable goods. Specifically it is: time inconsistency, rationing and information asymmetry.

Durability choice. In this part the main focus is how long the product should serve in order to make monopoly better off. To give more intuition about this problem, let's look at an example. An illustrative one would be the production of light bulbs. Let us think of a monopolist that produces light bulbs and his goal is to maximize his profit. What choice of output and durability should he make?

In the second half of the 1960s, a number of authors considered the question of whether a monopolist should select the same level of durability as a firm that operates in the competitive market. The main idea is that a monopolist should select some less degree of durability than in the competitive market (Kleiman and Ophir (1966);Levhari and Srinivasan (1969)). In our example with light bulbs, if a monopoly produces and sells light bulbs which will serve perpetually, in some period, there will be no demand in light bulbs and the monopoly will have to leave the industry.

Another paper belongs to Golovko (2010). He studies the simultaneous–move three-period model in which two firms choose the durability of their goods, how much to produce and whether to rent or sell the products. He shows that in this model the firm's profit tends to increase if it makes products less durable.

So, in order to avoid this problem, durable goods monopoly invests less in the quality of products than the efficient level. Since the used unit and a new one are substitutes, the price of the used unit constraints the price of a new good. If a monopolist reduces the quality of the used good below the efficient level, it will reduce the substitutability between new and used products as well. This enables the monopolist to increase the price of a new product, thus increasing it discounted profit.

Time Inconsistency and durable goods. The focus in this part is on difficulties that emerge because durable goods to be sold tomorrow influence tomorrow's value of units sold today. Durable goods produced today are also in use tomorrow because of that, the demand in the following period would be lower than the demand in this period. Rational consumer, who expects the demand to fall in the next periods, will not pay too much for the good in the current period.

Coase (1972), Bulow (1982) pointed out that it is more profitable for durable goods monopolistic firm to rent than sell. The intuition is that profit maximization condition for monopolist is that marginal costs equal to marginal revenue. However, since monopolist produces durable goods, the demand in the next period would be lower than in this period. It means that if consumers are rational, they will not pay too much for the good in the current period. Therefore, today's prices decrease and the monopoly will have less market power.

Since pure monopoly is very rare, several attempts are made to analyze other market structures. Saggi and Nicolaos (2000) analyze the asymmetric duopoly case, when firms rent and sell in each period. They show that the ratio between renting and selling depends on the production costs. If the cost of production increases, the ratio will also increase.

Board (2008) analyzed the case when durable goods monopoly faces the varying demand. He proposes the optimal set of prices and allocations, and determine how firm uses the time to discriminate between various generations. When incoming demand varies, prices changes, this provokes some consumers to postpone the purchases. However, he identified that, when a new demand grows, the prices rises quickly. On the other hand, when a new demand weaker, the prices falls slowly, since consumers postpone their purchases.

Sometimes in the durable goods markets, we can observe sales. Conlisk and Sobel (1984) analyzed this question in case of durable goods monopoly the infinite period, where new customers enter the market each period. The distribution of this customers assumed to be the same with a regard of their willingness to pay. Also they assumed that the good is infinitely durable, thus those who buy the good, leave the market. The conclusion is that in most periods monopoly sets the price which is equal to the willingness to pay of a high-valuation consumer. Additionally, when the amount of low-value consumers reach to a certain level, the monopoly lower the price in order to sell to low-value consumers. To sum up, in most periods, the monopoly's price is high, but periodic price reductions take place.

Information asymmetry. In some markets, especially in durable goods markets, consumers are unable to determine the quality of units offered for sale. One of the consequences could cause a problem of adverse selection, when firms withdraw high-quality products from the market because consumers are unable to notice high quality product and therefore, unwilling to pay a high price for it.

Akerlof (1970) analyzed asymmetric information and adverse selection. Originally, this work is not intended to be the analysis of durable goods. It is the study of the used car market. It can be considered as a supply-demand problem when suppliers are individuals, who own the car and know the real quality of it. Demanders are individuals, who want to buy the used car. Although they don't know the specific car quality, they know about the average quality offered for sale. Also, the author assumes that demanders have a higher value for used cars than suppliers. Akerlof shows that under these assumptions, not all used cars are traded. This happens because suppliers with a high-quality used car will not sell it at the price that is under the real price for such a car.

Hendel and Alessandro (1999) look into the model of new and used durable goods. Their results confirm the analysis provided by Akerlof; moreover, they find other interesting features. For example, they explain the fact that the secondhand market price affects the willingness of consumers to buy a new unit. If the price in the secondhand market increases, the willingness to buy a new unit product will also increase.

