ON THE INVESTIGATION OF
RESIDENTIAL HOUSING PRICE
BUBBLES IN MINSK

by

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Abstract

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The present research is devoted to the inspection of the existing techniques of detection of bubbles. These techniques are grouped into statistic and econometric ones and applied to a specific data set of the specially weighted offer prices and rents for residential property in Minsk. The underlying idea behind the research is to attempt to reveal the relevance of the techniques in their ability to identify rational bubbles at the ideal case or identify explosiveness of the series at the moderate case.

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Introduction

In his paper on real estate price bubbles Roehner (1999) assesses the magnitude of capital gain stemming from the real estate price bubble arising in Paris at the early 90-s of the previous century and finds that the gain was close to the magnitude of French GNP at that time. This allows him to make an inference on a serious impact of real estate price bubbles on a particular economy. Especially, it becomes evident when the bubbles burst harming financial system of an economy and raising the possibility of unfolding economic or financial crisis (Roehner, 1999).

The aim of the research to be conducted is revealing the existence of bubbles in the economy of Belarus and assessing their possible influence on GDP. The GDP growth is very strong in Belarus if compared to other NIS countries however the index of economic freedom is constantly being falling in the country. This situation is considered to be anomalous (the two indices are positively correlated in the rest of NIS), therefore it was given the name of "Belarusian Puzzle" (Daneiko (2003) et al., 2003,p.112). There are many evidences of exaggerating GDP growth figures in the Republic the main of which is manipulating prices. But many economists still wonder by how much the exaggeration is compensated with the unreported value added created in the "shadow economy" (Daneiko (2003) et al., 2003, p.112). So, the question of by how much the growth really persists in Belarusian Economy and by how much the growth is a mere exaggeration remains unanswered. In the thesis the price side of influencing GDP will be investigated. The direct consequence of the GDP exaggeration by increasing price level may be the creation of bubbles in certain sectors of the economy.

In the literature devoted to investigation of bubbles existence asset price bubbles are investigated. Moreover in the scope of most papers an examination of divergence between "real" values of assets and their actual price is examined.

Fortunately, there is an object for such investigation in Belarusian economy regardless to heavy influence of government on all markets including the stock market. The rationale for choosing residential housing market as the main object of research is stipulated by reasons of relatively less severe regulation of the market by the government, as well as relatively bigger attractiveness of the market for domestic and foreign investors. The former reason is beneficial for the purposes of the research to be conducted in the sense of relative reliability of the data available on the real estate market. The letter reason is beneficial in the sense of applicability of the existing analytical tools developed for market economies.

But the main reason of course is the perception of the market as merely the only investment opportunity in Belarus. Thus foreign exchange market appears to be too predictable with its constantly growing exchange rate for investors to gain from participating in it; money market appears to be heavily regulated by the Central Bank thus rising political and regulatory risks much higher in comparison to the expected profits; capital market may be characterized by the absence of corporate bonds issuing due to poor legislation as well as underdevelopment of the market for corporate shares due to the rule of "golden share" (i.e. the rule under which any share of government in an enterprise gives the government's representatives the voting right which dominates the rights of non government shareholders regardless to the magnitudes of shares of the latter in the company's stock). Such situation created prerequisites for strong speculative tendencies on the market for real estate which may be explained by a rush of investors for allocating hot money in this highly attractive market in the fear of growing possibility of Government intrusion to the market which may result in making it as unattractive as the markets mentioned above.

Another rationale for choosing residential housing market for investigation is the possibility of implementing achievements of the existing rational bubbles models, since the investigation of real estate prices in Belarus may well be fitted into the models. The similarity arises in very close nature of securities and residential housing trading, for example. Thus, there exist both primary and secondary markets for residential housing which operate similar to the markets for securities. Moreover, the rental payments obtained by residential housing owners may well be compared to the interest gained by holders of shares or bonds.

The rationale for using a bubble theory is even more attractive provided the general definition of the bubble stated by J. Stigliz: "if the reason that the price is high today is only because investors believe that the selling price will be high tomorrow-when "fundamental" factors do not seem to justify such a price-then a bubble exists" (Stiglitz, 1990). And there indeed exists a situation under which the residential housing prices in Belarus are growing well in accord with the property prices growth in the USA and certain European countries while the growth of consumers' income is far behind of that observed in the regions just mentioned (The Great Illusion, 2004).

The basic model will be based on rational expectations bubbles approach similar to that proposed by Blanchard and Watson in the simplest form of which the bubble term B_t follows an explosive path with certain growth rate a: $B_t=a\cdot B_{t-1}+\varepsilon_t$ with parameter $a>1$ and IID ε_t stochastic component with O mean and constant variance σ_t^2 (Lux et al., 2003).

Two different approaches which will be used for bubbles detection and will concern the estimation of the tails of distribution of real estate prices. These approaches will embody in actual estimation of the tails as well as in implementation of bootstrap technique for the same purposes due to not very sufficient number of observations (this fraction of research will be presented in the first two parts of the thesis).

For determining the presence of bubbles in real estate prices time series the GMM will be used (this part of research will be presented in the last part of the thesis).

In the recent time much attention has been turned to the real estate market in Belarus which resulted in enacting of a Presidential Act forbidding the reselling of residential property in the course of 2 years (Sinyak N., 2003). This was the reaction to consequences of permanent real estate price growth in Belarus the causes and consequences of which are still to be investigated. Thus the definitions given above reflect the current situation on the real estate market in Belarus quite precisely (Sinyak N., 2003). For the time being the index of real estate prices calculated for Minsk, the capital of Belarus, continues to grow with the forecasts of its slowing down from the year 2003 has not still become reality (Sinyak N., 2003).

L i t e r a t u r e   R e v i e w

In the literature review "from general to specific" approach is going to be used. In the very beginning the rationale for reviewing exactly that literature will be stated and then the review itself will follow.

Before introducing pure economic issues considered in the literature the attention is to be drawn to a more technical parallel between the former Soviet and the present Belarusian situation: the availability of relevant data. It is broadly believed that such information was collected in the former USSR for the purposes of proper planning and governing. But the information was not presented to the public. Only some ideologically transformed figures were made publicly available instead with suppression of negative social-economic data as well as data on shadow economy (Gur Ofer, 1987, p. 1770). The same situation may be observed today in the Republic of Belarus where official figures on economic performance look encouraging but the figures are broadly considered as not reliable (Daneiko, 2003, p.112).

Naturally, one of the main sources of doubtful information is that of GDP growth. Both in the former USSR and in today's Belarus the figures on GDP growth are reported to be substantial (Daneiko, 2003, p. 112; Gur Ofer, 1987, p. 1777). The sources of Soviet economic growth included among other things growing inputs accompanied with declining growth in productivity (Gur Ofer, 1987, p. 1782). The above factor may be well applied to the growth strategy of the present day Belarus (Daneiko, 2003, p. 118). But what is more important in the growth strategy of the former USSR and, possibly, present day Belarus is the exaggeration of growth rates by price increases (Daneiko, 2003, p. 116; Gur Ofer, 1987, p. 1784). This appears to be a very useful implication for the purposes of the research to be conducted. The reason for this is the possibility of asset price bubbles appearance in the economy of Belarus and, what is more fascinating, the appearance of possibility of investigation of their existence and their possible influence on growth figures.

There exists a reach selection of literature devoted to detecting the appearance of asset price bubbles in market economies. The actual thorough investigation of price bubbles started from the research conducted by Blanchard and Watson in late 70-s early 80-s of the previous century (Blanchard and Watson, 1982). And since then much controversy has appeared concerning the ability of the existing models to appropriately test for the existence of bubbles in certain time series. Thus a substantial part of empirical research has focused on explosive trends in asset prices time series. However, many authors doubt whether the tests applied for these purposes truly reflect the existence of bubbles (Lux Thomas, 2003, p. 589; Robert P. Flood et al., 1990, p. 87). But some of them still admit that the tests for bubbles are very important specification tests (Robert P. Flood et al., 1990, p. 99). For the present time three areas of bubbles tests implementation has been developed. Among them are securities market speculation bubbles, exchange rate bubbles and price level bubbles. For the models of the first type we have the following specification: qt=qtf+Bt where qt represents the current price of an asset, qtf represents its fundamental price calculated on the basis of discounted expected future returns using real interest rate, Bt is the bubble term which is the expected discounted value of a bubble (Robert P. Flood et al., 1990, p. 88). Interestingly, some researches admit that not rejecting of the existence of bubbles after applying testing procedure implies misspecification of the model (Robert P. Flood et al., 1990, p. 89). The interesting model on price level bubbles was developed by Flood and Garber (Robert P. Flood et al., 1980) under which authors could not reject the hypothesis of bubble existence but there were some specification inconsistencies as well (Robert P. Flood et al., 1990, p. 91). Another interesting approach concerning price level bubbles models includes two estimates of a parameter which may enclose bubbles and the application of the Hausman specification test which in effect becomes test for bubbles (Robert P. Flood et al., 1990, p. 92). Finally, the exchange rates bubbles models may be tested using the techniques described above, but the indeterminacy of whether such tests really discover bubble existence pertains in this case as well (Robert P. Flood et al., 1990, p. 94).

