TERM STRUCTURE OF INTEREST RATES ANLYSIS. THE CASE OF BELARUS

by

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Abstract

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The term structure of interest rates is a very important question in analyzing both financial markets and the conditions of the economy as a whole. This thesis provides the analysis of the term structure of interest rates on Belarusian government bonds by testing two theories: Pure Expectations Hypothesis and Liquidity Premium Theory. For this purpose yields to maturity and forward interest rates for bonds with maturity up to one year are calculated. The period investigated is 1999-2003. Pure Expectations Hypothesis is tested using the expectations of yield spreads, Liquidity Premium Theory – using the differences between forward and spot interest rates. The results of the study indicate that on average a yield curve for Belarusian GKO has a downward sloping structure. Pure Expectations Theory proved to be inconsistent with the data. Investigating of the liquidity premia indicated the presence of time varying negative premium.

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Chapter 1

INTRODUCTION

Term structure of interest rates is an important issue in analyzing financial markets and even macroeconomic parameters of countries. As Brown and Dybvig [1986] stated "The term structure of interest rates is important to economists because the relationship among the yields on default free securities that differ in their term maturity reflects the information available to the market about the future course of events". This course of events includes not only expectations on the stock markets and financial claims pricing at microeconomic level "the changes in expected future short rates are then further decomposed into portions attributable to changes in the expected future paths for inflation, the unemployment rate, and GDP growth and also to a fourth factor interpreted as changes in the "stance of monetary policy" (Clouse, 2004)at macroeconomic level.

From this point of view having the theory explaining the term structure of interest rates is essential. Modeling this structure is important for understanding the investor's behavior and the effectiveness of managing long-term and short-term government debt. Also difference between short-term and long-term interest rates (namely, the yield curve) is often happened to be a good instrument of prediction the economic activity in the country (Clinton, 1994).

The aim of this study is to construct the yield curves for Belarusian government bonds with maturity more than one year and to define whether one of "classic" theories of the term structure of interest rates could be applied to the financial market of the Republic of Belarus. Two theories tested are the

Pure Expectation Hypothesis and Liquidity Premium Theory. Pure Expectations Hypothesis is expected to be the only more consistent with real data; however a time-varying term premium also likely may exist. Market Segmentation Theory is not tested due to small history of financial market in Belarus and short time horizon of the analysis. Strict time preferences within one year period are unlikely to exist even on more developed markets (Taylor, 1992).

Although the term structure of interest rates has been widely discussed in the literature and research papers in developed countries, such a research is rare for the transition economies and CIS countries in particular. Among the post-soviet countries such studies have been done in Russia (Entov, Radygir Sinelnikov et al., 1998; Kryukovskaya, 2003) but never conducted in Belarus. Therefore, this research may be a significant contribution to the analysis of Belarusian financial market.

The general test of the theories is based on Kryukovskaya [2003] and Gordon [2003] analyses. For this purpose, the data on the deals with government bonds of different maturities up to one year during 1998-2003 is used. Correct estimation using this approach gives the opportunity to use it for understanding preferences of the public. Therefore, received results are to be applied for policy implications for financial market of Belarus.

The structure of this thesis is as follows. In Chapter 2 I give the review of literature on main theories and their tests and application relevant to my research. Chapter 3 consists of methodology necessary for the analysis. Chapter 4 contains data description. Results, conclusions and implications are presented in Chapters 5 and 6.

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LITERATURE REVIEW

The question of explaining the term structure of interest rates has been investigated for a long time already. The whole range of simple and complicated theoretical and empirical models is developed on this issue. There are two types of approaches to the explaining of the term structure. Micro level concerns the explaining of the term structure using only market information. There are some theoretical models described below developed on this. The main difference of them is the assumption about sellers' and buyers' preferences. Those models and their empirical testing are described in the first part of this review. Macro level means that the term structure of interest rates can be explained by different macro factors as GDP, inflation and unemployment. There are different (mostly empirical) models on this level. Their authors argue that the shape of yield curve is determined also by information outside the market. Some of these models are discussed in the second part of the review.

Let us move to the theoretical models on micro level. According to most of authors, there are three main theories on term structure of interest rates: Pure Expectations Theory, Market Segmentation Theory (Culbertson, 1957) and Liquidity Premium Theory (Hicks, 1946). The theories were developed rather long ago, but still are widely used as a basis for more complicated models.

There is no unique opinion about the originator of the Pure Expectations Theory. The essential statements can be found in papers of different economists of the first half of twentieth century. However, Shiller and

McCulloch [1987] attribute it to Fisher [1896]. The theory states that long term rates reflect the expectations of future short term interest rates, which implies that the return on long term bond is the same as the expected return on a series of short term bonds during the same period. In this case market should be efficient in the sense that no time arbitrage is available and therefore the bond pricing becomes an easy task. This theory is the most popular and the most empirically tested. However, not all authors show the support of this theory by data. Although Meiselmann [1962] found it quite reliable, Grant [1964], Buse [1967], Malliel and Kane [1969], Jorion and Mishkin [1991] and others found little evidence of this theory. The theory is however simple and can be used for constructing more complicated models. This may be the reason why it is tested again and again. The result is that the Pure Expectations hypothesis almost never holds for short-run changes of long term rates, but it is pretty often true for changes in short term rates for a long-run (Campbell and Shiller, 1991). The theory is also sometimes the only one to be used for emerging capital markets due to undeveloped financial instruments and absence of strict market segments (Drobyshevsky, 1999).

Liquidity Premium Theory developed by Hicks [1946] allows the long term interest rate deviate from the expected short term one. In this case the additional assumption on investors' preferences is made. The return on short term bonds is assumed to be more or less certain while the return on long term bonds (despite the name "risk-free") is not. In the long period some shocks can appear but long term bonds are not liquid enough to react to the shock immediately and this would lower the actual gain from holding the bond. Therefore investors would like to get additional interest called the liquidity premium for this uncertainty and long term interest rates deviate are higher than expected short term ones. The idea of Liquidity Premium Hypothesis is quite natural and indeed supported by data. A lot of empirical tests like done by Kessel [1965] and McCulloch [1975] sow the existence of the premium. However, there is no unique view what factors influence the liquidity premium and whether it varies over time. The opinions sometimes are mutually exclusive. For example, Cagan [1969] states the positive correlation between liquidity premium and the level of interest rates, Nelson [1972] agues the same relation to be negative whereas McCulloch [1975] finds no relation. The determinants of liquidity premium are still not unclear.

The Market Segmentation Theory introduced by Culbertson [1957] assumes that investors have strict maturity preferences. In this case pension funds with long term liabilities would invest in similar bonds while banks would operate in a shorter horizon. This implies existence of "separated" market segments each having interest rate determined by its own supply-demand interaction. The yield curves under this hypothesis are not even expected to be continuous over different maturity periods.

The listed theories all have their drawbacks and advantages. However, the researchers think that they are too narrow to explain the term structure completely. Therefore a number of models combining those theories are developed. The most widely mentioned is Preferred Habitat Theory (Modigliani and Sutch, 1966).