Rationing. Denicolo and Garella (1999) showed that a monopolist selling durable goods could gain from rationing. They focus on a two period model in which rationed buyers from the first period will carry their demand to the second period, increasing the second period demand. This would reduce the monopolist's incentive to cut the price in the second period. Knowing that, rational consumers would have less incentive to postpone purchase, improving the first period demand. This strategy improves the monopoly's ability to discriminate between high-willingness and low-willingness customers; as a result, this would increase the total discounted profit.

Boyer and Moreaux (1988) considered durable goods oligopoly model where one firm competes in price while another firm has a Stackelberg leadership. The authors conclude that the firm that moves first could find it attractive to ration demand for its good to make the rival react less aggressively. Clearly, the

Stackelberg duopoly model and two-period duopoly goods monopoly has some resembling features. Although the timing of moves is different: in duopoly case, only after observing prices the consumer buys, which is similar if we assume the possibility of full commitment in monopoly case.

Real life examples. Lets us now look at some real life examples. It would help us more deeply understand the intuition about the process that take place in the durable good markets.

Microsoft. It is clear that computer software is a durable good. One of the good example of a monopoly that operates in this type of industry is Microsoft. Lets look at the Microsoft's product frequent style changes and low-price strategies.

From the side of profitability, in order to make the used unit obsolete, the monopoly has incentives to introduce some kind of style changes. Now, let's consider the Microsoft Word program. The documents produced in Microsoft Word 2003 can easily opened in Microsoft Word 2010, however, sometimes those who use the 2003 addition can't open the documents produced by 2010's addition. Here we can notice that those who has the latest version is benefited, on the other hand, those who has the earliest version are worse off. This provokes customers to buy new versions of the product and monopoly keeps his sales.

The second aspect is the Microsoft low-pricing behavior. Schmalensee (1999) pointed out that if Microsoft followed the monopolistic strategy, it would extract the last dollar of profit from consumers by setting the price that would be hundreds of dollars more than it is. Hoppe and Lee (2002) explain this phenomenon based on the durable nature of the product that Microsoft offers. They pointed out that Microsoft charged such a low price in order to restrain entry. Charging a low price reduces incentives of potential rival to enter the market. The firm charges low prices, however, it sells to a large number of customers.

Aftermarket Monopolization. Aftermarket is the market where complementary goods and services such as maintenance can be purchased.
For instance, in Eastman Kodak Co. v. Image Technical Sevices (504 U.S. 451 [1992]), the Kodak refused to sell different parts of his product to alternative maintenance suppliers, thus the Kodak users has only one option but to purchase maintenance from Kodak. When consumers buy Kodak's product they are locked in and since the Kodak has a monopoly power in aftermarket it sets the price of maintenance higher than in competitive case.

Chapter 3

Two-period model

In this part I will briefly go through the main results of the Deniclo and Garella's paper. This will help us to understand the main intuition behind the process that take place.

Let's consider the monopolist selling a durable good in two periods. There are a lot of consumers and each of them is able to buy at most one unit of a good. Consumers are determined by the parameter of ν, which is their willingness to pay for this good. $F(\nu)$ is a distribution function of ν over the $[0, \hat{\nu}]$. The monopolist is not able to determine the consumer's willingness to pay as a result it cannot commit to a price sequence. Also, the model considers the following assumptions:

- Zero marginal cost ($MC = 0$);
- No resale market;

The consumer's utility from buying at date $t = 1, 2$ is:

$$u_t = \delta_b^{t-1}(\nu - p_t) \tag{3.1}$$

where:

- $\delta_b \in (0, 1)$ - the buyer's discount factor;
- p_t - price in period t.

After observing the first-period price of a good p_1, consumers have to decide whether to buy or wait until the next period. Since the monopolist cannot price discriminate, it sets up the second-period price p_2 in order to maximize the profit taking the first-period choices as given. Since we assume that consumers are rational, they can correctly anticipate the second-period price.

Let ν_1 define the value of ν, which makes the consumer indifferent between buying and not buying in the first period. It means that all consumers who have $\nu \geq \nu_1$ will buy the good in the first period. By knowing that, ν_1 can be easily found from (3.1) :

$u_1 = \nu_1 - p_1$ - the utility that consumer can gain if buys in the first period.