The most recent papers make an attempt not to confound the specification error of the model and the bubbles, but there is doubtful whether such endeavors lead to the desired result (Hooker, 2000).

In the light of the difficulties mentioned above Thomas Lux proposed a test for bubbles on a more elementary level by investigating the kurtosis of financial time series (Lux Thomas, 2003, p. 590). Interestingly, he showed that the critical value for coefficient indicating the existence of bubbles implies nonexistence of mean and variance of the data examined (Lux, 2003, p. 592).

However, in the present thesis an extensive use of a more applied coefficient, known as Hill estimator, will be exercised (Hill, 1975).

The estimator is used for detecting fatness of tails. And it has been increasingly used for applied research. In line with the direct application of the index more sophisticated methods have been developed. Thus, a modification of the index for small samples was proposed by Huisman et al. (Huisman, 2001). An even more advanced methodology of the estimate bootstrap technique was developed by Pictet et al. (Pictet, 1998). All the above the above techniques are to be implemented for the purposes of the current research.

Together with the approach mentioned above a purely econometric view of the problem is presented in the paper on investigation of residential price bubbles in Hong Kong (Chan, 2001). The authors use GMM for the purposes of the paper and their methodology will be thoroughly followed in the thesis.

Due to the devotion of core research to real estate price bubbles investigation the main set of data which is being examined is the index of average real estate prices in Minsk, the capital of Belarus, calculated by the Belarusian Society of Appraisers (BSA) on a monthly basis. The index represents the average price of a square meter of residential housing over the whole scope of the housing supply.

The actual values of the index have been available starting from the year 1999 but the weekly information is only available starting from the year 2000:

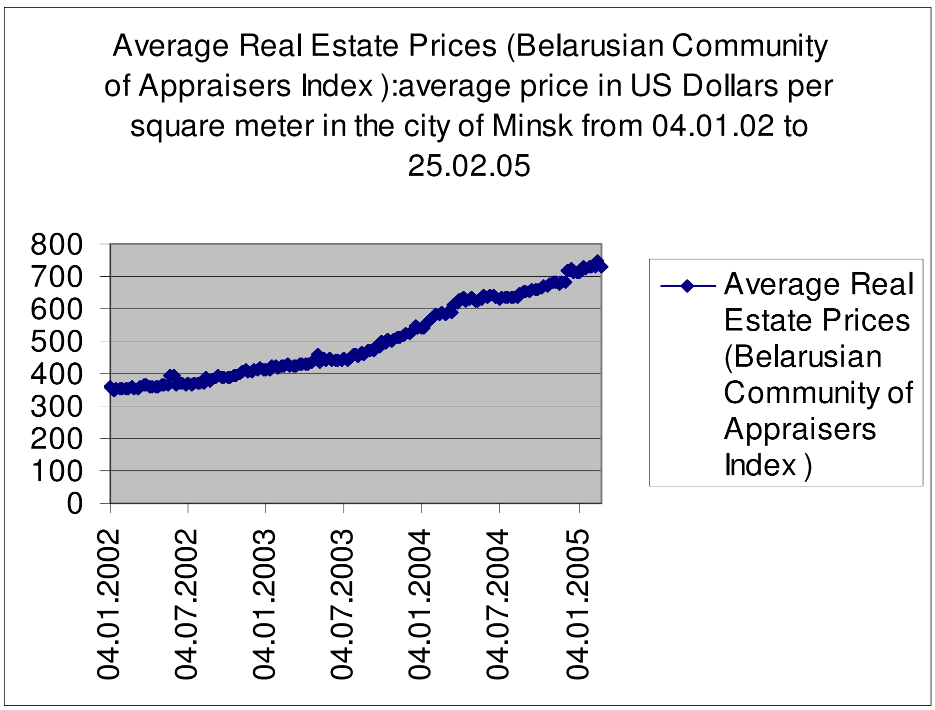

Figure 1

For the purposes of the current research the value of the index starting from the start of the year 2002 will be of particular interest since this period of real estate prices growth is suspected to contain a bubble. In the period mentioned weekly data will be used. Thus the period under investigation will contain 164 observations.

The next stage of research is going to be devoted not only to investigation of capital gains from holding residential property but also to rents on this kind of asset. The data on prices for renting apartments in Minsk is easily obtained from the relevant sites or newspapers. Hence, there are 164 observations on renting one room apartments and the dynamics is presented below:

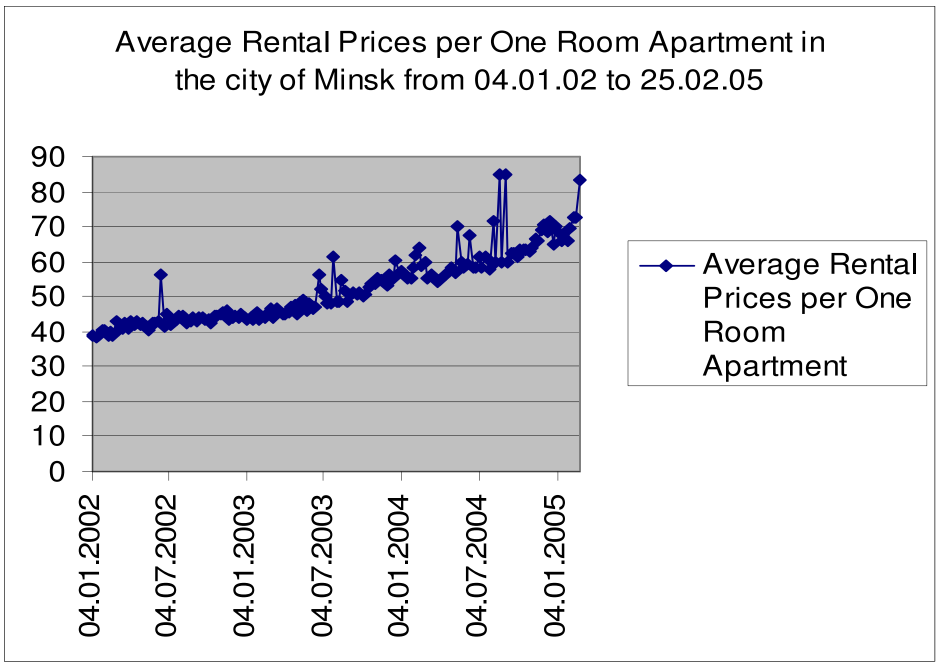

Figure 2

C h a p t e r 1

THE APPLICATION OF THE BOOSTRAP TECHIQUE TO ESTIMATING THE HILL INDEX OF THE TAILS FATTNESS OF REAL ESTATE PRICE INDEX (ITS) DISTRIBUTION

In the present paper a series of real estate prices index (ITS) is to be investigated for the presence of a rational bubbles. This index represents a weighted average of real estate prices (per square meter) in Minsk, the capital of Belarus, over the whole scope of supply of residential housing. The index is calculated by the Belarusian Society of Appraisers on a weekly basis. The calculation of the index (ITS-index of total supply) is implemented according to the following formula:

$$ITS = \frac{\sum_{i=1}^{n} Pi}{\sum_{i=1}^{n} Si} \tag{1}$$

, where

Pi - is the price of a specific apartment supplied in the whole sample of statistical data processed in a given period;

Si – is the total area of a separate apartment in the whole sample of the statistical data processed in a given period (Trifonov et. al., 2001).

