EMPIRICAL TESTING OF OPTION PRICING MODELS ON HIGHLY VOLATILE EMERGING MARKETS

by

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Abstract

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This study compares performance ofBlack-Scholes with volatility smile correction and non-linear GARCH option pricing models. Thelog of Mean Square Error ratio and wins ratio of one day out-of-sample forecast are used as measures of accuracy. The data that is used in this study comes from Russian derivatives exchange board. Market prices for the option on RTS index futures for twelve months of 2011 are considered.

Based onlog-MSE ratio criterion the conclusion is made about equal short-term forecasting powers of two pricing models.Wins ratio criterion implies that Black-Scholes model outperforms GARCH in calm market conditions, while the latter model produces more credible option price forecasts during market turmoil. This result suggests that GARCH option pricing model may be used along with convenient Black-Scholes model to estimate option prices during periods of high volatility in the emerging markets.

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Chapter 1

INTRODUCTION

The recent world financial crisis has significantly influenced investors' behavior. Uncertainty about the future of the Euro zone due to the instability of periphery economies has deteriorated the investors' perception of the risks inherent in certain developed markets with yields on short-term government debt exceeding 15% in Greece and Portugal. The massive flight to safe heavens has decreased the annual yields on relatively riskless investments down to roughly 2.5%, such as for US Treasury bills and German Eurobonds. Therefore, in a bipolar world, developing markets become increasingly attractive to the investors. Double-digit economy growth rates and large number of undervalued assets ensure that high yields on such markets stem not only from high risks but also from real economic potential.

Moreover, newly emerging and developing economies are strengthening their positions in the global economy. For instance, China currently is considered to be the driving force of future economic growth, while European economies have considerably increased their dependence on Russian commodity exports.

In the givencircumstances, development of financial markets in these countries becomes of paramount importance. In order to mobilize the vast cash inflow to the Russian economy from exports of natural resources government actively supports the development of spot and derivative financial markets. Same is true about China – its largest stock exchange in Shanghai is now ranked fifth in the world by market capitalization.

However, investors' life is not easy on such markets. Consider Russian market for example. While offering annual yields of up to 30% it is characterized by high level of information asymmetry and controversial legislature. Numerous cases of violation of rights of independent investors are reported in Russia. Corrupted courts and weak regulatory authorities make it possible for business owners to distort the disclosed information. When such cases become revealed to public stock prices unexpectedly fall and overallmarket volatility increases.

One more cause of high volatility in Russian financial market is its dependence on commodity prices. As it was already mentioned, the main drivers of Russian economy are exports of raw materials, such as oil and natural gas. Therefore, the financial performance (and consequently the stock price) of the largest members of Russian market is strongly dependent on world prices for raw materials. Thus, external shocks that happen to other resource exporters have significant impact on Russian market.

Therefore, such stock markets as Russian are highly volatile and difficult to forecast. Many investors are looking forways to hedge against the changes in prices of stocks, raw materials and final products. One of the possible solutions involves option trading. The holder of an option contract has a right to buy or to sell some asset at a predetermined price; consequently, the investor is able to fix the maximum possible loss.

Moreover, while requiring low initial investment, options allow betting on the future behavior of the underlying asset and may generate large profits.The developing markets are believed to be lucrative since they assume significant amounts of underpriced assets. Options allow betting on future growthand, therefore, help to attract new investors and increase market liquidity.

The largest futures and options exchange board on Russian market is FORTS (Futures and Options on RTS exchange).Base assets include futures on market indices, futures on stock prices of biggest corporations and futures on raw materials such as oil and gold.According to The Futures Industry Association, the number of derivative contracts traded in Russia in 2011 exceeded 1 billion with a 73.5% growth compared to 2010, which makes FORTS market tenth by the volume of traded contractsand third most rapidly developing derivative market in the world in the first quarter of 2011[1]. Thus, the demand for options on the Russian market is high.

Naturally, investors are interested in the fairness of option prices and their predictability in these markets. According to the DerEX (Derivative Experts)[2] agency the majority of investors in the Russian market employ the classic Black-Sholes approach with the price correction for volatility smile/smirk effects. However, many other option pricing schemes have been suggested in theoretical literature. Nevertheless, they are almost never tested on the developing or highly volatile markets. Therefore, it is reasonable to ask whether some other methods can produce better price estimates than Black-Scholes formula on Russian market and probably on markets of similar level of development. If the answer is positive, the application of these methods would help to reduce the level of price uncertainty on such markets and increase their attractiveness to investors.