Modigliani and Sutch [1966] extend the Market Segmentation Theory in the way that investor may deviate from their maturity preferences if compensated by higher yield. Basically, this means the mixing of Market Segmentation and Liquidity Premium Hypotheses. While testing main theories for the UK market Taylor [1992] rejects all of them except of Preferred Habitat.

The task of bond pricing on micro level forced the researchers to try to use different asset pricing models to determine the term structure of interest rates. Most of these attempts are empirical and use different simulations. However, Roll [1971] developed a theoretical mean-variance model and Merton [1974] offered the application of methodology used for option pricing. Despite all the research on this topic, the number of functions for bond prices is huge and no widely accepted theories are developed.

Let us now stick to macro level models. In this case the researchers assume that external factors also affect the decisions on the financial market and therefore could be used for explaining the term structure of interest rates. Those are usually multivariate and sometimes quite complicated models that could be used for predicting different macroeconomic variables. Also some researchers put the term structure into a macroeconomic model in order to look into the effect of macro factors in this term structure. Below both impact of macroeconomic disturbances and predictive ability of term structure arc discussed.

Turnovsky [1989] puts the term structure into stochastic macro economic model in order to find out the impact of fiscal and monetary policies on it The results are straightforward and easy understandable. The influence of macroeconomic disturbances is proved to exist and the directions of this impact are predictable. Unanticipated monetary expansions shocks, both permanent and temporary, lower short term and long term nominal and real interest rates. At the same time anticipated monetary changes do not affect interest rates or operate in the same direction as unanticipated ones. Unanticipated fiscal disturbances are shown to push interest rates in the direction opposite to monetary ones. Moreover, there not only value impacts but also structural implications in interest rates due to macroeconomic fluctuations.

As macroeconomic changes affect investors' preferences and decisions, the idea to reveal expectations using the behavior of agents on financial markets and consequently the term structure of interest rates is widely popular. Clinton [1994] stressed that the term structure appeared to be one of the best predictors of economic activity in the number of countries. His research for Canada also showed good predicting power of the term structure for GDP changes. However, it is not that good in predicting of GDP components, as Estrella and Hardouvelis [1991] also noticed for the United States. At the same time predictions of inflation changes based on the Expectations Theory are claimed not to work well.

Clouse [2004] however agues that Expectations Hypothesis fits for predicting main macroeconomic variables like unemployment rate, inflation and GDP growth using the term structure. His empirical model although deviating from the formal theoretical expectations structure proved to be well for the predicting task, which is important for policy implications. The most attractive feature of the model is the number of variables predicted: unemployment rate, inflation and GDP growth.

As it has been shown the number of theoretical and empirical studies on the term structure of interest rates is large. This topic relates to many economic issues both on micro and macro levels. The work in this direction is very important both for policy makers to see the future range of events and investors' in everyday task of bond prices.

Among the theories stated my research lies at micro level. Although information outside the financial market and macroeconomic factors are important for the term structure of interest rates, the availability of the data and quite short time horizon make the research at the macro level rather difficult. Therefore, assuming that the inside market information reflects more or less the outside factors, this work focuses on testing Pure Expectations and Liquidity Premium Theories for the government bond market of the Republic of Belarus. The result of my research might be used for further investigation of the term structure of interest rates and applying more complicated models to it.

Chapter 3

METHODOLOGY

1. Pure Expectations Theory. In this case the model of Kryukovskaya [2003] is used. She uses expectations hypothesis in two interpretations.

The first form is used for an investor buying a short security with expectation of return the same as for a long term bond sold after short period (1 or 3 months). This is explained by the following formula:

$$(1+Y_t(1)) = (1+Y_t(N))^N \cdot E_t\left\{(1+Y_{t+1}(N-1))^{-(N-1)}\right\} \quad (*)$$

where Y_t (1) – monthly return bond with 1 month to maturity at time t; Y_t (N) – monthly return bond with N month to maturity at time t and Y_{t+1} (N-1) – monthly return bond with N-1 month to maturity at time t+1

Second form is related to investor buying a long-term security and expecting the same return as of the series of buying short-term bonds and reinvesting income again:

$$(1+Y_t(N))^N = E_t\left\{(1+Y_t(1))\cdot(1+Y_{t+1}(1))\ldots(1+Y_{t+N-1}(1))\right\} = (1+Y_t(1))\cdot E_t\left\{(1+Y_{t+1}(N-1))^{N-1}\right\}$$

In general, these two interpretations are not equivalent. Due to the fact that this work is investigating bonds with maturity no more than one year the second type of the hypothesis cannot be applied. Even a bond maturing in one year cannot be considered in general as a long-term one. Moreover the number of deals with bonds of maturities 11 and 12 months is too small to apply econometric analysis of the second type of the hypothesis properly.

The main parameter to estimate is the yield spread:

$$S_t(N) = Y_t(N) - Y_t(1)$$

which is transformed using logarithmic form:

$$y_t(N) = \ln[1 + Y_t(N)]$$

$$s_t(N) = y_t(N) - y_t(1)$$

We can change the hypothesis statements to get the form for estimation:

Expressing expectation from (*) and taking logarithms, we get modified first type of hypothesis:

$$\ln[1 + Y_t(1)] = N \cdot \ln[1 + Y_t(N)] + E_t\left\{-(N-1) \cdot \ln[1 + Y_{t+1}(N-1)]\right\}$$
$$y_t(1) = (N-1) \cdot y_t(N) + y_t(N) - E_t\left\{(N-1) y_{t+1}(N-1)\right\}$$

as at time t $E_t(y_t) = y_t$:

$$E_t\left\{(N-1) \cdot y_{t+1}(N-1) - (N-1) \cdot y_t(N)\right\} = y_t(N) - y_t(1)$$
$$E_t\left\{y_{t+1}(N-1) - y_t(N)\right\} = \frac{y_t(N) - y_t(1)}{N-1}$$

$$E_t\{y_{t+1}(N-1)-y_t(N)\}=s_t(N)/(N-1)$$

For Pure Expectations Hypothesis to hold we need the following to be true:

$$y_{t+1}(N-1)-y_t(N)=E_t\{y_{t+1}(N-1)-y_t(N)\}$$

Therefore, if the following equation is estimated, the Pure Expectation Hypothesis is valid if α(N) is insignificantly different from 0 and β(N) is significantly close to 1.

$$y_{t+1}(N-1)-y_t(N)=\alpha(N)+\beta(N)\cdot(s_t(N)/(N-1))+\varepsilon(N)_t$$

The equation stated can be estimated by OLS using the Newey-West estimator of covariance matrix consistent with heteroskedasticity and autocorrelation.