The index has been experiencing considerable growth starting from the 3rd quarter 2000 but the actual weekly data is available only starting from the year 2002. There have been 164 observations available. This is a sufficient number of observations for making inference for the purposes of current research but there is still a possibility for a small sample bias of estimates. Thus, some additional computational techniques will be applied to enhance the inference. The description of the techniques will be provided later.

The theory of price bubbles started from the research conducted by Blanchard and Watson in late 1970-s early 1980-s (Blanchard et al., 1982). In the seminal work financial time series were given characteristics which are still considered as strongly attributable to financial data. One of these characteristics is leptokurtosis, or excessive fourth moment of financial returns (Blanchard et al., 1982, p.25; Lux et al., 2003, p. 590).

So, the natural way to start investigation for speculative bubbles is finding kurtosis coefficient for the first difference of the real estate prices index (ITS). The calculations were processed in Stata and the resulting coefficient was as follows:

- variable | kurtosis
- -----------------------
- IndexDif | 6.999065
- ------------------------

Table 1

, where IndexDif stands for the first difference of ITS and the number represents the index kurtosis.

As we can see from Table1, the kurtosis coefficient is well in access of the kurtosis coefficient for normal distribution the theoretical value of which is equal to 3 (Blanchard et al., 1982, p.25). So, an immediate conclusion follows that series under consideration is indeed leptokurtic.

The graphical representation of the above statement is even more convincing:

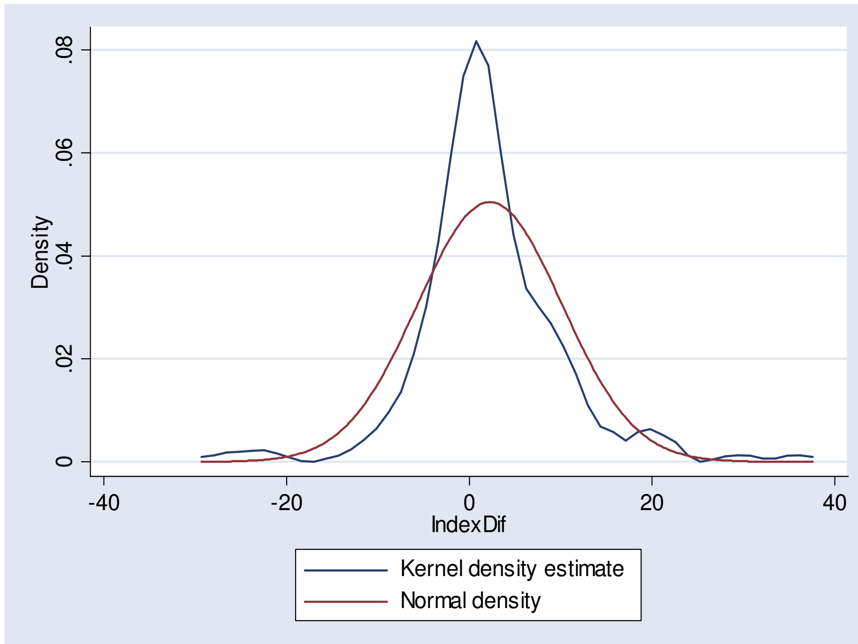

Figure 3

Using kernel density estimation of the series of the 1st differences of ITS index and comparing it with the normal distribution density confirms the excess kurtosis of the empirical time series (See Figure 3).

The less elementary characteristics of financial time series derived from the model a la Blanchard and Watson has given a rise for much controversy in conclusions concerning the ability of the existing models to appropriately test for bubbles in certain time series. Thus a substantial part of empirical research has focused on explosive trends in asset prices time series. However, many authors doubt whether the tests applied for these purposes truly reflect the existence of bubbles (Lux et al., 2003, p. 589; Flood et al., 1990, p. 87).

In the light of the difficulties mentioned above Thomas Lux and Didier Sornette proposed to test for bubbles on a more elementary level by investigating not only the kurtosis of financial time series but also the tails of their distribution (Lux et al., 2003, p. 590). The tails are investigated for being "fat", or for having more than exponential decline in probability mass (Lux et al., 2003, p. 590)[1]. The letter characteristic of the series is proposed to be investigated by using conditional maximum likelihood estimator known as Hill tail index (Lux et al., 2003, p. 603).

The index does not require any specific assumptions about the form of distribution but it requires sufficiently large samples since the inference is only asymptotically valid (Hill, 1975, p.1163). The Hill Index is calculated as follows:

in the first step the sample elements are put in descending order $x_n \geq x_{n-1} \geq \ldots \geq x_{n-k} \geq \ldots \geq x_1$ where k represents the number of parameters in the tail of distribution;

in the second step the index itself is calculated in accordance with the following formula:

$$I_H = 1 / \frac{1}{k} \sum_{i=1}^{k} [\ln x_{n-i+1} - \ln x_{n-k}] \tag{2}$$

(Lux et al, 2003, p. 603).

If the original distribution is fat-tailed it can belong to only one family of distributions:

$$G(x) = \begin{cases} 0 & x \leq 0 \\ \exp(-x^{-\alpha}) & x \succ 0, \alpha \succ 0 \end{cases} \tag{3}$$

, where $G(x)$ is the probability that $x_n > x$ and $\alpha = I_H$ is exactly the tail index which is the only parameter we have to estimate. The Hill Index is not a unique tail index estimator but it has been shown to be the best one in the family of such estimators (Pictet et al, 1998, p.289) for fat tailed distributions. The Index has also been shown to be consistent (Mason, 1982). Finally, the index has been proved to be biased (Pictet et al, 1998, p.297). But fortunately, the bias can be corrected for by using non trivial but theoretically justified technique of choosing the most appropriate k. This technique will also be more thoroughly explained

[1] For the graphical representation of "fat" tails the reader is referred to Picture1 again. It is evident from the Picture that the density of empirical distribution is higher then that of normal distribution as we approach both the most extreme positive and negative values in the tails. Picture 1.A in Appendix 1 represents kernel density estimation of the absolue value of the 1st differences of the ITS values.

below. So, these are the basic justifications behind choosing exactly Hill index for the purposes of current research.

Theoretically, the prevalence of a rational bubble component will lead to the magnitude of the index<1 which implies nonexistence of the mean and variance of the data (Lux et al., 2003, p. 592). However, empirical findings provide the magnitudes in the range between 2 and 4 (Lux et al., 2003, p. 590) which may partially be justified by the above mentioned bias.

At this stage all necessary explanations has been given and the most straightforward calculations can be exercised. These calculations concern the computation of the Hill Index for empirical data set of the 1st differences of ITS. Since, as it was mentioned earlier, the choice of *k* (the number of observations in the tails of the empirical distribution) is not trivial, we will calculate the index for each value of $k \in [8; 14]$ (for values of *k* smaller than 8 the magnitudes of the Index are even higher). The justification behind such a choice of *k* is simply Hill Index reaching its local minimum value on this interval (this is evident from Table 2 below where the Hill Index reaches its minimum value for *k* =12). An important note to be made here is that absolute values of the 1st difference of ITS has been used which is justified with the use of logarithms for computing the Hill Index (see expression (2)).

K	8	9	10	11	12	13	14
Hill Index	4.96875	3.281744	3.4347	2.684078	2.097045	2.234874	2.246568

Table 2

We can compare these results with that obtained from using Stata software. Here the index of regular variation2 is estimated with the help of Hill plot by using hillp function developed by Manuel G. Scotto, University of Lisbon, Portugal. The plot below shows the values of Hill estimator against different values of k. The value of the Hill Index estimator is to be chosen from

[2] this is just a more formal way of expressing the feature of equation (3) which can be denoted as regular variation with index -α assuming that $1-F(x) \sim C x^{-\alpha}$, where C, α >0 (Hall, 1990, p. 187; Lux, 2003, p. 590).

the *k*-region with roughly horizontal plot. In this case the estimate value fails to converge to any horizontal line.