The main hypothesis that will be addressedin this research is that GARCH option pricing model is more suitable model for highly volatile markets than Black-Scholes model. The reason for such claim is the following: by construction the

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1FORTS currency pairs are fastest growing contracts in the world over the Q1 2011, Accessed May 20, 2012, http://www.rts.ru/a22546/?nt=120

2DerEX is an analytical agency which studies Russian derivatives market and organizes practical courses for derivatives traders.

GARCH process incorporates historical market data to construct a specific volatility pattern, while Black-Scholes uses constant volatility estimates. Of course, in real-world applications volatility in the Black-Scholes formula is updated and corrected for the maturity and moneyness of the option. However, GARCH model captures these effects implicitly. Moreover, in GARCH modelmore parameters should be estimated which means that it has more degrees of freedom and should be more flexiblea priori (it should adapt faster to changing market conditions).On the other hand this flexibility is compensated by a more complicated process of estimation which is computationally demanding and may require more time. Moreover, the obtained model is not necessary credible.

The forecasting powers of GARCH model and Black-Scholes model will be compared according to the criterion of the accuracy of the out-of-sample price forecast. Two measures of accuracy will be used with one of them beingmean square error (MSE) and the other one is wins ratio. The first will capture the absolute difference in the forecasts while the second one will simply count the ratio of more precise forecasts.

The historical data will be used to estimate the parameters of GARCH model andto select the appropriate volatilityfor the Black-Sholes model. The conclusion about the applicability of GARCH method to pricing options on the emerging markets will be made.

The data about the dynamics of prices of call options on futures on RTS index and the corresponding base asset on Russian market will be used. The daily data would be collected for one year period betweenDecember 2010 and December 2011.

The next section will concentrate on the academic research on the topic. After that two pricing models will be described in details and algorithms for their estimation will be provided. Then the available data will be summarized. The discussion of the approaches for empirical testing and its results will follow. The conclusions on the performance of options pricing models will be presented in the last chapter.

Chapter 2

LITERATURE REVIEW

The most well-known and widely used model for options pricing was developed byBlack and Scholes(1973) and Merton (1973). These studies modeled the price of the base asset as a lognormal process and applied stochastic calculus to derive a formula for option pricing. Among the main properties of this model are its simplicity, ease of implementation and considerable preciseness. This study gave a significant impulse to the options trade all over the world, because it developed a widely applicable method to calculate option prices. Soon after a study by Cox, Ross and Rubinstein (1979) applied the idea of binomial trees to describe the behavior of the base asset and developed the simplest theoretical approach to option pricing.

However, the main limitation of these models is the assumption of constant volatility of the underlying asset price, which means that fluctuations of the price have fixed average amplitude. Empirical studies (Bollerslev, Chou and Krone, 1992) have shown that in the majority of real-world cases when option prices and risks or hedging portfolio should be estimated, variable volatility should be taken into the account. Therefore, various models, which incorporate time-varying or even stochastic volatility, were developed.

Moreover, market evidence suggests that implied volatilities (extracted from market prices) of the options written on the same underlying asset but with different strikes usually are different. Actually, the further the current price of an asset from the strike, the higher is implied volatility. This phenomenon is called 'volatility smile/smirk' and its first description in the academic literature is attributed to Rubinstein (1985).

The two predominant approaches that can be distinguished among the models that incorporate variable volatility are deterministic-volatility models and stochastic volatility models. The first group assumes that volatility can be estimated from the market data such as historical asset prices. The second group incorporates a more demanding approach which assumes that source of option price uncertainty is different from the uncertainty in the underlying asset price; however these uncertainties may be correlated. For example consider the variance gamma model described by Madan, Carr and Chang (1998) which modeled option contract price as a generalized Brownian motion in the form of a three-variable variance gamma stochastic process. The model incorporates the concept of volatility smile and being properly calibrated provides good in-sample estimation of option prices.

While obtainingprecise price estimates (Hull and White, 1987; Stein and Stein, 1991) stochastic volatility models are usually rather difficult to implement. Due to the fact that in most cases such models do not have a closed-form solution, specific numerical methods should be applied. These methods are usually time-consuming which eliminates the possibility of their application on real markets where the decision making process should be quick.

Therefore, the non-stochastic volatility models are considered to be the best choice in terms of trade-off between the accuracy and possibility of practical usage. These are various binomial tree models, generalized methods of moments (GMM) models and general autoregressive conditional heteroscedasticity (GARCH) models.