2. Liquidity Premium Theory. For testing this hypothesis the work of Gordon [2003] is used. The term premium is defined from decomposition of forward interest rate:

$f_{t,j}=E_t[r_{t+j}]+\alpha_{t,j}$, where $f_{t,j}$ – forward interest rate, $E_t[r_{t+j}]$ – expectation j-period ahead spot rate, $\alpha_{t,j}$ – term premium

This decomposition is valid on effective markets, that is $E_t[r_{t+j}]= r_{t+j}$ and in equation $f_{t,j}=\alpha_j+\lambda_j r_{t+j}+e_{t+j}$ λj is significantly close to 1. In this case r_{t+j} can be subtracted from forward rate and term premium is left on the right hand side of the decomposition:

$$(f-r)_{t,j}=\alpha_{t,j}+u_{t+j}$$

To test the theory I assume such rational expectations of agents on the market. Gordon [2003] tests three specifications of the term premium, which are also to be used in this work:

- constant: $\alpha_{t,j} = \overline{\alpha}_j$
- "non-stationary" (random walk): $\alpha_{t,j} = \alpha_{t-1,j} + \nu_t$
- "mean-reverting": $\alpha_{t,j} = c + \phi\alpha_{t-1,j} + \nu_t$

First two specifications are just the restricted versions of the third one. In the first case φ=0, in the second c=0 and ψ=1. Therefore the only equation to test in this work is the third one.

However thiespecification is to be changed due to different periodicity of the data: weekly instead of monthly. Therefore instead of one lag I will include from three to five of them to capture the same time horizon and avoid serial correlation. For detecting the serial correlation alternative Durbin-Watson test will be used. The equation estimated looks as follows:

$$\alpha_{t,j} = c + \phi_1 \cdot \alpha_{t-1,j} + \phi_2 \cdot \alpha_{t-2,j} + \ldots + \phi_k \cdot \alpha_{t-k,j} + \varepsilon_t$$

where k takes on values from 3 to 5 in order to capture one month lag horizon and avoid serial correlation.

As autoregressive conditional heteroskedastisity (ARCH) effects are expected, the estimation will be corrected on them. According to Drobyshevsky [1999] the model to apply is the ARCH-M (ARCH in mean) model. In this case mean of the premium depends on conditional variance. Conditional variance we denote as:

$$h_t = \gamma_0 + \gamma_1 \cdot \varepsilon_{t-1}^2 + \gamma_2 \cdot \varepsilon_{t-2}^2$$

And therefore $\varepsilon_t = v_t \sqrt{h_t}$

The software used for the analysis is Stata 8.2 by StataCorp LP.

Having discussed the methodological issues, I will proceed describing empirical part of my research.

Chapter 4

DATA DESCRIPTION

The data on the research is taken from everyday deals with government securities on Belarusian Currency and Stock Exchange (BCSE) in the period from 1998 to 2003. Variables used are yields to maturity constructed on the basis of bond prices:

$P = \frac{FV}{(1+Y)^n}$, where FV-face value of a bond, P – price of a bond, n – term to maturity in months and Y – yield to maturity

Yield spreads are counted from those yields. As terms to maturity are expressed in days the aggregation of bonds by maturity is necessary. Operating with daily expressed terms to maturity is too complicated and has little sense. To solve this problem standardizing of maturity periods is done. According to Drobyshevsky [1999] the distribution of bonds by maturity periods is taken as shown in Table 1.

Table 1: List of maturity correspondence used by aggregation

Aggregated period to maturity	Actual period to maturity, days
1 week	Up to 7
2 weeks	8-15
1 month	16-35
2 months	36-63
3 months	64-91
4 months	92-126
5 months	127-154
6 months	155-182

7 months	183-217
8 months	218-245
9 months	246-280
10 months	281-307
11 months	308-336
12 months	337-364

Although the number of observations for each maturity after aggregation by maturity periods is quite large (see Table 2), the range of bonds with different maturities traded in one day is usually small. Therefore, proper constructing of yield curves for any single day is impossible. To solve this problem the data was aggregated by weeks instead of days. Average prices of bonds with corresponding periods to maturity for each week were calculated using deals volumes as weights. This helped to get wider range periods to maturity for one deal period (week) and decrease statistical noise from daily variations of prices. Small trade volumes were also used. The descriptive statistics on trade volumes (in number of bonds) is represented in Table2.

Table 2: Descriptive statistics on trade volumes

Period to maturity	Number of observations	Mean value (Std. Dev)	Maximum value	Minimum value
1 week	560	168794 (722195.9)	7351120	1
2 weeks	857	152760.8 (749050.1)	11857286	1
1 month	2051	117187.2 (559094.1)	7500000	1
2 months	2708	116474.5 (631658.8)	18858490	1
3 months	2630	96031.1 (574950.9)	14461414	1
4 months	2855	123923.8 (734206.6)	16340017	1

5 months	1883	180425.8 (956249.4)	18035600	1
6 months	1697	200276.4 (1079445)	29838198	1
7 months	1677	115224.4 (779853.5)	17890990	1
8 months	1222	117830.3 (1085023)	25773282	1
9 months	1228	162604.8 (1110248)	19349491	1
10 months	639	22127.98 (100040.4)	1451570	1
11 months	632	68911.96 (344833)	4480096	1
12 months	303	19603.65 (150338.1)	2088000	1

Even after aggregation still exist weeks without any observations for some periods to maturity. In order to interpolate data, approximation by simple linear interpolation is used (only for gaps no more than one period):

$$Y_t(N) = \frac{Y_{t+1}(N) + Y_{t-1}(N)}{2}, \text{ if no observation for week t}$$

$$Y_t(N) = \frac{Y_t(N-1) + Y_t(N+1)}{2}, \text{ if no observation for maturity N}$$

Although the most popular method of interpolation in this case is using the approximation through splines, it is not used here. First, while using splines there is always the task of choice between smoothness of the approximating curve and the goodness of fit. Second, splines cannot be used for investigating of effects on the border of the yield curve, which is important for this work. Due to the aggregation the yield curves are quite smooth. However, linear interpolation allowed for further smoothing of yield curves and for obtaining some of the missing observations. Descriptive statistics of the data on interest rates received is expressed in Table 3.

Table 3: Descriptive statistics of the data on interest rates

Period to maturity	Variable name	Number of observations	Mean value (Std. Dev)
1 week	W1	195	0.8192 (2.2997)
2 weeks	W2	235	1.2998 (7.6537)
1 month	M1	266	0.7272 (0.4086)
2 months	M2	274	0.6629 (0.3894)
3 months	M3	278	0.6724 (0.3406)
4 months	M4	278	0.6606 (0.3217)
5 months	M5	253	0.6243 (0.3159)
6 months	M6	232	0.6289 (0.3261)
7 months	M7	223	0.6137 (0.3668)
8 months	M8	201	0.6041 (0.3866)
9 months	M9	193	0.5643 (0.3613)
10 months	M10	144	0.4697 (0.2507)
11 months	M11	114	0.3968 (0.1578)
12 months	M12	55	0.4215 (0.2116)

As it can be seen from the Table 3, the mean values of interest rates decrease with increasing of period to maturity. This means that a yield curve constructed using these values is downward sloping. However, yield curves for separate weeks have different shapes as shown in Figures 1,2.