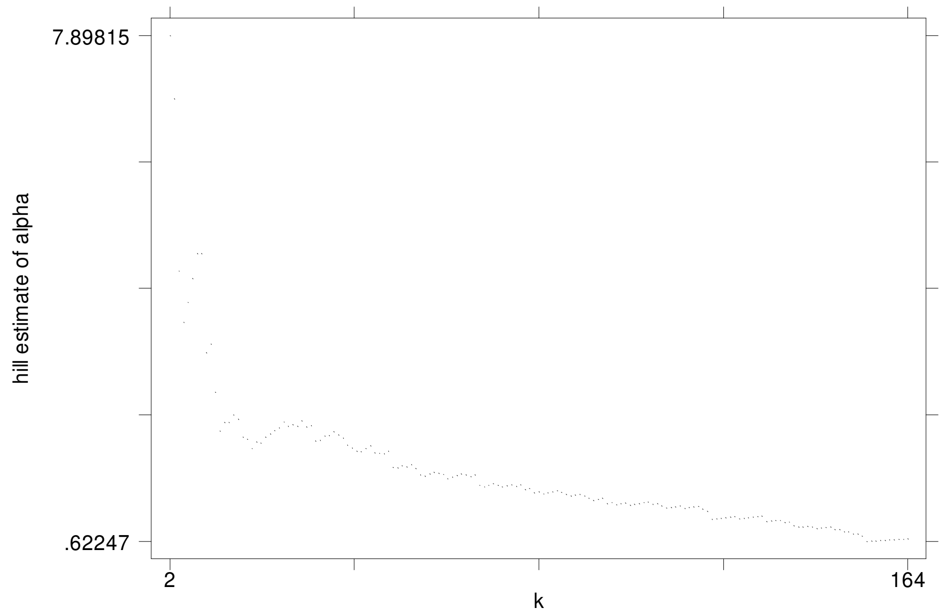

Figure 4

For the time being we obtained two alternative estimates of Hill Index. One of these estimates (α =0.62247; α <1) agrees in magnitude with theoretically predicted value while the other one (α =2.097045; 2< α <4) agrees with empirical findings. Such situation is quite predictable since Hill Index Estimator appears to be

first, only asymptotically valid while we have only 164 observations available;

second, even asymptotically the Hill Index Estimator remains biased. One way of challenging with this unfortunate situation is to use bootstrap method proposed by Pictet et al (1998). But before presenting the actual results I should give some theoretical background behind the method.

As it was already mentioned, the critical condition for obtaining the least biased Hill Index Estimator is choosing the optimal value of *k*. To show it mathematically we first need to present more careful formulas for expectation and variance of the estimator. For notational purposes lets denote $\gamma = 1/\alpha$. The exposition that ensues is an abridged version of a careful derivation presented in Pictet et al (1998).

For some observations X_i drawn from c.d.f. F(x) with density f(x)=dF/dx and for some y being the value of some *k* largest observation X_k, the unconditional expectation of $\gamma_{n,k}$, where *n* is the sample size, is

$$E[\gamma_{n,k}] = \frac{\int_{-\infty}^{\infty} p_{n,k}(y) E[\gamma_{n,k} \mid X_{(k)} = y] dy}{\int_{-\infty}^{\infty} p_{n,k}(y) dy} \tag{4}$$

,where $E[\gamma_{n,k} \mid X_{(k)} = y] = \int_y^x \ln \frac{x}{y} \frac{f(x)}{1 - F(x)} dx$, (4.1)

$$p_{n,k}(y) = \frac{n!}{(k-1)!(n-k)!} F^{n-k}(y)[1 - F(y)]^{k-1} f(y) \tag{4.2}$$

The expected variance of the Hill Index estimator around the mean of (4) is

$$E\{[\gamma_{n,k} - E[\gamma_{n,k}]]^2\} = \frac{\int_{-\infty}^{\infty} p_{n,k}(y) E[\gamma_{n,k}^2 \mid X_{(k)} = y] dy}{\int_{-\infty}^{\infty} p_{n,k}(y) dy} - E^2[\gamma_{n,k}] \tag{5}$$

(Pictet et al, 1998, p.292).

In asymptotic expansion equation (4) gives:

$$E[\gamma_{n,k}] = \frac{1}{\alpha}\{1 - \frac{\beta b^{`}}{\alpha + \beta} \frac{\Gamma(k + \frac{\beta}{\alpha})}{\Gamma(k)} [a(n-k)]^{-\frac{\beta}{\alpha}}\} = \frac{1}{\alpha} + B \tag{6}$$

, where $\Gamma(\ )$ is the gamma function, which is a continuous version of a factorial part of (4.2) and

$$b^{`} = b(1 + o(x^0)) \tag{6.1}$$

where b is a real number parameter of a specially assumed expansion of F(x) and β is a parameter in the following expansion: F(x)=1-ax $^{-\alpha}$ {1+ b` x $^{-\beta}$} (6.2)

with o meaning "of order higher than" (Pictet et al, 1998, p.297).

The B of equation (6) can be alternatively expressed as:

$$B = -\frac{1}{\alpha}\frac{\beta b}{\alpha+\beta} a^{-\frac{\beta}{\alpha}} (\frac{k}{n})^{\frac{\beta}{\alpha}} \{1 + \mathrm{O}(\frac{1}{k}) + \mathrm{O}(\frac{k}{n}) + o[(\frac{1}{k})^0 (\frac{k}{n})^0]\} \tag{7}$$

, here O means "of the same order".

Finally, the expected variance of the Hill index estimator is expressed as follows:

$$\mathrm{E}\{[\gamma_{n,k} \text{-} \mathrm{E}[\gamma_{n,k}]]^2\} = \frac{1}{\alpha^2 k}\{1 + \mathrm{O}(\frac{1}{k}) + \mathrm{O}[(\frac{k}{n})^{\frac{\beta}{\alpha}}]\} \tag{8}$$

So, the error of Hill Index estimator results into the following expression:

$$\mathrm{E}\{[\gamma_{n,k} - \frac{1}{\alpha}]^2\} = \left(-\frac{1}{\alpha}\frac{\beta b}{\alpha+\beta} a^{-\frac{\beta}{\alpha}} (\frac{k}{n})^{\frac{\beta}{\alpha}} \right)^2 + \frac{1}{\alpha^2 k} \tag{9}$$

(Pictet et al, 1998, p.298).

Obviously, the error can be minimized wrt *k*.

$$\text{Such } \bar{k} = [\frac{\alpha(\alpha+\beta)^2}{2\beta^3 b^2}]^{\frac{\alpha}{\alpha+2\beta}} (an)^{\frac{2\beta}{\alpha+2\beta}} \tag{10}$$

Putting the value of $\bar{k}$ back into (9) we obtain the minimum error variance (Pictet et al, 1998, p.299).

Fortunately, equation (10) is empirically computable. The exact procedure of this computation is described in Hall (1990). Hall proposed to minimize wrt k_1 the following expression: $\min_{k_1} E[\{\gamma^*_{n_1,k_1} - \gamma_0\}^2 \mid F_n]$ (11)

$\gamma^*_{n_1,k_1}$ is computed from the bootstrap resamples of size $n_1 < n$ and different values of k_1 which are chosen so that to minimize (11); γ_0 is some full sample estimate with reasonably chosen *k* (in our case it will be the inverse of the Hill Index from Table 1 for *k* = 12); and F_n is empirical distribution function.

The value of k_1 which minimizes (11) is denoted $\bar{k}_1$ and then it is used to estimate $\bar{k}$ which can be found from equation (10) by the following formula:

$$\hat{k} = \bar{k}_1 \left(\frac{n}{n_1} \right)^{\frac{2\beta}{2\beta+\alpha}} \tag{12}$$

, where $\hat{k}$ is the bootstrap estimate of $\bar{k}$ (Pictet et al, 1998, p.299).

