FIRMS' HETEROGENEITY MONOPOLISTIC COMPETITION IN DIFFERENTIATED PRODUCTS AND INTERNATIONAL TRADE: A GENERALIZED MODEL

by

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Abstract

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The intent of the paper is to construct the model of costly international trade in differentiated products and perform the welfare analysis in its framework to investigate whether Ukraine will gain at all stages of a reduction in trade barriers, or are there potential problems along this way. It was shown that a two-way trade barriers results in continuous welfare improvement even for the country with higher cost industry such as Ukraine even though owners of the firms operating in the monopolistically competitive sector of the less efficient country suffer losses from trade liberalization.

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Chapter 1

INTRODUCTION

The intent of the new Ukrainian government is to sign a trade agreement with European Union (EU) as envisioned by the concept of the "EU neighbourhood." This draws attention to the potential benefits from trade liberalization. Many elements enter into the determination of whether it would be beneficial for Ukraine to enter into such an agreement with the EU and put more effort in acceleration of WTO accession. This thesis extends international trade theory to explore a crucial dimension of the trade liberalization issue.

International trade theory based on comparative advantage and perfect competition suggests that countries receive gains from trade. There are, however, winners and losers from trade liberalization. Yet comparative advantage arising from relative differences in factor endowments and technology cannot fully explain international trade relationships between countries. The importance of intra-industry trade based on scale economies and imperfect competition suggests that world has become more complicated. Due to the inefficiencies of imperfect competition it is possible for a country to experience losses in moving from autarky to free trade and from limited mutual trade to trade liberalization. Indeed, in the countries in transition one often hears the argument that a country as the whole may lose from trade liberalization since its firms are not sufficiently strong to engage in the competition with foreign ones. The analysis of Montagna [2001] contradicts this statement. She shows that it is more efficient country that should worry about a potential reduction in national welfare as a consequence of free trade because its consumer price index tends to move rather up than down. My analysis is able to extend this result by considering incremental reductions in trade barriers from initial trade equilibrium. In both analyses, the owners of monopolistically competitive firms in the less efficient country constitute vested interests that are adversely affected by a further opening to trade, but, the country as a whole typically gains. At least in the long run context, therefore, Ukraine's national interests may be well served by trade liberalization with more efficient entities such as EU.

The intent of this thesis is to construct a model of costly international trade in differentiated products and perform a welfare analysis in its framework. The key question is to investigate whether the theoretical model of intra-industry trade suggests that Ukraine will gain at all stages of a reduction in (two-way) trade barriers, or determine if there are any potential problems along the way.

To be able to conduct the analysis, I will start from the model proposed by Montagna [2001] that assumes technical and therefore cost heterogeneity among firms. While Montagna allows for different intensity of preferences for variety, this thesis will revert to the standard love of variety approch. Iceberg-type transportation costs will be introduced, which allows a complete analysis of the reduction in transport costs rather than the movement from the autarky to free trade. From the conventional trade theory we know that free trade is better than autarky. This is not very helpful or reassuring, however. Ukraine is not at autarky and even if it entered the EU some (true transport cost) barriers would remain. Therefore, the model including non-prohibitive barriers to trade will be much more instructive than simple comparison of two extreme cases.

The justification of the chosen theoretical framework is the following. In the case of Ukraine, the former command system did not offer much variety. While there have been important increases in the availability of different varieties, the transition has been difficult and average incomes remain low, which continues to restrict access to a wide variety. Overall, there is a strong argument that trade-induced increases in variety would be beneficial. The heterogeneity across firms introduced by Montagna (2001), however, is of utmost importance. There is a strong presumption that in the EU and other advanced economies, the strongest firms have much lower cost than Ukrainian. Therefore, it will be useful to retain heterogeneous firms as in Montagna [2001], but return to the standard love of variety approach that uses a simple CES utility. Further, I intent to model transportation costs rather than tariffs to avoid the unimportant complications associated with tariff revenues.