Figure 1

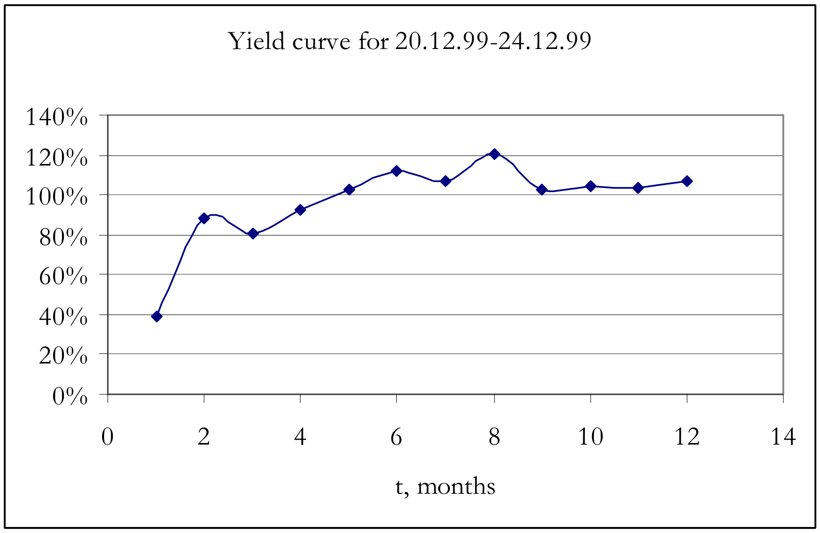
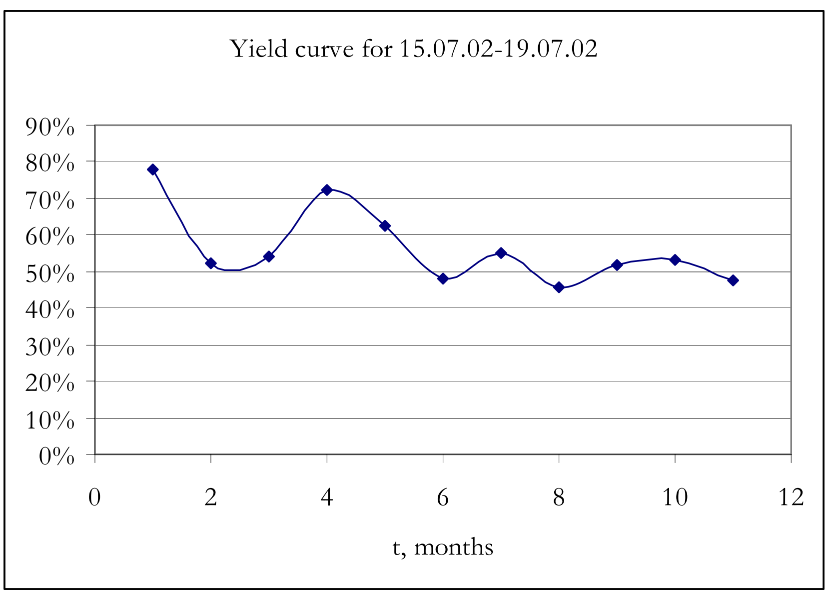


Figure 2



Logarithmic transformation of yields and yield spreads is done according to the methodology described in Chapter 3. The descriptive statistics of variables obtained is shown in Appendix 1, Tables 1,2

Forward rates: $f_{1,N} = \frac{Y_t(N) - Y_t(1)}{1 + Y_t(1)}$, where $f_{1,N}$ – forward interest rate, $Y_t(N)$, $Y_t(1)$ – rates in period 1 and N

Term premia for every period are calculated as $\alpha_N = f_{1,N} - Y_t(N)$. Descriptive statistics of term premia is shown in Table 3. Statistics for forward interest rates is given in Appendix 1, Table 3.

Table 4: Descriptive statistics for calculated term premia

Period ahead	Variable name	Number of observations	Mean value (Std. Dev)
2 months	a2	265	-0.6810 (0.3425)
3 months	a3	265	-0.6851 (0.3248)
4 months	a4	260	-0.6786 (0.3188)
5 months	a5	231	-0.6574 (0.3273)
6 months	a6	209	-0.6593 (0.3374)
7 months	a7	196	-0.6538 (0.356)
8 months	a8	170	-0.6419 (0.3764)
9 months	a9	158	-0.5975 (0.3485)
10 months	a10	106	-0.5142 (0.2448)
11 months	a11	82	-0.4542 (0.2013)

Negative values of the term premia are related to the negative liquidity premia that are some times exist on emerging markets determining the downward sloped yield curves. Although rarely investigated negative liquidity premia are possible (Fernandez, 2002) and mean that investors are ready to buy and hold longer term bonds despite the lower return on them.

As deals with bonds maturing in one year and longer are rare and the correspondence of maturity periods is essential, for the analysis the most actively traded part of bonds with maturities from 1 to 11 months is chosen.

Chapter 5

EMPIRICAL ANALYSIS AND RESULTS

Pure Expectations Hypothesis. The hypothesis is tested using the OLS method with Newey-West estimator of covariance matrix consistent in the presence of heteroskedasticity and autocorrelation in accordance with Kryukovskaya [2003]. The equation to estimate is as following:

$$y_{t+1}(N-1) - y_t(N) = \alpha(N) + \beta(N) \cdot (s_t(N)/(N-1)) + \varepsilon(N)_t$$

The hypothesis will be valid if β(N) is equal to 1 and α(N) is equal to 0. The estimated values of coefficients are presented in Table 5. The complete results of regression are shown in Appendix 2.

Table 5: Empirical results of Pure Expectations Theory testing

	α(N)	F-stat for H_0: **α(N)=0** (Prob>F)	β(N)	F-stat for H_0: **β(N)=0** (Prob>F)	F-stat for H_0: **β(N)=1** (Prob>F)
N=2	-0.0086 *(0.0159)*	0.29 (0.5905)	-0.7902 (0.1247)	40.14 (0.0000)	205.99 (0.0000)
N=3	-0.0071 *(0.0155)*	0.21 (0.6479)	-0.4849 (0.1955)	6.16 (0.0137)	57.71 (0.0000)
N=4	-0.0009 *(0.0127)*	0.00 (0.944)	-1.1455 (0.2971)	14.86 (0.0001)	52.14 (0.0000)
N=5	0.0187 *(0.0142)*	1.73 (0.1894)	-0.2707 (0.3361)	0.65 (0.4214)	14.30 (0.0002)
N=6	-0.0361 *(0.0158)*	5.24 (0.0231)	-1.4971 (0.5322)	7.91 (0.0054)	22.02 (0.0000)