To clarify the procedure behind estimating (11) lets denote the following:

$$\hat{MSE}(n_1, k_1) = E[\{\gamma^*_{n_1,k_1} - \gamma_0\}^2 \mid F_n]$$

$$= E(\gamma^{*2}_{n_1,k_1} \mid F_n) - 2\gamma_0 E(\gamma^*_{n_1,k_1} \mid F_n) + \gamma_0^2 \tag{13}$$

Since γ_0^2 is a constant we should only minimize the first two terms of (13) wrt k_1 which in alternative notation can be expressed as $\min_{k_1} \mathrm{A}_2 - 2\gamma_0 A_1$

, where

$$\mathrm{A}_1 = E(\gamma^*_{n_1,k_1} \mid F_n) = n_1 k_1^{-1} n^{-1} \sum_{i=1}^{n-1} i I_{1-in^{-1}}(n_1 - k_1, k_1)\Delta_{ni} \tag{13.1}$$

$$\mathrm{A}_2 = E(\gamma^{*2}_{n_1,k_1} \mid F_n)$$

$$= k_1^{-2} \sum_{1\le i \le} \sum_{j \le n-1} (2-\delta_{ij})\{n_1(n_1-1) ijn^{-2} \times I_{1-jn^{-1}}(n_1 - k_1, k_1 - 1) +$$

$$+ n_1 in^{-1} I_{1-jn^{-1}}(n_1 - k_1, k_1)\}\Delta_{ni}\Delta_{nj} \tag{13.2}$$

$\Delta_{ni} = \log(X_{ni} / X_{n,i+1})$; δ_{ij} is the Kronecker Delta which takes the values of 1 for i=j and 0 otherwise; $I_x(a,b) = \{B(a,b)\}^{-1} \int_0^x t^{a-1}(1-t)^{b-1} dt$, $0<x<1$, is the incomplete beta function (Hall, 1990, p.188-189).

When applying this methodology to the actual data I took a natural question arise: what value of n_1 is to be chosen? In Hall (1990) this issue is not addressed at all and the only restriction on n_1 is that its value should be less than the sample size. Pictet et al (1998) mention that n_1 is to be of the different order of magnitude as compared to the sample size. Since the sample under consideration contains only 164 observations, I decided to choose subsamples form 20 to 60 observations each with the step of 5 observations. Since it was questionable which of observations to choose, an average value has been taken from each subsample of 20 to 60 observations starting from the first 20 to 60 observations and ending with last 20 to 60 observations with the shift of 1 for each estimation. I bootstrapped the sub samples by making 100 replications then I took average value of each observation over these replications and used these values for minimizing (13) with γ_0 = 0.476861488 (see Table 2). Than each k^i was substituted into (12) and the resulting average value over all k^i's was chosen as the estimate of $\hat{k}$. When computing $\hat{k}$ from (12), the only unknown parameter left was β which Pictet at al (1990) recommend setting equal to 1 (Pictet et al, 1998, p.304). Thus, we obtain the following values of $\hat{k}$:

n_1	20	25	30	35	40	45	50	55	60
$\hat{k}$	12,799	15,028	17,531	17,973	20,183	21,008	20,441	20,089	20,746
$\hat{\alpha}$ (95% conf. interval, (8))	2,2348 (±0,234)	2.2979 (±0,22)	2.0038 (±0,23)	2.0038 (±0,23)	1.9611 (±0,223)	1.9442 (±0,22)	1.9611 (±0,223)	1.9611 (±0,223)	1.9442 (±0,22)

Table 3

So, the bootstrap technique provides an estimated value of $\hat{\alpha}$ approximately between 2 and 2,3.[3] Of course there are further speculations possible with the values presented in the table above for obtaining the desired estimate. Thus, the most frequently reported estimate could be chosen or an

[3] Most part of the computations has been conducted in MatLab, the detailed codes presented in the Appendix2.

average of the most frequently reported estimates may be computed. Finally, the average $\hat{k}$ can be found and then a single estimate of $\hat{\alpha}$ computed. But this gives basically same results.

Interestingly, there has been found another method of choosing optimal *k* in the course of writing making estimations for this section. Thus, *k* may be chosen from the region on the Picture 1 where the tails of the sample distribution become "fatter" than those of normal distribution (I also refer the reader to the Picture 1.A in Appendix 1 for one tail representation). Thus, by visual inspection of both Figure 3 and Figure 3.A it becomes clear that the tails of our sample distribution become fatter for the values of first differences in ITS index slightly less then 20. The three candidate values with corresponding values of *k* and $\hat{\alpha}$ are presented in the table below:

The Value of the First Difference of ITS Price Index	The Corresponding Value of *k*	The Corresponding Value of $\hat{\alpha}$
17,6	11	2,684078
17,9	10	3,4347
20,3	9	3,281744

Table 4

Obviously, the bootstrap technique gave the lower values of $\hat{\alpha}$ (which implies fatter tales) and approximately two times longer tales. But the technique reported in Table 4 may be at least applicable for choosing the "reasonable" value of $\hat{\alpha}$ to be used in estimation of equation (13).

The results presented in Table 4 will appear even more interesting after comparing them with the estimates of the following section. In the following section another technique of obtaining estimates especially for small samples will be examined. This technique was proposed by Huisman et al. who developed a modified Hill Estimator which is calculated as a special kind of average (OLS) and allows avoiding the problem of choosing k (the number of parameters in the tail of distribution). This alternative estimator is feasible only for relatively small samples (more than 100 observations, which is exactly our case) (Huisman et al., 210).

Chapter 2

SMALL SAMPLE ESTIMATE OF THE HILL INDEX

In the previous section an elimination of small sample bias has been conducted via the implementation of bootstrap technique. This section is devoted to alternative technique which has been developed especially for obtaining small sample estimates of the Hill Index (Huisman et al, 2001). The latter approach avoids the problem of the choice of optimal *k* by extracting the unbiased estimate from the values of Hill index calculated for the whole set of *k*-s (Huisman et al, 2001, p.209). The resulting estimate is simply the weighted average of the set of Hill indexes each conditioned on particular *k*.

From formula (7) of the previous section we saw the exact expression for bias of the Hill index estimator for the fat-tailed class of distribution functions approximated by expression (6.2). The bias is not zero for all fat-tailed distribution except for Pareto distribution which is expressed in (3). Finally, the variance of the estimator is expressed in (8). All these formulas combined show that small *k* reduces the bias but large *k* reduces the variance. From Picture 2 of the previous section we can see an almost linear increase in Hill estimator for more than a half of *k* values which allows approximating for a bias by linear function. The last opportunity justifies the choice of $\alpha=\beta$ restriction in (7). Thus, for comparatively small samples the trade-off between bias and efficiency is reconciled by controlling for bias and allows expressing (6) as follows:

$$I_H=\beta_0+\beta_1\mathbf{k}+\varepsilon(\mathbf{k}),\ \mathbf{k}=1,\ldots,\varkappa \qquad (14)$$

where $\varkappa$ denotes the threshold value of *k* for which linear increase in Hill estimator is observed (Huisman et al, 2001, p. 210). In equation (14) a vector of conventional Hill indexes is regressed on the corresponding values of *k*. Since β_0 is the value obtained after adjusting for bias it is exactly the unbiased estimator of Hill index (Huisman et al, p.210).

Expressing (14) in matrix notation we obtain the following expression:

$$I_H^*=Z\beta+\varepsilon, \tag{15}$$

where I_H^* is a vector of Hill indexes for values of k chosen as described above; Z is 2X$\varkappa$ vector with ones in the first column and k-s from 1 to $\varkappa$ in the second column. Huisman et al argue that the variance of the Hill index estimators is inversely related to k one should use WLS instead of OLS for estimating (15). They propose the matrix of weights W_{kXk} have the values of $\sqrt{k}$ on the main diagonal and zeros elsewhere. As a result the following 2X1 vector is obtained:

$$b_{wls}=(Z'W'WZ)^{-1}Z'W'WI_H^* \tag{16}$$

with the first element of b_{wls} being the unbiased Hill index estimator which appears to be a simple weighted average of Hill indexes in I_H^* (Huisman et al, 2001, p.210).

To provide the robustness of results the necessary computations were performed in EViews and Matlab for WLS of (14) and matrices of (16) respectively. The following table reports results from estimation in Eviews and

Dependent Variable: HILL
Method: Least Squares
Date: 04/28/05 Time: 21:32
Sample: 1 76
Included observations: 76
Weighting series: SQRTK

Variable	Coefficient	Std. Error	t-Statistic	Prob.
K	-0.022841	0.001659	-13.76895	0.0000
C	2.843871	0.089731	31.69324	0.0000
Weighted Statistics				
R-squared	-0.005479	Mean dependent var		1.828554
Adjusted R-squared	-0.019067	S.D. dependent var		0.273150
S.E. of regression	0.275742	Akaike info criterion		0.287259
Sum squared resid	5.626476	Schwarz criterion		0.348594
Log likelihood	-8.915831	F-statistic		189.5840
Durbin-Watson stat	0.321142	Prob(F-statistic)		0.000000
Unweighted Statistics				
R-squared	0.513487	Mean dependent var		2.121484
Adjusted R-squared	0.506913	S.D. dependent var		0.995816
S.E. of regression	0.699263	Sum squared resid		36.18373
Durbin-Watson stat	0.283555			

Table 5

Matlab results will be reported later, but here it should be mentioned that there are identical to those obtained in EViews. Here $\varkappa$ was chosen to be equal to $n/2$ with n be the number of observations in the sample.