Introduced by Engle (1982) autoregressive conditional heteroscedasticity (ARCH) models were used for general time-series analysis. The main idea of these models is that in each period the error term is assumed to be a function of the previous values of error terms with appropriate weights assigned. The application of GARCH for the options pricing problem was introduced by Duan (1995) who used this approach to simulate returns and volatility of the underlying asset. GARCH has proven to be a good estimation approach for time-varying volatility, giving accurate in-sample and out-of-sample estimates. Moreover, it was theoretically proven by Duan (1996) that bivariate diffusion methods are limits of the GARCH option pricing model when the discretization of time approaches zero. This means that the large group of stochastic volatility methods which are much less straightforward and harder to estimate can be approximated with GARCH models.

The description of the one of the most convenient numerical algorithms for GARCH simulation was developed by Duan and Simonato (1998).Authors employ the traditional Monte-Carlo simulation pattern and also address the problem of simulated option price being out of natural boundaries. In such a case it is impossible to find the implied volatility of an option contract and simulation should be repeated once again. Authors suggest using Empirical Martingale Simulation (EMS) process which performs martingale adjustment on each step of the algorithm.

One of the most recent theoretical studies on GARCH option pricing models was published by Heston and Nandi (2000). Authors employ specification of non-linear GARCH which slightly differs from the model of Duan (1995). The main result that authors obtain is the closed-form solution for such model. However, for this solution to be applied, GARCH parameters should be estimated from market data. Authors suggest using maximum likelihood method which, however, may fail to produce credible model as log-likelihood function may be too flat to find the global minimum within reasonable time.

Considering the empirical tests of GARCH approach, the study byDuan and Zhang (2001)on GARCH option pricing metodshould be mentioned. Authors compared GARCH model to the Black-Scholes model with the correction for volatility smile using the data from the Hong-Kong Stock exchange before and during the Asian financial crisis in 1998. It turned out that GARCH gives more precise estimates of option prices even during market turmoil. Liu and Morley (2009) have also used Hang Seng Index options data from Hong Kong stock exchange to assess the effectiveness of GARCH processes for volatility forecasting. Comparing GARCH to the historical averaging models they have obtained results similar to Duan and Zhang (2001) – in the out-of-sample forecasting GARCH gave the most precise estimates among the models which were compared.

The only published academic study on options market in Russia is attributed to Morozova (2011). Author tries to construct an option pricing model which incorporates the 'true' statistical distribution of option prices. As this model appears to produce the estimates which are different from the observed market prices, the conclusion is made about informational inefficiency of Russian options market. Although such conclusion sounds reasonable for the developing market, the employedapproach is doubtful as the credibility of 'true' pricing model can hardly be assessed. Therefore, the option pricing problem on Russian and similar emerging market remains open.

Thus, this study will contribute to the existing literature by assessing the applicability of GARCH model and Black-Scholes model with the correction for volatility smile/smirk on Russian market. Yet, GARCH option pricing model was not tested on the data from developing markets. The existing evidence on the effectiveness of this method for price forecasting applied to Asian crisis market data (Duanand Zhang, 2001) makes it a reasonable candidate for testing on Russian market.

Chapter 3

METHODOLOGY

In this study a nonlinear generalized autoregressive conditional heteroscedastic (NGARCH) process is used to model the behavior of log-return of base asset.Such model is characterized by discrete time intervalswhich correspond to daily frequency of market data used in this study. The risk-neutral version of suchNGARCH(1,1)model was developed by Duan(1995), whose notations will be followed from now on.

In the general specification, one-period rate of return on futures is assumed to be conditionally lognormally distributed as in equation (1), and its error term follows the process described by equation (2):

$$\frac{\ln F_{t+1}}{F_t} = r - y_{t+1} + \lambda\sigma_{t+1} - \frac{1}{2}\sigma_{t+1}{}^2 + \sigma_{t+1}\varepsilon_{t+1} \qquad (1)$$

$\varepsilon_{t+1} \sim N(0,1)$

$$\sigma_{t+1}{}^2 = \beta_0 + \beta_1 \cdot \sigma_t{}^2 + \beta_2 \cdot \sigma_t{}^2 \cdot (\varepsilon_t - \theta)^2 \qquad (2)$$

HereF_t is a price of the futures (base asset) at moment t,r is a risk-free rate of return, λ is risk premium parameter,θ is leverage parameter and $\sigma_{t+1}{}^2$ is conditional variance.