N=7	-0.0612 (0.0191)	10.32 (0.0015)	-3.2021 (0.7166)	19.97 (0.0000)	34.39 (0.0000)
N=8	-0.0212 (0.0204)	1.07 (0.3014)	-2.2125 (0.8898)	6.18 (0.0139)	13.03 (0.0004)
N=9	-0.0429 (0.0232)	3.41 (0.0667)	-0.1255 (1.2601)	0.01 (0.9208)	0.80 (0.3732)
N=10	-0.0064 (0.0322)	0.04 (0.8422)	0.2292 (1.8826)	0.01 (0.9034)	0.17 (0.6831)
N=11	-0.0617 (0.0239)	6.70 (0.0115)	-1.9035 (1.4508)	1.72 (0.1934)	4.00 (0.0488)

Although $\alpha(N)$ in most of the cases is not significantly different from zero, $\beta(N)$ shows the estimates far from unity. The hypothesis $\beta(N)=1$ cannot be rejected only for longer maturities 9-11 months. However, tests show that also a hypothesis of $\beta(N)=0$ cannot be rejected in these cases with even higher probability. This may be explained by the higher volatility (standard deviations of $\beta(N)$ are much higher for maturities 9-11 months). Moreover, for N=9 and N=11 we can reject the hypothesis $\alpha(N)=0$. Therefore, the Pure Expectations Theory cannot be accepted for the tested data.

Practically all values of $\beta(N)$ estimators are negative. This is not the rare case when testing the Pure Expectations Hypothesis in different countries. Kryukovskaya [2003] found similar results for Russian market. Her possible explanation of this fact is that high volatility of short term rates could lower the rates for longer period bonds. However, in my case volatility of rates although decreasing with longer period to maturity is not very high even for short periods (see Table 3) Therefore, possible explanations of decreasing rates may be preferences of investors or continuous mispricing of longer term bonds due to small number of their issues.

Liquidity Premium Theory. The theory is tested using ARCH-M method according to Gordon [2003] and Drobyshevsky [1999]. The equation to estimate is:

$$\alpha_{t,j} = c + \phi_1 \cdot \alpha_{t-1,j} + \phi_2 \cdot \alpha_{t-2,j} + ... + \phi_k \cdot \alpha_{t-k,j} + v_t$$

with k equal from 3 to 5 in order to capture one month lag horizon and avoid serial correlation.

The results of alternative Durbin-Watson tests for serial correlation are shown in Table 6. The results of testing are presented in Table 7. The coefficients shown are significant at 1% significance level in most of the cases. Complete regressions are represented in Appendix 3.

Table 6: The results of alternative Durbin-Watson tests for serial correlation

	Number of lags	χ^2	Prob> χ^2
j=2	5	0.395	0.5295
j=3	3	0.564	0.4525
j=4	4	1.152	0.2832
j=5	3	1.974	0.1600
j=6	5	0.867	0.3519
j=7	5	3.229	0.0723
j=8	3	0.542	0.4615
j=9	3	0.812	0.3674
j=10	3	0.186	0.6660
j=11	3	0.127	0.7221

Three lags are found to be the most frequent specification. This means that investors take into account the most recent time period in their decisions. All the lagged values have positive impact on current term premium, which was expected.

However, in most cases the impact of lagged premia is more or less equal for all lags. This is rather surprising because intuitively investors should put more weight on the latest (lag one) value of a term premium. A possible explanation to this is that investors change their decisions less frequently compared to the frequency of observations used. In this case all information inside of this time interval should influent investors' decision equivalently.

Table 7: Results of testing the Liquidity Premium Theory.

	c	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5
j=2	-0.6832 (0.0684)	0.2757 (0.0621)	0.1582 (0.0609)	0.2205 (0.0584)	0.1837 (0.0572)	0.1219 (0.0425)
j=3	-0.5230 (0.0737)	0.2955 (0.0776)	0.3221 (0.0489)	0.2516 (0.0437)		
j=4	-0.8019 (0.0592)	0.4023 (0.0654)	0.0854* (0.0687)	0.3001 (0.0501)	0.1660 (0.0602)	
j=5	-0.7467 (0.0465)	0.3414 (0.0735)	0.2490 (0.0719)	0.3539 (0.0532)		
j=6	-0.6844 (0.0474)	0.1617 (0.0606)	0.0409* (0.0627)	0.2855 (0.0509)	0.2031 (0.0565)	0.2816 (0.0526)
j=7	-0.6494 (0.0424)	0.2669 (0.0542)	0.0522* (0.0543)	0.2766 (0.0458)	0.1386 (0.0548)	0.2419 (0.0569)
j=8	-0.7499 (0.0412)	0.3174 (0.0722)	0.2937 (0.0950)	0.3311 (0.0632)		
j=9	-0.7204 (0.0474)	0.3882 (0.0792)	0.3324 (0.0809)	0.2459 (0.0667)		
j=10	-0.6169 (0.0391)	0.3254 (0.1051)	0.3232 (0.1085)	0.2726 (0.0898)		
j=11	-0.5057 (0.0480)	0.1834* (0.1348)	0.3642** (0.1478)	0.2297** (0.1091)		

* – coefficient is insignificant
** – coefficient is significant at 5% significance level

As almost all coefficients are significant, the hypothesis of a time varying term premium is supported. The premium itself is however negative. This coinsides with downward sloping yield curves that are rarely observed on developed markets but sometimes appear on emerging ones. It may mean that investors would like to hold longer term papers despite their lower pay-off. Consequently we can accept the Liquidity Premium Theory despite its unusual form in this particular case.

Chapter 6

CONCLUSIONS AND IMPLICATIONS

In this work I tested the Pure Expectations Hypothesis and Liquidity Premium Theory for the term structure of interest rates on government bonds of the Republic of Belarus. For this purpose the data on deals with bonds of maturity up to one year during period 1998-2003 was used. Using this data I calculated yields to maturity for different time periods, forward interest rates, term premia and constructed yield curves. The methods of econometric analysis used are OLS corrected for heteroskedasticity and autocorrelation and ARCH-M regression

Constructing of yield curves and testing of two hypotheses of term structure of interest rates for the government bond market of the Republic of Belarus has shown some interesting results.

First of all, a yield curve on this market is appeared to be on average downward sloping. This phenomenon although being rare for developed countries has been observed on some emerging financial markets like Chile (Fernandez, 2002). The reasons for such yield curve shape have not been investigated much and therefore no widely accepted explanation is developed. One possible reason could be the existence of negative liquidity premium.

Testing of the Pure Expectations Hypothesis has shown that it cannot be rejected only for time horizons 9-11 months. However even for these periods the results are quite controversial and do not prove the validity of the hypothesis. Expectations of the investors on the market mostly do not represent rationality needed to fit the Expectations Hypothesis. This also supported by the fact of downward sloping yield curve.

The term pemia calculated for the market of the Republic of Belarus are negative. This represents the willingness of investors to hold longer term bonds even despite lower return. This may be explained by low number of issues of long term bonds. As a result the volumes of long term papers traded do not serve the needs of the financial market. At the same time the financial market of Belarus has a short history and still undeveloped. Therefore another explanation for the negative premia is the mispricing of long term bonds due to low qualification and experience of agents and small volumes issued and traded.