One important note to be made here is that neither OLS nor WLS account for the resulting autocorrelation of the error term which stems from autocorrelation of conventional Hill estimators which are calculated on common observations. Huisman et al propose asymptotic standard errors which take into consideration this peculiarity. They also show that the standard errors with asymptotically plausible properties hold sufficiently well for comparatively small samples like the sample under consideration (Huisman et al., 2001, p. 210).

Huisman et al. take y as increasing order statistics forming a vector of the size $((\varkappa+1)X1)$ with $y(i)=\ln(x(i))$ and $i=n-\varkappa,\ldots,n$. From (2) we can see that the Hill index estimator is a linear combination of y's which allows to express its inverse as $(I_H^*)^{-1} = Ay$. The transformation matrix A is of the size $((\varkappa\times\varkappa)+1)$. Then, to estimate covariance matrix for the set of the inverses of the Hill index estimates Ω we need to estimate the covariance matrix of order statistics in y, which is denoted Σ. The expression which connects the two matrices is $\Omega=A\Sigma A'$.

In general, increasing order statistics $z(i)$, $i=1,\ldots,\varkappa+1$, from the sample of size n asymptotically have multivariate normal distribution with mean $\mu(i)$ and covariances between order statistics $z(i)$ and $z(j)$ $(j\geq i)$ of $\nu(i,j)$ (Cox and Hinkley, 1974). Here,

$\mu(i)=\ln((1-p(i))^{-1/\alpha})$, where $p(i)$ is approximated by i/n (17)

$$\nu(i,j)=\frac{p(i)(1-p(j))}{n\alpha(i)\mu(i)^{-(1+\alpha(i))}\alpha(j)\mu(i)^{-(1+\alpha(j))}} \text{ for } j\geq i. \quad (18)$$

In formulas (17), (18) Pareto distribution is assumed with cdf $F(x)=1-x^{-a}$, $x>0$, for which $\alpha = I_H$ and corresponding pdf's are expressed in the denominator of (18) (Huisman et all, 2001, p. 215).

Formulas (17), (18) completely define the covariance matrix Σ. Matrix A can be shown to have the following form:

$$\begin{bmatrix} 0 & ... & 0 & 0 & 0 & -1 & 1 \\ 0 & ... & 0 & 0 & -1 & 1/2 & 1/2 \\ 0 & ... & 0 & -1 & 1/3 & 1/3 & 1/3 \\ ... & ... & ... & ... & ... & ... & ... \\ -1 & 1/\kappa & 1/\kappa & 1/\kappa & 1/\kappa & 1/\kappa & 1/\kappa \end{bmatrix} \quad (19)$$

(Huisman et all, 2001, p. 215).

Thus, we can now compute Ω=AΣA' which is then used in the following expression:

$$Cov(b_{wls})=(Z'W'WZ)^{-1}Z'W'W\Omega W'WZ(Z'W'WZ)^{-1} \quad (20)$$

Here matrices Z and W are the same as those of expression (16). The results of the estimates obtained from matrix expressions (16) and (20) are reported in Table 6 below:

b_{wls}	$Cov(b_{wls})$	
2.8439	0.00696453754079	0.00006667141430
-0.022841	-0.00004156417093	-0.00000039789319

Table 6

Here we can see that redefined standard errors allow for even more significant results as compared to Table 4 above.

Thus, after applying a number of alternative techniques for obtaining Hill index estimates it has now become possible to interpret the first results. Two major estimates for the tail index are respectively those presented in Tables 3,4 of the previous section and $\hat{\alpha} = 2.8439$ of the present section. The bootstrap estimates of the previous section indicate fatter tail which is closer in theoretical value to the rational bubble tale ($\hat{\alpha}$ <1) while estimates of Table 4 of the previous section and the estimate of the current section are close to the empirical findings for alternative time series tested for bubbles (Lux, 2003).

It must be admitted that the bootstrap technique applied in the Section 1 may not allow for the elimination of the bias and the resulting estimators are slightly downward biased. This possible failure to fully eliminate bias is unambiguously due to small sample. This finding agrees with those of Huisman et al, which confirm that conventional Hill estimator when obtained by applying the technique of choosing *k* by minimization of MSE overestimates the fatness of tails in comparison to the modified one (WLS based) in small samples (Huisman et al, 2001, p. 212). So, the estimate of the current section seems to be more reliable. But what is more surprising, it is very close to the estimates obtained by simple visual inspection of probability density function plot (Figures3, 3.A). The value of $\hat{\alpha} = 2.8439$ is closest to the value of $\hat{\alpha} = 2.684078$ form Table 4 with *k*=11. So, I conclude that the first difference of ITS time series has the tails close to those of Student-t distribution with 2 to 3 degrees of freedom (the theoretical Hill Index for Student-t distribution equals the number of degrees of freedom of the distribution (Pictet et al, 1998)). Thus, the probability mass is indicative of comparatively fat tails in the distribution underlying ITS time series which leaves the possibility for the existence of abnormal deviations in housing prices but does not allow for rational bubbles as defined by Blanchard and Watson (1982) in the data.

Chapter 3

ECONOMETRIC MODEL FOR DETECTION OF RATIONAL BUBBLES IN RESIDENTIAL HOUSING PRICES

In the previous two sections a more elementary approach has been exercised which did not allow to detect rational bubbles in the returns for holding residential property but did allow to detect fat tails in the underlying ITS time series. This section will differ in two aspects: first, a more sophisticated econometric analysis will be applied; second, prices in levels, not in first differences, will be analyzed in combination with rents on holding residential property. These differences arise from the inappropriateness of the methods of the previous two sections when applied to non-stationary time series (ITS in levels, see Picture 1) due to stationarity of the underlying time series implied by the Hill index estimates (Hall, 1990).

As it was mentioned in the literature review, the model of rational bubbles a la Blanchard and Watson gave rise to three areas of its implementation: securities markets bubbles, exchange rate bubbles and price level bubbles. The models which are developed for these three areas are frequently interchangeable since they stem from one common model and they may be extended for other areas in a straightforward way, provided the structure of pricing assets is similar to that of the three areas mentioned above. So, in what follows, an as close as possible replication of the procedure applied by Chan et al (2001) will be applied. In their paper Chan et al detect price bubbles in Hong-Kong residential housing market with the extensive use of the techniques originally developed for analyzing Cagan hyperinflation model (Durlauf and Hooker, 1994).

In the paper by Chan et al (2001) property is treated as an investment the current price of which is determined as the present value of expected current rate and expected next period price:

$P_t=\delta E(D_t+P_{t+1}|\Omega_t)$ (21)

here P_t denotes real current value of property which is obtained by discounting rationally expected (E) rent D_t and next period price P_{t+1} conditional on the currently available information set Ω_t at ex ante constant real rate δ.

By recursively substituting for $E(P_{t+1+i}|\Omega_{t+1})$ into (21), using the law of iterated expectations and imposing transversality condition $\lim_{i\to\infty}\delta^i$ $E(P_{t+1-i}|\Omega_{t+1})=0$ on (21) the market fundamental solution for P_t is obtained:

$$P_t^f=\sum_{i=0}^{t}\delta^i E(D_{t+1}\mid\Omega_t) \quad (22)$$

If the actual price is consistent with the market fundamental solution the market fundamental hypothesis can not be rejected. The hypothesis however can be rejected if some deviations from the fundamental price occur. These deviations can be explicitly incorporated into model specification. Thus, a random process

$B_{t+1}=\delta^{-1}B_t+e_{t+1}$ (23)

if added to the fundamental solution of (22) will still solve equation (21) because fundamental solution is only particular solution to the problem (Chan et al, 2001, p.63). In (23) B_t stands for rational bubble provided that $\delta^{-1}>1$, and e_{t+1} stands for bubble innovation term which is assumed to be orthogonal to the information set Ω_t.