It was shown by Duan (1995) that the risk-neutral version of this model is constructed by setting$\xi_{t+1} = \varepsilon_{t+1} + \lambda$ (ξ_{t+1} becomes standard normal variable under risk-free measure). Moreover, the dividend yield and risk-free rate can be neglected as the base asset is futures with same maturity as the option; therefore, these two factors are already captured by futures price (Lieu, 1990). Thus, the specification described by equations (3) and (4) is used.

$$\frac{\ln F_{t+1}}{F_t} = -\frac{1}{2}\sigma_{t+1}{}^{2} + \sigma_{t+1}\xi_{t+1} \qquad (3)$$

$\xi_{t+1} \sim N(0,1)$

$$\sigma_{t+1}{}^{2} = \beta_0 + \beta_1 \cdot \sigma_t{}^{2} + \beta_2 \cdot \sigma_t{}^{2} \cdot (\xi_{t+1} - \theta - \lambda)^{2} \qquad (4)$$

The four parameters $(\beta_0, \beta_1, \beta_2, \theta + \lambda)$ of these equations areestimated with the help ofMonte-Carlo simulations and numerical minimization process. On the first step initial values of parameters are selected and 10,000 futures prices paths are generated according to equations (3) and (4). On the second step these prices are normalized in order to ensure that they obey martingale property. This procedure is called Empirical Martingale Simulation and its detailed description was given by Duan and Simonato (1998).On the third step for each of simulated futures prices at maturitythe price of futures-style call option is calculated as shown in equation (5).

$$\widetilde{c_t} = E_t^M(\max(F_T(T) - K, 0)) \qquad (5)$$

Here E_t^M is the mathematical expectation with respect to risk-neutral measure and K is the strike value of the option. The call prices are then averaged on the step four to get the estimated for the option price.

Steps 1-4 are repeated for each of the options in the sample that is used for calibration. The obtained prices are then plugged into the Black-Scholes formula (6), which is solved to get the implied volatilities. Finally, these volatilities are compared to the implied volatilities derived from market prices and value of the objective function (7) is calculated. Then the numerical minimization algorithm is used to update the parameters of GARCH model until minimum of (7) is attained.

$$c_t = F_t(T) \cdot N(d_1) - K \cdot N(d_1) \cdot e^{-r(T-t)}$$

$$d_1 = \frac{\frac{\ln(F_t(T))}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad d_2 = d_1 - \sigma\sqrt{T-t} \tag{6}$$

Finally, when parameters of GARCH model are obtained it is used for price forecasting as described above in steps 1-4.

$$V = \min \sum_{i=1}^{m} (\sigma_t^{GARCH} - \sigma_t^{MKT})^2 \tag{7}$$

Black-Scholes formula for European-style call option (6) is used as a benchmark in this research. The main challenge that practitioners face is to estimate the volatility parameter σ. Moreover, there exists strong empirical evidence that σ should be adjusted by the moneyness of the option – higher difference between current price of base asset and strike implies higher volatility.

In this research the approach for implied volatility forecasting which is similar to the one described byDumas, Fleming and Whaley (1998) will be followed. Different estimators for volatility will be used for in the money options $\left(2.5\% \leq \frac{F_t - K}{K}\right)$, near the money options $\left(-2.5\% \leq \frac{F_t - K}{K} \leq 2.5\%\right)$ and out of the money options $\left(\frac{F_t - K}{K} \leq -2.5\%\right)$. For each class the implied volatilities will be averaged across in-sample market option prices in order to get volatility estimates for out-of-sample forecasting. The average volatilities are than used for price forecasting of options of similar class.

Chapter 4

DATA

The market data for this study comes from Russian derivatives exchange board FORTS. In particular, this research focuses on the option contractswhich base asset is futures on RTS index. The reason for such selection is the highest liquidity of these contracts among all options on FORTS board. RTS index consists of 50 Russian stocks which are weighted according to their capitalization. Corresponding futures are written on 1,000 shares of RTS index and mature each quarter.

Option contracts which are written on RTS index futures and mature on the same date with futures are selected for this study. All options are of American type, therefore only call options are considered to eliminate the possibility of early execution (Hull, 2009). The daily data on the market prices of futures and corresponding option contracts for the period between 15 December 2010 and 8 December 2011 is obtained from the official site of RTS exchange. The dataset includes contracts with four maturity dates: 15 March 2011, 15 June 2011, 15 September 2011 and 15 December 2011;on each trading day only contracts with earliest maturity are, however, considered. The last 5 trading days for each option are excluded from the sample to eliminate near-maturity bias.