Testing of the Liquidity Premium Theory showed that the premia exist and being negative vary over time. In making decisions about term premium investors take into account the information about premia over last month (3-5 weeks). However, premia of every week of this month appeared to have more or less equal impact on the decision of investors.

Issuing bonds a government increases its debt. From the point of view of the government it is always preferable to borrow at lower rate and on longer period. However, the Belarusian government prefers to borrow on short periods: from 470 government bond issues over the testing period only 91 have maturity 1 year and more. This causes inconveniences for the government because in case of problems with repayment it has to roll over the debt issuing new short term securities. In the end this could lead to a default like in Russia 1998 if the policy makers are not wise enough and have big appetites.

At the same time on the financial market of the Republic of Belarus there is the situation when bonds with longer periods to maturity pay lower interest than with short periods. This means that when issuing long term securities the government fulfils the task of borrowing cheaper on longer time period. No doubt that if government will start to issue long term bonds actively, the market will sooner or later come to "normal" condition with upward sloping yield curve. However the government should take the advantage during this period to finance necessary projects by cheap borrowing.

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Appendix 1: Descriptive statistics of constructed variables

Table 1. Descriptive statistics of the yield spread in logarithms

Variable	Variable name	Number of observations	Mean value (Std. Dev)
$y_{t+1}(1)-y_t(2)$	y12	273	0.0084 (0.3178)
$y_{t+1}(2)-y_t(3)$	y23	277	-0.0025 (0.2674)
$y_{t+1}(3)-y_t(4)$	y34	282	0.0097 (0.2243)
$y_{t+1}(4)-y_t(5)$	y45	281	0.0707 (0.2598)
$y_{t+1}(5)-y_t(6)$	y56	256	0.0307 (0.2810)
$y_{t+1}(6)-y_t(7)$	y67	249	0.0325 (0.3320)
$y_{t+1}(7)-y_t(8)$	y78	231	0.0668 (0.3233)
$y_{t+1}(8)-y_t(9)$	y89	215	0.0181 (0.3281)
$y_{t+1}(9)-y_t(10)$	y910	199	0.1445 (0.3425)
$y_{t+1}(10)-y_t(11)$	y1011	146	0.0737 (0.2855)

Table 2. Descriptive statistics of the yield spread expectations

Variable	Variable name	Number of observations	Mean value (Std. Dev)
$s_t(2)/1$	s12p	266	-0.0235 (0.2589)
$s_t(3)/2$	s13p	265	-0.0097 (0.0871)
$s_t(4)/3$	s14p	260	-0.0076 (0.0566)
$s_t(5)/4$	s15p	231	-0.0089 (0.0420)
$s_t(6)/5$	s16p	209	-0.0054 (0.0318)
$s_t(7)/6$	s17p	196	-0.0059 (0.0286)

$s_t(8)/7$	s18p	170	-0.0041 (0.0259)
$s_t(9)/8$	s19p	158	-0.0036 (0.0219)
$s_t(10)/9$	s110p	106	-0.0052 (0.0185)
$s_t(11)/10$	s111p	82	-0.0067 (0.0153)

Appendix 2: Tests for heterskedasticity, autocorrelation and complete results of regressions testing Pure Expectation Hypothesis.

Table 1: Breusch-Pagan / Cook-Weisberg test for heteroskedasticity by running simple OLS

	χ^2	Prob> χ^2
N=2	23.91	0.0000
N=3	28.27	0.0000
N=4	5.40	0.0201
N=5	0.79	0.3755
N=6	2.44	0.1183
N=7	2.19	0.1392
N=8	1.50	0.2213
N=9	0.57	0.4505
N=10	16.39	0.0001
N=11	0.41	0.5224

Table 2: Durbin-Watson d-statistics by running simple OLS

	d-stat
N=2	1.8959
N=3	1.6700
N=4	0.9990
N=5	1.3003
N=6	1.3726
N=7	1.0442
N=8	0.7237
N=9	0.6851
N=10	0.4223
N=11	0.8890

Complete regressions results:

N=2

```
Regression with Newey-West standard errors          Number of obs  =       264
maximum lag: 0                                      F(  1,   262)  =     40.14
                                                    Prob > F       =    0.0000

------------------------------------------------------------------------------
             |             Newey-West
         y12 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        s12p |  -.7902398   .1247356    -6.34   0.000    -1.035852   -.5446279
       _cons |  -.0085983   .0159605    -0.54   0.591    -.0400255    .0228288
```

N=3

```
Regression with Newey-West standard errors          Number of obs  =       263
maximum lag: 0                                      F(  1,   261)  =      6.16
                                                    Prob > F       =    0.0137

------------------------------------------------------------------------------
             |             Newey-West
         y23 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        s13p |  -.4849621   .1954663    -2.48   0.014    -.8698538   -.1000703
       _cons |  -.0070807   .0154882    -0.46   0.648    -.0375784     .023417
```

N=4

```
Regression with Newey-West standard errors          Number of obs  =       259
maximum lag: 0                                      F(  1,   257)  =     14.86
                                                    Prob > F       =    0.0001
------------------------------------------------------------------------------
             |             Newey-West
         y34 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        s14p |   -1.14547   .2971327    -3.86   0.000    -1.730595   -.5603451
       _cons |  -.0008947   .0127328    -0.07   0.944    -.0259686    .0241791
```

N=5

```
Regression with Newey-West standard errors          Number of obs  =       230
maximum lag: 0                                      F(  1,   228)  =      0.65
                                                    Prob > F       =    0.4214

------------------------------------------------------------------------------
             |             Newey-West
         y45 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        s15p |  -.2707065   .3360653    -0.81   0.421    -.9328974    .3914844
       _cons |    .018729   .0142285     1.32   0.189    -.0093071    .0467652
```

N=6

```
Regression with Newey-West standard errors          Number of obs  =       207
maximum lag: 0                                      F(  1,   205)  =      7.91
                                                    Prob > F       =    0.0054

------------------------------------------------------------------------------
             |             Newey-West
         y56 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        s16p |  -1.497123   .5321704    -2.81   0.005    -2.546352   -.4478937
       _cons |   -.036142   .0157912    -2.29   0.023     -.067276    -.005008
```

N=7

```
Regression with Newey-West standard errors          Number of obs  =       193
maximum lag: 0                                      F(  1,   191)  =     19.97
                                                    Prob > F       =    0.0000

------------------------------------------------------------------------------
             |             Newey-West
         y67 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        s17p |  -3.202099   .7165709    -4.47   0.000    -4.615508   -1.788691
       _cons |  -.0612478   .0190613    -3.21   0.002    -.0988454   -.0236502
------------------------------------------------------------------------------
```