$u_2 = \delta_b(\nu_1 - p_2)$ - the utility that consumer can gain if buys in the second period.

$$u_1 = u_2 \Rightarrow \nu_1 - p_1 = \delta_b(\nu_1 - p_2) \tag{3.2}$$

$$\nu_1 = \frac{p_1 - \delta_b p_2}{1 - \delta_b} \tag{3.3}$$

Lets us now look at possible cases:

1. $p_1 = p_2$, from (3.3): $\Rightarrow \nu_1 = p_1$;
2. $p_1 > p_2$, the $\nu_1 = \frac{p_1 - \delta_b p_2}{1 - \delta_b}$;
3. $p_1 < p_2$, from (3.2): $u_1 > u_2 \Rightarrow p_1 = \nu_1$.

To summarize:

$$\nu_1 = \begin{cases} p_1, \text{ if } p_2 \geq p_1 \\ \dfrac{p_1 - \delta_b p_2}{1 - \delta_b}, \text{ if } p_2 < p_1 \end{cases} \tag{3.4}$$

Only those consumers that have $\nu \geq p_2$ will buy a good in the second period. The monopolist's discounted profit would be:

$$\pi = p_1 x_1 + \delta_s p_2 x_2 \tag{3.5}$$

where

- δ_s is the seller's discount factor;
- $x_1 = 1 - F(\nu_1)$ is the first-period output;
- $x_2 = F(\nu_1) - F(p_2)$ is the second-period output.

Consequently, in no-rationing case the equilibrium is the combination of (p_1, p_2) that maximizes π given (3.2) and having p_2 to maximize the $\pi_2 = p_2 x_2$.

Chapter 4

Properties Of Rationing equilibria

Clearly, rationing is not useful in the second/final period since it can not affect the price. So if the monopoly operates in n-periods, the rationing would be useful in $n-1$ periods. The rationing scheme is a function $\gamma(\nu) : [0, \bar{\nu}] \to [0, 1]$ (it determines the $\nu-$ the proportion of consumers not served in the 1-st period), $\gamma(\nu) = 1$ for $\nu \leq \nu_1$. In case of proportional rationing $\gamma(\nu) = \mathit{const}$ in the interval $\nu \geq \nu_1$; with the efficient rationing $\gamma(\nu)$ would have the following structure:

$$\gamma(\nu) = \begin{cases} 0, \text{ for } \nu \geq F^{-1}(1 - x_1) \\ 1, \text{ for } \nu < F^{-1}(1 - x_1) \end{cases} \tag{4.1}$$

The equilibrium rationing strategy is the price combination (p_1, p_2) and $\gamma(\nu)$ with $\int_{\nu_1}^{\hat{\nu}} \gamma(\nu) dF(\nu) > 0$ and p_2 is the price that maximizes the 2-nd period profit. This equilibrium brings more profit than the no rationing equilibrium. The first period demand x_1 in case of rationing would be:

$$x_1 = \int_{\nu_1}^{\hat{\nu}} (1 - \gamma(\nu)) dF(\nu) \tag{4.2}$$

while total demand in the second period would be:

$$x_2 = \int_{\max\{p_2, \nu_1\}}^{\hat{\nu}} \gamma(\nu) dF(\nu) + \max\{0, F(\nu_1) - F(p_2)\} \tag{4.3}$$

The cases of proportional and efficient rationing is depicted in the Figure 1. Assuming the uniform distribution over the interval [0,1], we can notice that in case of the proportional rationing, the second-period demand has a kink at $p_2 = \nu_1$. It is important to notice that the lower part of the second-period demand function depends only on the total quantity sold in the first period,

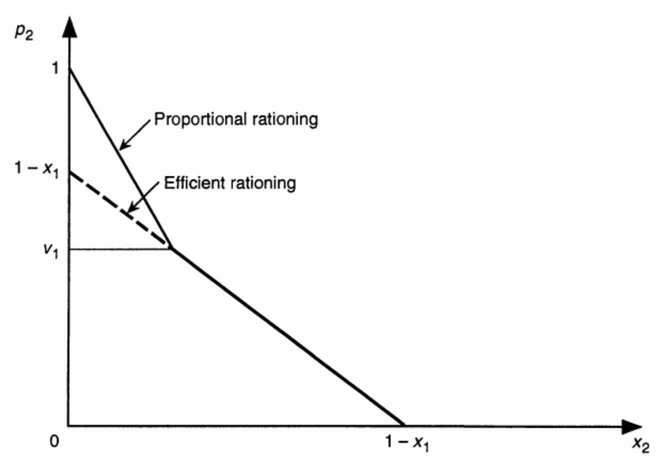

Figure 4.1. The cases of proportion and efficient rationing

thus if it is optimal to set $p_2 < \nu_1$, it would be also optimal to eliminate the rationing in the first period. The following proposition could be made:

Proposition 1. If there is a rationing equilibrium, then it entails $p_2 > p_1$.