For each option contract up to 18 strikes with step of 5,000 rubles are considered. However, only from 6 up to 14 most actively traded strike quotes are selected each day in order to avoid problems which may arise from low liquidity.

The number of option contract pricesfor each moneyness category is provided in Table 1. The largest number of price observationsin the sample is for out-of-the-money options, while the smallest part is for near-the-money. No specific timepattern within any category is found upon comparison offirst half of the sample and second half of the sample for each maturity.

Table 1.The number of option contracts prices by moneyness

Maturity		**OTM**	**NTM**	**ITM**	**Total**
March 2011	First half	230	47	98	375
	Second half	194	47	149	390
	Total	424	94	247	765
June 2011	First half	260	57	176	493
	Second half	329	58	123	510
	Total	589	115	299	1003
September 2011	First half	230	62	266	558
	Second half	370	56	132	558
	Total	600	118	398	1116
December 2011	First half	354	35	104	493
	Second half	291	45	174	510
	Total	645	80	278	1003

The review of futures prices for the obtained dataset has shown that significant shock has struck Russian market in August 2011. During the period between 3 August and 11 August 2011 futures on RTS index has lost 20% due to global markets turmoil caused by the outbreak of Eurozone crisis. The 2011 historical price movement of futures on RTS index is shown in the Figure 1.

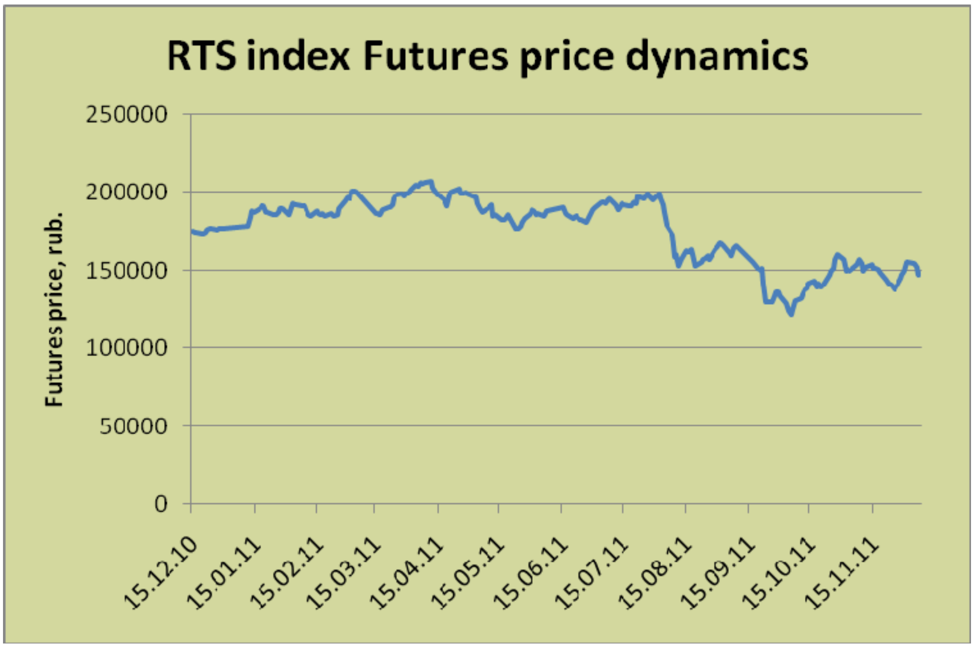

Figure 1. Historical prices of RTS index futures

Thus, the obtained data contains a possible structural break which provides an opportunity to compare the performance of two option pricing models not only in calm market conditions but also during the market turmoil.

Chapter 5

EMPIRICAL RESULTS

For the empirical testing of Black-Scholes and GARCH models Matlab software was used. In order to estimate moneyness-dependent volatility in the Black-Scholes model historical prices for five previous trading days were used. Such time interval is required to ensure that options of all three types (out-of-the-money, near-the-money and in-the-money) are present in the historical sample and average implied volatilities in each class can be calculated.

The summary statistics for averaged implied volatilities estimates is provided in the Table 2. The minimum annual volatility observed in the sample is roughly 22% while at maximum it reaches 55% during August turmoil. The average value of implied volatility increases for the contracts which mature in September and remains higher than 50%for the contracts which mature in December. However, the standard deviation of implied volatility is only high during the turmoil (roughly 10%), while it falls to 3% for contracts which mature in December. This evidence supports the initial assumption about time-varying volatility.