N=8

```
Regression with Newey-West standard errors          Number of obs  =       166
maximum lag: 0                                      F(  1,   164)  =      6.18
                                                    Prob > F       =    0.0139

------------------------------------------------------------------------------
             |             Newey-West
         y78 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        s18p |  -2.212523   .8898263    -2.49   0.014    -3.969516   -.4555299
       _cons |  -.0211617   .0204126    -1.04   0.301    -.0614671    .0191437
------------------------------------------------------------------------------
```

N=9

```
Regression with Newey-West standard errors          Number of obs  =       155
maximum lag: 0                                      F(  1,   153)  =      0.01
                                                    Prob > F       =    0.9208

------------------------------------------------------------------------------
             |             Newey-West
         y89 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        s19p |  -.1255401   1.260119    -0.10   0.921    -2.615019    2.363939
       _cons |  -.0429007   .0232273    -1.85   0.067    -.0887883    .0029868
```

N=10

```
Regression with Newey-West standard errors          Number of obs  =       103
maximum lag: 0                                      F(  1,   101)  =      0.01
                                                    Prob > F       =    0.9034

------------------------------------------------------------------------------
             |             Newey-West
        y910 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       s110p |   .2291609   1.882591     0.12   0.903    -3.505394    3.963716
       _cons |  -.0064186   .0321666    -0.20   0.842    -.0702285    .0573913
```

N=11

```
Regression with Newey-West standard errors          Number of obs  =        80
maximum lag: 0                                      F(  1,    78)  =      1.72
                                                    Prob > F       =    0.1934

------------------------------------------------------------------------------
             |             Newey-West
       y1011 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       s111p |  -1.903457   1.450832    -1.31   0.193    -4.791843    .9849283
       _cons |  -.0617708   .0238721    -2.59   0.012    -.1092966    -.014245
------------------------------------------------------------------------------
```

Appendix 3: Complete results of regressions testing Liquidity Premium Theory.

j=2

```
ARCH family regression -- AR disturbances

Sample:  41 to 311, but with gaps               Number of obs      =       265
                                                Wald chi2(6)       =    996.99
Log likelihood =  97.35512                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
a2           |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
a2           |
_cons        |  -.6832486   .0684242    -9.99   0.000    -.8173574   -.5491397
-------------+----------------------------------------------------------------
ARCHM        |
sigma2       |  -.0079904   .2338616    -0.03   0.973    -.4663507    .4503699
-------------+----------------------------------------------------------------
ARMA         |
ar           |
          L1 |   .2756918   .0620859     4.44   0.000     .1540057     .397378
          L2 |   .1582162   .0609171     2.60   0.009      .038821    .2776115
          L3 |   .2205345   .0583721     3.78   0.000     .1061273    .3349417
          L4 |   .1837265   .0572371     3.21   0.001     .0715439    .2959091
          L5 |   .1219253   .0424806     2.87   0.004     .0386648    .2051858
-------------+----------------------------------------------------------------
ARCH         |
arch         |
          L1 |   .5641483   .1284086     4.39   0.000     .3124721    .8158245
          L2 |   .2798062   .0935467     2.99   0.003      .096458    .4631544
_cons        |   .0100233   .0020586     4.87   0.000     .0059886    .0140581
------------------------------------------------------------------------------
```

j=3

```
ARCH family regression -- AR disturbances

Sample:  41 to 311, but with gaps              Number of obs      =       265
                                               Wald chi2(4)       =    678.33
Log likelihood =  51.56606                     Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
a3           |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
a3           |
_cons        |  -.5230194   .0737142    -7.10   0.000    -.6674966   -.3785422
-------------+----------------------------------------------------------------
ARCHM        |
sigma2       |  -.1041337   .1461129    -0.71   0.476    -.3905097    .1822423
-------------+----------------------------------------------------------------
ARMA         |
ar           |
          L1 |   .2954616   .0776026     3.81   0.000     .1433634    .4475598
          L2 |    .322128   .0489328     6.58   0.000     .2262215    .4180345
          L3 |   .2515519   .0437258     5.75   0.000     .1658508     .337253
-------------+----------------------------------------------------------------
ARCH         |
arch         |
          L1 |   .6227645   .1276619     4.88   0.000     .3725517    .8729773
_cons        |   .0242577   .0023323    10.40   0.000     .0196865    .0288288
```

j=4

```
ARCH family regression -- AR disturbances

Sample:  41 to 311, but with gaps               Number of obs      =       260
                                                Wald chi2(5)       =    642.74
Log likelihood =  72.45823                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
a4           |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
a4           |
_cons        |  -.8019513   .0591931   -13.55   0.000    -.9179678   -.6859349
-------------+----------------------------------------------------------------
ARCHM        |
sigma2       |   -1.00271   .7893663    -1.27   0.204     -2.54984    .5444192
-------------+----------------------------------------------------------------
ARMA         |
ar           |
          L1 |   .4022808   .0654278     6.15   0.000     .2740447    .5305169
          L2 |    .085439   .0687066     1.24   0.214    -.0492235    .2201014
          L3 |   .3001816   .0501479     5.99   0.000     .2018935    .3984697
          L4 |   .1660221   .0602014     2.76   0.006     .0480296    .2840147
-------------+----------------------------------------------------------------
ARCH         |
arch         |
          L1 |   .1567557   .1173464     1.34   0.182     -.073239    .3867505
          L2 |   .2663545    .096986     2.75   0.006     .0762653    .4564436
_cons        |   .0216568   .0038192     5.67   0.000     .0141712    .0291423
```

j=5

```
ARCH family regression -- AR disturbances

Sample:  41 to 311, but with gaps              Number of obs      =       231
                                               Wald chi2(4)       =    814.46
Log likelihood =  54.31129                     Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
a5           |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
a5           |
_cons        |  -.7467051   .0464922   -16.06   0.000    -.8378281   -.6555822
-------------+----------------------------------------------------------------
ARCHM        |
sigma2       |  -.5557358   .2488449    -2.23   0.026    -1.043463   -.0680087
-------------+----------------------------------------------------------------
ARMA         |
ar           |
          L1 |   .3414451   .0735271     4.64   0.000     .1973346    .4855557
          L2 |   .2490426   .0719697     3.46   0.001     .1079847    .3901006
          L3 |   .3538681   .0532265     6.65   0.000      .249546    .4581902
-------------+----------------------------------------------------------------
ARCH         |
arch         |
          L1 |    .495261   .1268177     3.91   0.000     .2467029    .7438191
          L2 |   .4654968   .1192267     3.90   0.000     .2318167    .6991768
_cons        |   .0114412    .003419     3.35   0.001     .0047401    .0181423
------------------------------------------------------------------------------
```