From this proposition it can be implied that $\nu_1 = p_1$ in any rationing equilibrium. Also we can notice that efficient rationing is not optimal. In case of efficient rationing, the second period demand is similar to the market-clearing pricing.

Chapter 5

Linear Demand Function

In this part we discuss the specific case with the following assumptions:

- ν is uniformly distributed across the interval $[0, 1]$.
- distribution function $F(\nu) = \nu$
- linear static demand function $x = 1 - p$

No rationing: In order to calculate the no rationing equilibria the backward induction is used. Lets start with the optimal choice in the second period. After serving the x_1 customers in the first period, the monopolist would face the demand of $x_2 = 1 - x_1 - p_2$ customers. Based on the monopoly profit optimization condition ($MR = MC = 0$), $p_2 = \frac{1-x_1}{2}$ that gives profit in second period $V_2 = \frac{(1-x_1)^2}{4}$. Thus, no rationing total profit would be:

$$\pi = p_1 x_1 + \frac{\delta_s (1 - x_1)^2}{4}$$

Using $x_1 = 1 - \nu_1$ and the equation (3.3) we have:

$$\begin{aligned} x_1 = 1 - \nu_1 = 1 - \frac{p_1 - p_2 \delta_b}{1 - \delta_b} \\ p_1 = (1 - \frac{\delta_b}{2})(1 - x_1) \end{aligned}$$

Thus, the no rationing profit is

$$\pi^{NR}(x_1) = \left[\left(1 - \frac{\delta_b}{2} \right) (1 - x_1) \right] x_1 + \delta_s \frac{(1 - x_1)^2}{4} \tag{5.1}$$

After maximization of (5.1) with respect to x_1 the non-rationing equilibria can be calculated:

$$\pi^{NR} = \frac{(2-\delta_b)^2}{4(4-2\delta_b-\delta_s)} \tag{5.2}$$

Rationing: As mentioned above, with proportional rationing, $\gamma(\nu)$ is a constant function in the interval $\nu \geq \nu_1$. To prove that rationing could be optimal, lets first calculate the optimal time-consistent strategy with rationing and after that compare profits with rationing and no rationing. We proceed again by the backward induction. In the second period the monopolist would set p_2, given the first period output. Since rationing can not be optimal unless $p_2 > p_1$, from (3.2) it follows that $\nu_1 = p_1$. Thus the second period demand function is

$$x_2 = \begin{cases} \gamma(1-p_2), \text{ if } p_2 > p_1 \\ \gamma(1-p_1) + p_1 - p_2, \text{ if } p_2 < p_1 \end{cases} \tag{5.3}$$

For the first situation, using the monopoly maximization condition we get $p_2 = \frac{1}{2}$ while $x_2 = \frac{\gamma}{2}$ which leads to the second period profit $V_2 = \frac{\gamma}{4}$. However, this can be a part of the subgame-perfect equilibrium if the calculated profit would be at least as great as in case $p_2 < p_1$. For the case of $p_2 < p_1$ the $x_1 = 1 - p_1 - \gamma(1-p_1)$, $p_2 = \frac{\gamma(1-p_1)+p_1}{2}$ and $x_2 = \frac{\gamma(1-p_1)+p_1}{2}$. Based on these results, the profit would be equal to the case with no rationing ($V_2 = \frac{(1-x_1)^2}{4}$). So, for rationing to be a part of the subgame-perfect equilibrium, it is necessary that $V_2 = \frac{(1-x_1)^2}{4} \leq \frac{\gamma}{4}$, or

$$\gamma \geq (1-x_1)^2 \tag{5.4}$$

Thus, we have to maximize the $\pi^R = x_1 p_1 + \frac{1}{4}\delta_s\gamma$ subject to (5.4). The inequality (5.4) has to be satisfied for rationing to have an effect. As previously mentioned

$$x_1 = 1 - p_1 - \gamma(1 - p_1)$$

$$x_1 = (1 - p_1)(1 - \gamma)$$

and based on the constraint (5.4), we get

$$\pi^R(x_1) = \frac{(1 - x_1)}{(2 - x_1)} x_1 + \delta_s \frac{(1 - x_1)^2}{4} \tag{5.5}$$

where π^R-the optimal profit with rationing. By maximizing this equation the following lemma can be prooved.