For the estimation of GARCH model there is no need to use historical data forthe period longer than one day. Minimization process yields parameters of GARCH process such that volatilities of GARCH-priced options are as close as possible to the implied volatilities of historical one-day sample. Root mean square error which is simply the standard deviation of error of GARCH volatility estimate is the goodness-of-fit measure for this model. The average RMSE does not exceed 3%, which is significantly lower than minimum market volatility of 22%. This implies that GARCH is able to produce credible estimates for option implied volatiles and prices. For the summary statistics of GARCH parameters refer to the Table 3.

It should be noted that in the risk-free specification of GARCH model neither risk-premium (λ) parameter, nor leverage (θ) parameter can be separately estimated. However, this is not a problem as the main purpose of GARCH model in this research is to generate credible paths for asset returns.

Table 2. Summary statistics of implied volatility estimates

Maturity		**OTM**	**NTM**	**ITM**
March 2011	Minimum	0.2255	0.2369	0.2571
	Maximum	0.2732	0.2776	0.3465
	Mean	0.2463	0.2606	0.2856
	S.D.	0.0140	0.0108	0.0184
June 2011	Minimum	0.2274	0.2424	0.2421
	Maximum	0.2862	0.2868	0.2842
	Mean	0.2516	0.2612	0.2663
	S.D.	0.0196	0.0094	0.0110
September 2011	Minimum	0.2225	0.2379	0.2455
	Maximum	0.5109	0.5579	0.5218
	Mean	0.3195	0.3343	0.3236
	S.D.	0.1069	0.1098	0.0965
December 2011	Minimum	0.3812	0.4190	0.3958
	Maximum	0.5153	0.5720	0.5645
	Mean	0.4399	0.4770	0.4818
	S.D.	0.0310	0.0416	0.0481

To compare out-of-sample prediction power of two models one-day option price forecasts are considered. Regarding the fact that Black-Scholes model needs at least five days of historical data to be estimated, the forecasts are calculated starting from the day 6 of each sampling period and continue up to the end of the period. Both models are updated in each subsequent period: the average implied volatilities in Black-Scholes model are recalculated and GARCH parameters are re-estimated; therefore, any new market information is immediately captured by both models.

Table 3. Summary statistics of GARCH coefficients

Maturity	**Coef.**	β_0	β_1	β_2	$\theta + \lambda$	**RMSE**
March 2011	Min	7.9E-05	0.370	1.07E-07	116.18	0.0054
	Max	0.000126	0.533	1.64E-07	299.03	0.0964
	Mean	9.92E-05	0.458	1.26E-07	198.28	0.0211
	S.D.	1.18E-05	0.045	1.58E-08	50.419	0.0178
June 2011	Min	7.36E-05	0.371	9.63E-08	111.65	0.0041
	Max	0.000135	0.547	3.05E-07	326.05	0.0506
	Mean	9.82E-05	0.461	1.78E-07	204.13	0.0176
	S.D.	1.44E-05	0.042	5.52E-08	64.928	0.0089
September 2011	Min	7.21E-05	0.371	9.87E-08	116.18	0.0033
	Max	0.000145	0.863	1.89E-07	246.14	0.0449
	Mean	0.000101	0.581	1.41E-07	195.09	0.0160
	S.D.	2.03E-05	0.159	2.47E-08	34.861	0.0091
December 2011	Min	0.000107	0.371	2.81E-08	106.86	0.0087
	Max	0.000334	0.716	1.07E-07	246.72	0.0549
	Mean	0.000251	0.542	4.19E-08	152.55	0.0291
	S.D.	4.62E-05	0.079	1.19E-08	42.165	0.0125

The option price forecasts are compared using two measures. Firstof them calculates the Mean Squared Error between implied volatilities of the price forecast and actual market price as shown in equation (8).

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(\sigma_i^{FCST} - \sigma_i^{MKT}\right)^2 \qquad (8)$$

Here n is the number of option prices that are forecasted during one period, σ_i^{FCST} is the implied volatility of the forecasted option price and σ_i^{MKT} is the implied volatility which corresponds to the market option price.