j=6

```
ARCH family regression -- AR disturbances

Sample:  41 to 311, but with gaps               Number of obs      =       209
                                                Wald chi2(6)       =    655.40
Log likelihood =  59.02947                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
a6           |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
a6           |
_cons        |  -.6843759   .0473889   -14.44   0.000    -.7772565   -.5914953
-------------+----------------------------------------------------------------
ARCHM        |
sigma2       |  -.4122745   .2250804    -1.83   0.067     -.853424     .028875
-------------+----------------------------------------------------------------
ARMA         |
ar           |
          L1 |   .1616733    .060632     2.67   0.008     .0428367    .2805099
          L2 |   .0408905   .0626888     0.65   0.514    -.0819773    .1637583
          L3 |   .2855329   .0509437     5.60   0.000      .185685    .3853808
          L4 |   .2031825   .0564878     3.60   0.000     .0924684    .3138965
          L5 |    .281647   .0526388     5.35   0.000     .1784768    .3848173
-------------+----------------------------------------------------------------
ARCH         |
arch         |
          L1 |    .563576   .1438648     3.92   0.000     .2816062    .8455458
          L2 |   .2795271   .1214661     2.30   0.021     .0414578    .5175963
_cons        |   .0124098   .0031824     3.90   0.000     .0061725    .0186472
```

j=7

```
ARCH family regression -- AR disturbances

Sample:  41 to 311, but with gaps               Number of obs      =       196
                                                Wald chi2(6)       =    962.92
Log likelihood =  76.76474                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
a7           |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
a7           |
_cons        |  -.6494237   .0424192   -15.31   0.000    -.7325637   -.5662836
-------------+----------------------------------------------------------------
ARCHM        |
sigma2       |  -1.206572   .4842066    -2.49   0.013      -2.1556   -.2575446
-------------+----------------------------------------------------------------
ARMA         |
ar           |
          L1 |   .2668626   .0541707     4.93   0.000       .16069    .3730352
          L2 |   .0521654   .0542987     0.96   0.337    -.0542581     .158589
          L3 |   .2765638    .045761     6.04   0.000     .1868739    .3662536
          L4 |    .138559    .054807     2.53   0.011     .0311393    .2459788
          L5 |    .241951    .056991     4.25   0.000     .1302507    .3536513
-------------+----------------------------------------------------------------
ARCH         |
arch         |
          L1 |   .2430566   .1200601     2.02   0.043     .0077431    .4783702
          L2 |   .7210687   .1702349     4.24   0.000     .3874144    1.054723
_cons        |   .0077329   .0025655     3.01   0.003     .0027046    .0127612
------------------------------------------------------------------------------
```

j=8

```
ARCH family regression -- AR disturbances

Sample:  41 to 311, but with gaps                 Number of obs      =       170
                                                  Wald chi2(4)       =   1099.60
Log likelihood =  54.68038                        Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
a8           |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
a8           |
_cons        |  -.7499052   .0412087   -18.20   0.000    -.8306727   -.6691378
-------------+----------------------------------------------------------------
ARCHM        |
sigma2       |  -1.220812   .3851145    -3.17   0.002    -1.975622   -.4660011
-------------+----------------------------------------------------------------
ARMA         |
ar           |
          L1 |   .3173715   .0721705     4.40   0.000     .1759199    .4588231
          L2 |   .2937201   .0950365     3.09   0.002     .1074519    .4799883
          L3 |   .3311156   .0631501     5.24   0.000     .2073437    .4548875
-------------+----------------------------------------------------------------
ARCH         |
arch         |
          L1 |   .3972184   .1335734     2.97   0.003     .1354194    .6590174
          L2 |    .528677   .1517928     3.48   0.000     .2311685    .8261855
_cons        |    .008523   .0020821     4.09   0.000     .0044422    .0126038
```

j=9

```
ARCH family regression -- AR disturbances

Sample:  41 to 311, but with gaps               Number of obs      =       158
                                                Wald chi2(4)       =   1234.42
Log likelihood =  52.80308                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
a9           |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
a9           |
_cons        |  -.7203621    .047379   -15.20   0.000    -.8132232    -.627501
-------------+----------------------------------------------------------------
ARCHM        |
sigma2       |  -.2546876     .30289    -0.84   0.400    -.8483412     .338966
-------------+----------------------------------------------------------------
ARMA         |
ar           |
          L1 |   .3881729   .0792312     4.90   0.000     .2328826    .5434632
          L2 |   .3323753   .0808508     4.11   0.000     .1739107    .4908399
          L3 |   .2458648   .0667488     3.68   0.000     .1150396      .37669
-------------+----------------------------------------------------------------
ARCH         |
arch         |
          L1 |   .7847497   .2023612     3.88   0.000     .3881291     1.18137
          L2 |   .5290073   .1932176     2.74   0.006     .1503078    .9077068
_cons        |   .0048918   .0028207     1.73   0.083    -.0006365    .0104202
```

j=10

```
ARCH family regression -- AR disturbances

Sample:  41 to 311, but with gaps               Number of obs      =       106
                                                Wald chi2(4)       =    299.94
Log likelihood =  45.36332                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
a10          |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
a10          |
_cons        |  -.6169356   .0390587   -15.80   0.000    -.6934893   -.5403818
-------------+----------------------------------------------------------------
ARCHM        |
sigma2       |  -.7185317   .5029163    -1.43   0.153     -1.70423    .2671662
-------------+----------------------------------------------------------------
ARMA         |
ar           |
          L1 |   .3254466   .1051132     3.10   0.002     .1194286    .5314646
          L2 |   .3231504   .1085384     2.98   0.003     .1104191    .5358817
          L3 |   .2725519   .0897672     3.04   0.002     .0966115    .4484924
-------------+----------------------------------------------------------------
ARCH         |
arch         |
          L1 |   .2463938   .2156914     1.14   0.253    -.1763536    .6691411
          L2 |   .9023862   .2614546     3.45   0.001     .3899445    1.414828
_cons        |   .0048104   .0039503     1.22   0.223    -.0029319    .0125528
```

j=11

```
ARCH family regression -- AR disturbances

Sample:  41 to 311, but with gaps               Number of obs      =         82
                                                Wald chi2(4)       =      71.99
Log likelihood =  42.73048                      Prob > chi2        =     0.0000

------------------------------------------------------------------------------
             |                 OPG
a11          |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
a11          |
_cons        |  -.5056703    .048027   -10.53   0.000    -.5998015   -.4115391
-------------+----------------------------------------------------------------
ARCHM        |
sigma2       |   -1.11885   .9323658    -1.20   0.230    -2.946254    .7085533
-------------+----------------------------------------------------------------
ARMA         |
ar           |
          L1 |   .1833947   .1347613     1.36   0.174    -.0807325     .447522
          L2 |   .3642176   .1478223     2.46   0.014     .0744913     .653944
          L3 |   .2297049   .1091314     2.10   0.035     .0158113    .4435984
-------------+----------------------------------------------------------------
ARCH         |
arch         |
          L1 |   .4835253   .2511631     1.93   0.054    -.0087452    .9757959
          L2 |   .8463908   .3152299     2.68   0.007     .2285515     1.46423
_cons        |   .0017974   .0022162     0.81   0.417    -.0025462     .006141
------------------------------------------------------------------------------
```