Lemma 1: π^R *does not depend on* δ_b*. It is increasing with* δ_s*, it tends to the static monopoly profit* $\frac{1}{4}$ *when* δ_s *goes to one, and it tends to* $3 - 2\sqrt{2}$ *when* δ_s *goes to zero.*

Proposition 2: For all $\delta_s < 1$ *there exist* $\hat{\delta}_b(\delta_s) < 1$ *such that* $\pi^R > \pi^{NR}$ *for* $\delta_b > \hat{\delta}_b(\delta_s)$*.*

Thus in case of rationing, some part of costumers would carry their demand to the second period, increasing the second period demand. As a result monopolist has less incentives to cut the second period price. The above analysis prove that, there could be the cases when the monopolist can gain from this strategy.

Chapter 6

Three periods

The rationing strategy takes place in the first period, in the second period the monopolist has a full monopoly power and he can increase the price and extract monopoly rents from rationed consumers. One can doubt this results in a longer periods, since the second-period price cannot be set as the optimal static level because of the effect of subsequent price cuts. Lets consider the case with three-period durable monopoly, in which rationing can occur in the first period, the second, or both. Three possible strategies are take place.

Lets begin with the case when rationing takes place in both periods. In this case price increasing over time and the third-period price equals the optimal static price. The first period rationing is determined such that the monopolist has no incentive to cut the second-period price. The same happens with the second period rationing. In the third period the monopolist sets static monopolist price. This strategy may be sustain in the three-period case.

Now, lets consider the case with rationing only in the second period. This strategy most closely replicates the rationing equilibrium analyzed for the two-period case. If rationing had not occurred in the first period, the truncated distribution function in the second period would still uniform. The only difference would be in the set of consumers. Thus the presence of the third period creates only a scale effect that does not change the incentive to ration.

The third case with rationing in the first period. In this case, during the second-period the monopolist serves only those consumers that were rationed in the first period, but in the third period the monopolist have to reduce the price in order to serve low-value consumers. Thus, the price rises only in the second period. In this rationing strategy, the price eventually falls, but comparing with no rationing it falls slowly. Deniclo and Garella (1999) in their paper show that the third type of rationing strategy is not optimal when $\delta_b = \delta_s$. It may be optimal when $\delta_b > \delta_s$. This happens because, the durable good producer's monopoly power is lower when the number of periods increases.

Chapter 7

Discount model

In this part I am going to develop the model that for some cases gives more profit that the non-rationing and rationing models.

I consider the similar model as in the previous part, with same assumptions. However, the game will be different. The game will be held as follows:

In the first period, the monopoly announces the first period price and the probability of getting the product in the first period q. Also he sets the second period discount price p_2 and the second period price for consumers who did not try to buy the good in the first period. There would be four types of consumers.

1. Those who try to buy the product in the first period and actually buy it.
2. Those who try to buy the product in the first period and fail to get one. (In the second period they will prefer not to buy the product at p_3).
3. Those who try to buy the product in the first period and fail to get one. (In the second period they buy at p_3).
4. Those who postpone the purchase until the second period and buy the product in the second period at price p_2.

So the intuition here is that the monopoly by using rationing in the first period, keeps in mind those consumers who came in the first period and did not get the product. It sells them at a price that would be different than the second period price. Those who postponed the purchases and come in the second period would get the product at p_2. It is clear that the p_3 has to be higher than p_2 in order to force consumers not to risk in the first period. Clearly monopolist by rationing get some information about customer's and this actually helps him to price discriminate.

Knowing the (p_1, p_2, p_3, q), the consumers decide, whether to risk and try to buy the product in the first period, or to postpone the purchases until the second period sets in. This strategy helps monopoly to better discriminate between consumers.

The attentive reader can notice that if we set $p_3 = p_2$ in the discounting model, we will get the simple rationing case. This means that the simple rationing is just one of the cases of discounting model. Another two cases of discounting model are :

1. When p_3 is such that nobody can afford the product for this price.
2. When p_3 somewhere between p_2 and 1. There are high-value customers who will buy the product at this price.

In order to understand better this model let us consider the first case. For this case those consumers that try to buy the good in the first period and did not get one, would not be able to buy it in the second period. From, Proposition 1 it is reasonable to assume that $p_1 < p_2$.