Although such measure does not compare forecasted prices directly, it has the advantage of being independent of the absolute values of option prices. This means that pricing errors are accounted fairly and options with large premiums do not bring any distortion. In order to compare models with each other, natural logarithm of the ratio of two MSE is calculated as shown in equation (9).

$$DMSE = \ln\left(\frac{MSE_{GARCH}}{MSE_{BS}}\right) \qquad (9)$$

According to the construction of log-MSE ratio, its negative value means that GARCH provides more accurate price estimates while positive value implies that Black-Scholes model outperforms GARCH model. The natural logarithm of MSE ratio is calculated for each daily set of forecasted prices. The obtained values of log-MSE ratios for contracts of four different maturities are presented in Table 4 while the average values, descriptional statistics and results of significance tests for DMSE are presented in the Table 5.

Table 4. Log-MSE ratio values for each trading day

Trading day	March 2011	June 2011	September 2011	December 2011
6	1.028422	0.879604	-0.61454	-0.24964
7	1.202181	0.101239	0.590889	-0.62646
8	1.335988	-0.09652	0.380049	-0.30465
9	1.43789	-0.18662	1.321054	0.444541
10	-0.13236	0.822577	-0.43582	0.385932
11	0.745789	1.131998	0.728724	-0.12274
12	1.254344	1.012363	1.193262	0.466285
13	1.046297	0.942873	1.09631	-0.43068
14	-2.96619	1.361673	1.195399	-1.08342
15	0.369693	1.7944	0.884429	-1.3784
16	0.207604	1.976638	1.863333	-0.45913
17	1.163543	2.055229	0.105668	0.565679
18	0.80733	1.897435	0.970331	-0.59661
19	1.628862	1.281712	1.623985	-0.52488
20	1.419904	1.065811	0.895698	-0.59582
21	1.142679	1.903616	0.347677	0.003516
22	-0.3876	1.554643	-0.27625	0.929877
23	-1.56363	0.172646	0.953779	0.595069
24	-0.8355	2.012652	-0.67458	0.773568
25	0.118654	-0.99522	2.278952	0.503618
26	0.057868	0.448866	1.019188	0.351457
27	0.580324	0.688551	-0.11135	0.648042
28	0.185975	0.513846	1.44763	-0.01234
29	1.547654	0.225964	0.755677	0.462736
30	-0.37266	0.28205	-0.0749	0.164773
31	-1.12225	0.976166	-0.39016	0.229139
32	0.848005	0.722617	0.71756	-0.87943
33	1.736988	0.092708	-0.02016	-1.05279
34	-0.7605	0.205213	0.630522	-0.61397
35	-0.55483	-0.38684	0.097435	-1.08621
36	-0.68006	-0.15056	-0.98907	-0.36069
37	0.569286	-2.68725	-0.53608	0.433836
38	0.210928	-1.17767	-0.34583	-0.08484
39	1.449687	0.310775	-0.97298	0.997232

Table 4. Log-MSE ratio values for each trading day - Continued

Trading day	March 2011	June 2011	September 2011	December 2011
40	0.080783	0.315228	-1.05826	1.178631
41	-0.48031	1.281994	-0.53968	-0.3106
42	-0.64104	-0.35288	-0.97167	0.785318
43	0.992733	0.034348	-0.51754	-0.08046
44	0.580537	0.047625	-0.47258	0.821462
45	0.509509	-0.06583	-3.01003	0.322151
46	0.272501	-0.12789	-3.0467	0.062997
47	0.488792	-0.38089	-1.58153	-1.29523
48	-0.73663	0.679989	0.487152	-1.53264
49	0.598409	0.087119	0.643188	-0.91622
50	-0.01148	-0.48782	0.287369	-0.42991
51	0.32878	0.039124	0.075407	0.467701
52		0.151433	-0.29081	0.223416
53		0.132033	-0.50739	0.241568
54		0.556149	-0.15647	0.345498
55		-0.28426	-0.22039	-0.10036
56		-0.4007	-0.54658	-1.32914
57		-0.64066	-1.62846	-0.51369
58		1.074302	-1.14018	0.164308
59		0.036789	0.482652	-0.40997
60			-1.17726	
61			1.308908	
62			0.183062	

Positive average log-MSE ratio for the three of four maturities suggests that on average Black-Scholes model produces slightly better forecasts than GARCH model.However, average log-MSE ratio for the second half of the sample is negative in three of four cases, which may be an indicator that GARCH model produces more accurate estimates than Black-Scholes model for the option contracts that are closer to maturity.