Thus, in addition to particular market fundamental solution to equation (21) the general solution can be added:

$P_t^g=P_t^f+B_t$ (24)

Up to this moment the exposition was well in line with that of Blanchard and Watson (1982). A novel term to their set up is the unobserved component of the price levels S_t, which denotes misspecification error. This misspecification term denotes further deviations of actual price P_t from fundamental solution P_t^g and allows explicitly representing the actual price as the sum of the three components:

$P_t=P_t^f+B_t+S_t$ (25)

where the last two terms B_t and S_t denote the total model noise.

Durlauf and Hall (1989) and Durlauf and Hooker (1994) developed the technique of noise extraction via implementation of flow and stock tests.

For carrying out the flow test let us consider the perfect foresight fundamental price (see (22)):

$$P_t^* = \sum_{i=0}^{t} \delta^i D_{t+1} \tag{26}$$

$E(P_t^* | \Omega_t) = P_t^f$ and since P_t^f is orthogonal to Ω_t by construction, so is P_t^* and the stochastic term by which they will differ (ν_t-rational expectation forecast error) thus is also orthogonal to the information set Ω_t and

$$P_t^* = P_t^f + \nu_t \tag{27}$$

Expressing P_t^f from (27) and substituting it into (25) yields:

$$P_t - P_t^* = B_t + S_t - \nu_t \tag{28}$$

The quasi difference $\varphi(P_t - P_t^*)$ of (28) is taken in order to eliminate the predictable part of the bubble component (B_t), where $\varphi = -(1-\delta L^{-1})$. And the quasi difference is expressed as follows:

$$r_{t+1} = \varphi(P_t - P_t^*) \tag{29}$$

By rearrangement the following expressions can be obtained:

$$r_{t+1} = \varphi(S_t - \nu_t) + \delta e_{t+1} \tag{30}$$

$$r_{t+1} = \delta P_{t+1} - P_t + D_t \tag{31}$$

Due to expression (31) r_{t+1} is known as the excess return on holding property.

Now, since the information on r_{t+1} is available (we have average prices (ITS) and average rents (D_t) and δ will be calculated a bit later), the flow test can be conducted. Let $L_t(x)$ denote the information set available at time *t* and be a subset of Ω_t. By projecting r_{t+1} onto $L_t(x)$ we can capture the fitted value of $\varphi(S_t)$. This becomes possible because ν_t and e_{t+1} of (31) are orthogonal to $L_t(x)$ by construction. Thus, if the projection of r_{t+1} onto $L_t(x)$ is not zero we can claim that specification error is present in the data. Putting it in other words, failing to reject the null hypothesis of zero projection of excess return on holding property to the information set signals about the presence of specification error S_t.

The last thing to be done before the implementation of the flow test is obtaining δ, which is ex ante constant discount factor. Due to the use of orthogonality conditions in constructing the model this is naturally done via using GMM and imposing the following orthogonality condition:

$E[L_t(x)'r_{t+1}(\delta)]=0$.

The results are presented in the Table 7 below.

Dependent Variable: Implicit Equation
Method: Generalized Method of Moments
Date: 05/15/05 Time: 21:01
Sample(adjusted): 1/04/2002 2/18/2005
Included observations: 164 after adjusting endpoints
Kernel: Bartlett, Bandwidth: Fixed (4), Prewhitening
Simultaneous weighting matrix & coefficient iteration
Convergence achieved after: 6 weight matrices, 7 total coef iterations
C(1)*PRICE(+1)-PRICE+DIV
Instrument list: PRICE DIV

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.887058	0.001254	707.5187	0.0000
Mean dependent var	0.000000	S.D. dependent var		0.000000
S.E. of regression	9.632724	Sum squared resid		15124.67
Durbin-Watson stat	1.759282	J-statistic		0.068242

Table 7

In Table 7 $L_t(x)$ is represented by the current values of housing prices (represented by ITS index) and rents (both in levels) which are used as instruments. And $r_{t+1}(\delta)$ is simply the RHS of (31) where C(1) stands for the δ which is to be estimated.

The procedure behind the estimation is designed so as to minimize the correlation between $L_t(x)$ and r_{t+1} as much as possible which is alternative to varying δ so as to minimize J statistic (bottom right corner of the Table) (Hansen, 1982). In line with Chan et al (2001) an automatic bandwidth selector has been chosen (Newey and West, 1994). The prewightening procedure has been used before applying GMM to reduce autocorrelations in the parameters.

The estimated orthogonal constant discount factor δ is indeed a very plausible assumption provided that the real interest rate on national currency term deposits in Belarus has been constant starting from the year 2001 (NBB, 2004). A more controversial result is that for the period of ITS index calculation real rate of return on residential housing market appears to be 12.7% per week which is abnormally high if compared to money deposit rate of 0.7% per month (NBB, 2004). However, the underlying procedure aims at finding orthogonal δ and not at estimating the existing real rate if return.

To test the appropriateness of over-identifying restrictions for GMM estimation J-test is applied (Hansen, 1982). Under the hypothesis that the restrictions are satisfied multiplication of J-statistics by the number of usable

Elements of $L_t(x)$	OLS estimates of slope coefficients		Standard errors		Wald test for orthogonality (the slope coefficients are jointly equal to 0)
ΔP_t, ΔD_t	-0.351011	0.546010	0.084786	0.115447	18.80331
ΔP_t, ΔD_t,	-0.393881	0.703875	0.085477	0.163802	11.75149
ΔP_{t-1}, ΔD_{t-1}	-0.198867	0.265182	0.084913	0.162735	
ΔP_t, ΔD_t,	-0.387237	0.742866	0.085343	0.165421	8.659480
ΔP_{t-1}, ΔD_{t-1},	-0.226711	0.368557	0.086349	0.212940	
ΔP_{t-2}, ΔD_{t-2}	-0.160331	0.142334	0.084725	0.164183	
ΔP_t, ΔD_t,	-0.383229	0.768681	0.081818	0.172229	6.432158
ΔP_{t-1}, ΔD_{t-1},	-0.221102	0.394837	0.084150	0.222486	
ΔP_{t-2}, ΔD_{t-2},	-0.170208	0.138970	0.083905	0.218818	
ΔP_{t-3}, ΔD_{t-3}	-0.060862	-0.001599	0.084389	0.176132	

Table 8

observations (N) is asymptotically χ^2 with degrees of freedom equal the number of the over-identifying restrictions. For the specification of Table 7 the value J*N constituted approximately 11 with *p*-value equal to 0.010733, which does not support the J-test. However, increasing the number of usable observations does not change the estimated value of δ by much but only results in higher J value.

After the estimation of δ it becomes possible to construct r_{t+1} and conduct the flow test. In order to construct the information set we check the orthogonality of r_{t+1} to the information set which in line with Durlauf and Hall (1989) is taken to be the first; first and second; first, second and third; and first, second, third and fourth differences of D_t and P_t (here ITS stands for P_t). The results are presented in Table 8 above. Here it should be mentioned that fro the estimation the sub series of 143 values has been taken (up to October 8, 2004) in order to retain the last 21 observations for the construction of perfect foresight price of stock test below.

For all four information sets the orthogonality hypothesis is rejected at high level of significance. Thus, the flow test provides us with the evidence of that specification error is present in the data. But in order to obtain the full evidence we also need to conduct the stock test.

After conducting the flow test and detecting specification error, it is now necessary to conduct the stock test which is designed so as to make inference about the presence of bubble. The stock test is based on equation (28). The logic behind the test is as follows: if projection of P_t- P_t^* on the information set is zero, then neither a specification error nor a bubble is contained in the time series (this is the null hypothesis); failing to accept the null hypothesis implies that the total model noise created by the fitted values of specification error and bubble is present in the data.

Unfortunately, we cannot observe P_t^*, but it is possible to construct the empirical analog of its values by imposing certain conditions on the infinite sum in (26). This approximation method was proposed by Shiller (1981). And the resulting observable analog of P_t^* is:

$$P_t^{'} = \sum_{i=0}^{T-t-1} \delta^i D_{t+i} + \delta^{T-t} P_T \qquad (32)$$

The most critical aspect about the above expression is the condition that the terminal value of P_T should not contain a bubble because in this case the bubble will be canceled out by P_t' and the stock projection will not allow to detect it (Flood at al. 1986).