Let us find the ν_1 - the value that makes consumer indifferent to buy in the first period or in the second period.
$u_1 = q(\nu_1 - p_1)$ - expected utility of a consumer if he buys in the first period.
$u_2 = \delta_b(\nu_1 - p_2)$ - utility of a consumer if he buys in the second period.

$$u_1 = u_2 \Rightarrow \nu_1 = \frac{qp_1 - \delta_b p_2}{q - \delta_b}$$

Let us find out where would be the ν_1 with the regard of p_1 and p_2

$$\frac{qp_1 - \delta_b p_2}{q - \delta_b} - p_2 = \frac{qp_1 - \delta_b p_2 - p_2 q + \delta_b p_2}{q - \delta_b} = \frac{q(p_1 - p_2)}{q - \delta_b} > 0 \Rightarrow \nu_1 > p_2 > p_1$$

Lemma 1: In case when $p_3 \to \infty$, there are no customers with $\nu \in (p_1, \nu_1)$ who would have incentive to buy in the second period, they all try to buy in the first period. Those who have $\nu > (\nu_1, 1)$ would wait and buy in the second period.

Proof of Lemma 1

$\forall\, \epsilon > 0$, the customer with $\nu = \nu_1 + \epsilon$ would prefer to buy in the second period. In order to show that let's, compare utilities in case of buying in the first period and in the second period.

$$u_1 = q(\frac{qp_1 - \delta_b p_2}{q - \delta_b} - p_1 + \epsilon) = q(\frac{qp_1 - \delta_b p_2 - p_1 q + \delta_b p_1}{q - \delta_b} + \epsilon) \tag{7.1}$$

$$= q\epsilon + \frac{\delta_b(p_1 - p_2)}{q - \delta_b} = q\epsilon + u(\nu_1) \tag{7.2}$$

Where $u(\nu_1)$ - utility for indifferent customer. Similarly: $u_2 = \delta_b * \epsilon + u(\nu_1)$. Since $q < \delta_b \Rightarrow u_2 > u_1$. Thus for $\nu \in (\nu_1, 1) : u_2 > u_1$. Analogously, it can be proven that for $\nu \in (p_1, \nu_1) : u_1 > u_2$. Q.E.D.

Now we will determine whether we can get higher profit than in case of non-rationing and rationing. For this purpose the following analysis proposed.

Let us now look at possible values q. As mentioned above, $p_2 \leq \nu_1 \leq 1$.

$$\frac{qp_1 - \delta_b p_2}{q - \delta_b} \leq 1 \Rightarrow q \leq \frac{\delta_b p_2 - \delta_b}{p_1 - 1}$$

and

$$\frac{qp_1 - \delta_b p_2}{q - \delta_b} \geq p_2 \Rightarrow q \geq 0$$

This leads to $q \in [0, \frac{\delta_b p_2 - \delta_b}{p_1 - 1}]$

Let us find the q that maximizes the π^D given the p_1 and p_2:

The first and second periods' profit would be:

$\pi_1^D = p_1 q(\nu_1 - p_1); \quad \pi_2^D = p_2(1 - \nu_1)$

The total profit:

$$\pi^D = \pi_1^D + \delta_s \pi_2^D = p_1 q \frac{qp_1 - \delta_b p_2}{q - \delta_b} - p_1^2 q + \delta_s p_2 - \delta_s p_2 \frac{qp_1 - \delta_b p_2}{q - \delta_b}$$

$$\frac{\partial \pi^D}{\partial q} = -\frac{\delta_b(p_1 - p_2)(\delta_b p_1 - p_2 \delta_s)}{(b - \delta_s)^2}$$

Since $p_1 < p_2$, the sign of the derivative depends on the sign of $\delta_b p_1 - p_2 \delta_s$.

Now we can consider two cases:

1. If $p_1 \delta_b > p_2 \delta_s \Rightarrow$ the derivative is positive, meaning that if the probability of getting product in the first period increases, it will increase π^D, given p_1 and p_2. Thus the q that maximizes the π^D in this case is $q_{max} = \frac{\delta_b p_2 - \delta_b}{p_1 - 1}$. This value of q associated with $\nu_1 = 1$. Which leads us to the following expression:
$$\pi^D(q_{max}) = (1 - p_1)\tfrac{\delta_b p_2 - \delta_b}{p_1 - 1} p_1 = \delta_b(1 - p_2)p_1.$$
2. If $p_1 \delta_b < p_2 \delta_s \Rightarrow$ the derivative is negative, meaning that if the probability of getting product in the first period increases, it will decrease π^D, given p_1 and p_2. Thus the q that maximizes the π^D in this case is $q_{max} = 0$. This value of q associated with $\nu_1 = p_2$. Which leads us to the following expression: In this case the $q_{max} = 0$ with $\nu_1 = p_2$, that leads to:
$$\pi^D(q_{max}) = (p_2 - p_1)0p_1 + \delta_s(1 - p_2)p_2 = \delta_s(1 - p_2)p_2$$