Nevertheless, none of the values of t-statistics exceeds the critical value which means that all log-MSE ratios are statistically insignificant. This leads to the conclusion that Black-Scholes and GARCH option pricing methods are equally powerful in out-of-sample forecasting, according to log-MSE ratio measure.

Table 5. Comparison of forecasting power based on Mean Squared Errors

Sample		**March 2011**	**June 2011**	**September 2011**	**December 2011**
First Half	Min	-2.97	-1.00	-0.67	-1.38
	Max	1.63	2.06	2.28	0.93
	Mean	0.43	0.91	0.63	-0.03
	S.D.	1.09	0.78	0.75	0.61
	t-stat	0.39	1.17	0.84	-0.05
Second Half	Min	-1.12	-2.69	-3.05	-1.53
	Max	1.74	1.28	1.31	1.18
	Mean	0.21	-0.08	-0.58	-0.15
	S.D.	0.79	0.73	0.97	0.73
	t-stat	0.26	-0.11	-0.59	-0.21
Total	Min	-2.97	-2.69	-3.05	-1.53
	Max	1.74	2.06	2.28	1.18
	Mean	0.32	0.42	0.04	-0.09
	S.D.	0.95	0.89	1.05	0.67
	t-stat	0.34	0.46	0.04	-0.13

The second measure that is used to assess the performance of two option pricing models is wins ratio. It is calculated as ashare of option contracts in the total sample for which the GARCH model has produced a more accurate price forecast. Though, the value of wins ratio below 0.5 indicates that Black-Scholes model has performed better, while the ratio above 0.5 ensures that GARCH has produced more credible estimates.

Wins ratio approach is different from log-MSE ratio comparison as it ensures that none of the models performs better only due to the overfitting. The calculatedwins ratios for each maturityand the corresponding values of z-test for the equality to 0.5 are presented in the Table 5.

Table 6. Comparison of forecasting power based on wins ratio[a]

		March 2011	June 2011	September 2011	December 2011
First Half	wins	0.35	**0.32***	**0.32***	0.53
	z-stat.	-1.48	-1.99	-2.01	0.37
Second Half	wins	0.39	0.48	**0.66†**	0.53
	z-stat.	-1.04	-0.21	1.91	0.37
Total	wins	**0.37***	0.40	0.53	0.53
	z-stat.	-1.75	-1.47	0.40	0.52

[a]–"*" indicates wins ratio that is statistically lower 0.5 under 5% confidence level (one-tailed test), "†" indicates wins ratio that is statistically higher than 0.5 under 5% confidence level (one-tailed test).

Wins ratio test shows that the number of more accurate forecasts is statistically higher for Black-Scholes for the whole set of contracts that matured in March. This model has also produced better price estimates during the first half of the sample of contracts which mature in June. However, when turmoil started, GARCH model managed to outperform Black-Scholes with wins ratio of 0.66 for the second half of the sample of contracts that matured in September.Further on, the performance of two models is very similar with wins ratio being statistically indifferent from 0.5.

Thus, the two accuracy measures did not reveal the model which performs better under any circumstances. Two pricing methods produce statistically similar results within an MSE framework, while wins ratio test implies that GARCH option pricing model performs better during the periods of high volatility.

Chapter 6

CONCLUSION

This study addresses comparison of option pricing models on Russian derivatives market. The two models that are considered are the widely used Black-Scholes model with correction for volatility smile effects and GARCH option pricing model. Although academic evidence exists that the latter is able to outperform BS model in the developed markets, the out-of-sample forecast comparison of market data from Russian market suggests that two models have relatively similar prediction power on this market.

The comparison based on the mean square errorcriterion does not reveal better performing model, as all results are statistically indistinguishable. On the other hand, wins ratio comparison suggests that Black-Scholes model is more suitable if the market situation is calm, while GARCH produces better forecasts under high volatility.

Thus, the possibility of application of GARCH model for option pricing on Russian market was empirically proven. Although it did not manage to outperform Black-Scholes model and thus it cannot serve as the only option pricing instrument on the markets similar to Russian, it has an advantage of being more precise than Black-Scholes model during the market turmoil. This feature may be interesting to the investors on Russian market.

Further research on this topic may concentrate on other specifications of GARCH processes for option pricing. Moreover, the closed-form solution of the GARCH model developed by Heston and Nandi (2000) may be estimated if the way to overcome log-likelihood estimation problems is found.

Finally, other option pricing models may be considered for applying on Russian market. For example, although being hard to calibrate, stochastic volatility models may appear to produce precise results.

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