In some papers the price values of post-bubble period are taken (Hansen and Sargent, 1999). In the present paper there is no luxury of obtaining such values because the housing prices under inspection have not fallen dramatically by the present moment. On the other hand, taking values which are assumed not to contain bubbles put additional restrictions on the model. So, we use the last observation of the data in line with Chan et al (2001) but instead of weekly prices a quarterly price is taken as a terminal value. As a result, the actual projection on $L_t(x)$ is P_t- P_t'. And the estimation outputs are presented in the table below:

Elements of $L_t(x)$	OLS estimates of slope coefficients		Standard errors		Wald test for orthogonality (the slope coefficients are jointly equal to 0)
ΔP_t, ΔD_t	0.624426	0.044898	0.592621	0.109242	4080.973
ΔP_t, ΔD_t,	1.002448	-0.122632	1.043550	0.707366	2523.869
ΔP_{t-1}, ΔD_{t-1}	0.682733	-0.069913	1.062315	0.696509	
ΔP_t, ΔD_t,	3.089963	-0.970640	1.128492	0.653997	2489.955
ΔP_{t-1}, ΔD_{t-1},	1.444609	-1.468862	0.926161	0.911962	
ΔP_{t-2}, ΔD_{t-2}	1.100820	-0.583285	0.922211	0.627844	
ΔP_t, ΔD_t,	2.586702	-0.798914	2.459332	-1.293493	2399.215
ΔP_{t-1}, ΔD_{t-1},	2.792074	-1.780277	2.729542	-2.180552	
ΔP_{t-2}, ΔD_{t-2},	1.460316	-1.417255	1.769175	-1.761500	
ΔP_{t-3}, ΔD_{t-3}	0.361693	-0.355872	0.413553	-0.632581	

Table 9

As we can see from Table 9, the null hypothesis is rejected for all four information sets.

To sum up, there have been three possible inferences based on the flow and stock tests above:

1) If we failed to reject the flow test hypothesis and rejected the stock test hypothesis then the rational bubble would be detected in the data;
2) If we failed to reject both stock and flow test hypothesis, then neither specification error nor bubble would be detected in the data and fundamental solution would be followed by the data;
3) If both stock and flow test nulls are rejected (which is our case), then we can state that specification error is surely present in the data, but the existence of bubbles needs further inspection.

Thus, if there is bubble term which is in charge for the rejection of the null, than some degree of nonstationarity is to be present in the series P_t- $P_t^{'}$ (Durlauf and Hall, 1994). The following results of the augmented Dickey Fuller Unit Root Test present a mixed evidence of non stationarity:

ADF Test Statistic	-3.533124	1% Critical Value*	-4.5348
		5% Critical Value	-3.6746
		10% Critical Value	-3.2762

*MacKinnon critical values for rejection of hypothesis of a unit root.

Table 10

As we can see from the Table, the null is rejected somewhere on the half-way between 5% and 10% level of significance. So, we can conclude that there is some weak possibility of nonstationarity. In order to get an even deeper insight we examine the ΔP_t- $\Delta P_t^{'}$ for nonstationarity. The results are presented in Table 14:

ADF Test Statistic	-5.990917	1%	Critical Value*	-3.8572
		5%	Critical Value	-3.0400
		10%	Critical Value	-2.6608

*MacKinnon critical values for rejection of hypothesis of a unit root.

Table 11

The overall conclusion is that there might be some form of nonstationarity in the P_t- $P_t^{'}$ series, but not in the ΔP_t- $\Delta P_t^{'}$ series. So, if any bubble solution is present to the model, it is non linear one and does not correspond to the solution assumed in (23).

Summary and Conclusions

The current thesis revealed the difficulties connected with the implementation of bubble tests for the purposes of empirical research. Thus, econometric approach heavily relies on the assumptions which stipulate the resulting inference. While the statistic approach provides much more clear evidence of deviations in data distribution but puts restrictions on the minimum size of the data set. In any case, the research conducted above allowed revealing the abnormal behavior of the residential housing prices in their extreme realizations (fat tails). This allows concluding on their explosiveness and increased volatility. This conclusion provides the possibility of strong jumps in prices in the future. If several such jumps will have downward direction during comparatively prolong time period, there will be possible to call the phenomenon a price bubble. Rational price bubbles a la Blanchard and Watson have not been revealed in the data, but this is only one of many more possible specifications of bubbles available. Thus, if the bubbles are redefined as some degree of explosiveness in the data we can not reject their presence (here explosiveness is defined as increased probability of extreme realizations of the data).

As for the influence of such extreme realizations on GDP, it is unambiguously possible that the bursting bubble may considerably dampen the real growth but the exact mechanism of such influence may be a good direction for further research. Finally, the abnormal behavior of prices in the case of Belarusian economy may be caused not by rational expectations but for example by increased difficulty to enter such a prosperous market as that of residential housing and its increased concentration which also may lead to unstable path of pices.

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Appendix 1

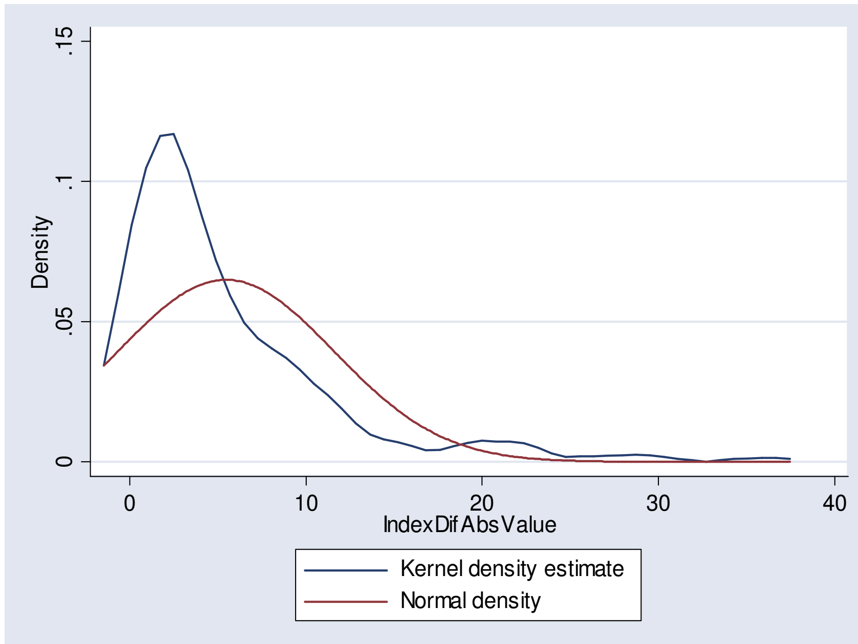

Figure 3.A

```
[bootstat,bootsam] = bootstrp(100,'hill1',clipboarddata);
meanbootstat=mean(bootstat);
bootstat1=meanbootstat';
[Answer]=hill2(164,154,bootstat1)
```

, where **hill1** is a simple function sorting bootstrap resamples:

```
function [c]= hill1(b)
c=sort(b);
end;
```

hill2 is a self made function on the basis of equations (13, 13.1, 13.2):

```
function [Answer]= hill2(n,n1,N)
r1=1;i=1;j=1;II=(1:n1-1);I1=(II'*II)*(1/n)^2*n*(n-1);
while i<=n1-1;r1<=n1-1;j<=n1-1;IIII(i)=((n1/n)*1/II(i));A8=diag(IIII);
A2=(2*ones(n1-1)-diag(ones(size(1:n1-1))));
J(i)=betainc(1-i/n1,n1-r1,r1-1);J1(i)=betainc(1-i/n1,n1-r1,r1);
I(i)=betainc(1-i/n1,n1-r1,r1); A4(i)=log(N(i)/N(i+1));
III(i)=(1/II(i))^2;
j=j+1; i=i+1; r1=r1+1; end;

A9=A4'*I;A10=II*A9';A11=A8*A10';
A6=triu(A4'*A4), A3=I1*J(1:n1-1)'+diag(II)*J1(1:n1-1)'*n1*(1/n), A7=diag(III)*((triu(A2)*A3)'*A6)',

k=1;C=min(A7-2*0.476861488*A11); while (A7(k)-2*0.476861488*A11(k))>C; Answer=k, k=k+1;end;
end;
```