Thus:

$$\pi^D = \begin{cases} \delta_b(1-p_2)p_1, \text{ for } p_1\delta_b > p_2\delta_s \\ \\ \delta_s(1-p_2)p_2, \text{ for } p_1\delta_b < p_2\delta_s \end{cases} \tag{7.3}$$

Here we can notice that these two possible expressions of π^D is maximized when $p_1 = p_2 = 0,5$, given any value of δ_s and δ_b. It means, that if the monopoly chooses the discounting model, in order to maximize his profit it will set the $p_1 = p_2 = 0,5$, and he will choose to sell only in the first period, or only in the second period. The decision when to sell will depend on the sigh between δ_s and δ_b. If $\delta_s > \delta_b$ it will sell only in the second period. If $\delta_s < \delta_b$ it will sell only in the first period.

Proposition 3: $\forall\ \delta_b > \frac{2}{3}\ \exists\ \hat{\delta_s}(\delta_b) :\ \forall\ \delta_s < \hat{\delta_s} :\ \pi^D > \pi^{NR}$

Proof of Proposition 3

According to the (5.2):

$$\pi^{NR} = \frac{(2-\delta_b)^2}{4(4-2\delta_b-\delta_s)}$$

Let's consider the case when $p_1\delta_b > p_2\delta_s$. As mentioned above $p_1 = p_2 = 0,5$ and $\pi^D = \delta_b(1-p_2)p_1$, so:

$$\pi^{NR} - \pi^D = \frac{(2-\delta_b)^2}{4(4-2\delta_b-\delta_s)} - \delta_b(1-p_2)p_1 =$$

$$= \frac{4-4\delta_b+\delta_b^2-\delta_b(4-2\delta_b-\delta_s)}{4(4-2\delta_b-\delta_s)}$$

As we can notice, the denominator is positive, thus we should look at the numerator and find out where it becomes negative:

$$4+3\delta_b^2+\delta_b\delta_s-8\delta_b < 0$$

The solution is: for $\delta_b > 0$, $\delta_s < \frac{-3\delta_b^2+8\delta_b-4}{\delta_b}$. However, the expression $\frac{-3\delta_b^2+8\delta_b-4}{\delta_b}$ becomes positive only for $\delta_b > \frac{2}{3}$. For $\delta_b = \frac{2}{3}$ the δ_s must be equal to 0 in order to have $\pi^{NR} - \pi^{D} < 0$. Thus we proved that for every $\delta_b > \frac{2}{3}$ exist $\hat{\delta_s}(\delta_b)$ that for every $0 \leq \delta_s < \hat{\delta_s}$, the discount model will give us at least the same profit as in no-rationing case.

Now let's consider the case when $p_1\delta_b < p_2\delta_s$:

$$\pi^{NR} - \pi^{D} = \frac{(2-\delta_b)^2}{4(4-2\delta_b-\delta_s}) - \delta_s(1-p_2)p_2 =$$

$$= \frac{4-4\delta_b+\delta_b^2-\delta_s(4-2\delta_b-\delta_s)}{4(4-2\delta_b-\delta_s)}$$

Let's look at the numerator:

$$4+\delta_b^2+2\delta_b\delta_s+\delta_s^2-4(\delta_b+\delta_s) = (\delta_s+\delta_b-2)^2 > 0$$

Thus in case when $\delta_s > \delta_b$, $\pi^{NR} > \pi^{D}$. Q.E.D.

Next step is to prove that in some cases, π^D gives more profit than π^R

$$\max_{x_1} \pi^R = \left(\frac{1-x_1}{2-x_1}\right) x_1 + \delta_s \frac{(1-x_1)^2}{4}$$

Propostion 4: $\forall\ \delta_s\ \exists\ \ \hat{\delta_b}(\delta_s):\ \forall\ \delta_b > \hat{\delta_b}(\delta_s):\ \pi^D > \pi^R.$

Proof of Proposition 4

For $\delta_s = 0$ the above maximization problem leads to $\pi^R = 3 - 2\sqrt{2}$. Let us find $\hat{\delta_b}:\ \pi^D = \pi^R$. From (7.3)

$$\pi^D = \delta_b(1-p_2)p_1 = \frac{\delta_b}{4}$$

.

$$3 - 2\sqrt{2} = \frac{\hat{\delta}_b}{4} \Rightarrow \hat{\delta}_b = 4(3 - 2\sqrt{2})$$

Thus $\forall\ \delta_b > \hat{\delta}_b\ : \pi^D > \pi^R$.

For $\delta_s = 1$ the non-rationing profit has its greatest value which is $\frac{1}{4}$. In this case, from the equation (7.3) : $\forall\ \delta_b \in (0,1) :\ \pi^D = \frac{1}{4}$.

In the general case, when $\delta_s \in (0,1)$, the approach how to find $\hat{\delta}_b$ is the same as in the case with $\delta_s = 0$. First we find the maximum rationing profit, second we calculate the $\hat{\delta}_b$ that equates $\pi^D = \pi^R$. Finally for all $\delta_b > \hat{\delta}_b$: $\pi^D > \pi^R$. Q.E.D.

It is important to notice that for some values of δ_b and δ_s, the π^D gives more profit for the monopolist. This means that the proposed strategy works better in some cases.

Chapter 8

Conclusions

In this paper we propose the strategies which can be of gain for durable goods monopolist. We generalize the two-period model proposed by Deniclo and Garella. In this model the monopolist charge different prices for those who was rationed and those who just postpone the purchases in the first period. The rationing case appears to be one of the possible scenarios of the modified model, specifically, when the second period price for rationed customers is the same as for those who just wait until the second period.

I analyzed the second case of this model. More precisely, when the price for rationed customers in the second period is such that they are not interested in buying product. This strategy helps monopoly make high-willingness consumers to wait until the second period and pay higher price for the product. I find the combinations of δ_b and δ_s, when this strategy is preferable than non-rationing and rationing.

Future research. The topic for future study is the analysis of the third case of this model, when the second price for those customers who postpone the purchase is lower than for those who were rationed. The intuition behind this case is that some customers would prefer to wait until the second period and buy for discount price rather risk in the first period.

Another possible topics could be:

- Extension to the three and more periods case.
- The case with demand which varies over time.
- Another strategies that could help monopoly to improve his performance

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Appendix A

The proof of Proposition 1 and 2 follows.

Proof of Proposition 1: By contradiction: suppose that there is a rationing equilibria with $\hat{p_1} \geq \hat{p_2}$. From (3.2) it can be easily shown that $\hat{\nu_1} \geq \hat{p_1} \geq \hat{p_2}$. In the light of (4.3)

$$x_2 = \int_{\hat{\nu_1}}^{\hat{\nu}} \gamma(\nu) dF(\nu) + F(\nu_1) - F(p_2)$$

Hypothesis: If we change the distribution of consumers on $[\hat{\nu_1}, \hat{\nu}]$, by holding the sales in first period constant, it will not change the optimal price and profits in the second period.

Let us consider an alternative strategy with $\tilde{\gamma}(\nu) = 0$ and with the increased first period price $\tilde{p_1} > \hat{p_1}$ with the appropriate value $\tilde{\nu_1}$:

$$\int_{\hat{\nu_1}}^{\hat{\nu}} \hat{\gamma}(\nu) dF(\nu) + F(\nu_1) = F(\tilde{\nu_1})$$

Since $\tilde{p_1} > \hat{p_1} \Rightarrow \tilde{\nu_1} > \hat{\nu_1}$. By the assumption $\hat{p_2}$ maximizes profit in the second period in case of rationing strategy, so in case of alternative strategy $\tilde{\gamma}(\nu) = 0$ the profit-maximizing price would be the same $\hat{p_2}$. Thus the second period profit and price would be the same. Since $\tilde{p_1} > \hat{p_1}$ and the output in the first period is the same, therefore the total profit in case of alternative strategy must be higher. $\Rightarrow$ Contradiction.

Proof of Proposition 2: We show that $\pi^R > \pi^{NR}$ for $\delta_b = 1$ and $\pi^R < \pi^{NR}$ for $\delta_b = 0$. Then the results follow by continuity. When $\delta_b = 0$, from (5.2) we get $\pi^{NR} = \frac{1}{(4-\delta_s)} \geq \frac{1}{4} > \pi^R$, where the latter inequality follows by Lemma 1 and $\delta_s < 1$. When $\delta_b = 1$, comparing (5.1) and (5.5) it is clear that for all values of

$x_1 > 0, \pi^R(x_1) > \pi^{NR}(x_1)$, which obviously implies $\pi^R > \pi^{NR}$. Q.E.D