Therefore, the contribution of the current research involves depiction of the intermediate cases of limited trade and adjustments in two trading countries during the partial reduction of the trade barriers instead of just comparison of the two limiting cases of pure free trade and autarky which is quite common in the field of international trade.

Henceforth, the organization of the paper is the following: in the next chapter I consider the development of theoretical views on the determination of the trade pattern and distribution of welfare gains from trade among the trading partners. In Chapter 3, I develop the basic model similar to set up proposed by Montagna [2001] but I include transportation costs and use the standard assumption about preferences for variety. The subsequent two chapters will represent the main implications from the general equilibrium model, and Chapter 6 will deliver the important conclusions of the thesis.

Chapter 2

LITERATURE REVIEW

The early developments of the "pure" as opposed to monetary theory of international trade was associated with such issues as determination of the pattern of inter-industry trade, factor price equalization and impact of the trade policy on the equilibrium (Bhagwati [1964]). The most famous models in this theoretical framework were the Ricardian and Hecksher-Ohlin models. These approaches to determination of the pattern of international trade in two goods between two countries were later extended to the case of many goods and many countries (Bhagwati [1964]). As a result of difficulties and controversies with the empirical verification of both models in conjunction with the huge growth of trade in similar goods between the industrialized countries after the World War II (Grubel [1967]), have stimulated further lines of research in this field.

In reality countries simultaneously imported and exported products of the same industries[1]. However, the conventional theory of comparative cost advantage was not good at predicting and explaining such patterns of intra-industry trade. In case of increasing returns and monopolistic competition in differentiated products, it was proved that intra-industry trade among the countries with similar levels of economic development actually could take place, and welfare could increase due to lower prices and the greater variety of products available on the market (Grubel [1976]).

Rapid progress in the explanation of intra-industry trade under increasing returns to scale was not possible until the late 1970s when Dixit and Stiglitz

[1] Here the term "industry" is used in its broad meaning.

[1977] provided a general equilibrium framework for analysing monopolistic competition that

- allowed modelling of international trade, and particularly intra-industry trade, in manufactured products among developed countries (Krugman [1980]);

- enabled researchers to explain the existence of trade in intermediate products (Ethier [1982]);

- facilitated trading block analysis (Frankel, Stein and Wei [1993]) as well as the analysis of multinationals (Markusen and Venables [1996]).

The applications of the Dixit and Stiglitz findings in the field of International Trade appeared almost immediately (Krugman [1980], Krugman [1981], Helpman [1981], Lancaster [1980], Ethier [1982]). The Chamberlinean model of monopolistic competition even received an application in the modelling of endogenous growth (Romer [1987]) and real business cycles (Chatterjee and Cooper [1993]), notwithstanding that initially Dixit and Stiglitz were concerned with the questions of industrial organization (namely, social optimality of the Chamberlinean equilibrium).

The main stress here, actually, must be made on "general equilibrium analysis," which became possible in the new setup. In contrast to the partial equilibrium approach, it helped researcher define how pattern of international trade was determined, describe the welfare implications of policy measures on the equilibrium, and how that equilibrium changed if the technology or consumers' preference altered.

As a result, monopolistic competition and IRS were shown to be a source of international trade, and the trade was welfare-improving since consumers gained from the wider variety of products (Krugman [1980]). However, openness to international trade left production pattern unaffected even after transportation costs were introduced to the Krugman's model. The only impact of costs of transportation was the possibility of wage inequality across trading partners. It is worth noting that, generally, factor price equalization is a standard simplifying (but at the same time quite restrictive) feature of models of international trade under monopolistic competition, since once trade becomes costly, the factor price equalization set turns out to be of reduced dimension. In the latter case producers become more oriented towards "home" markets and concerned more with the domestic supply of production factors (Markusen and Venables [1996]).

Nevertheless, introduction of the transportation costs into the model allowed analysis of the impact of country size on the volume and direction of international trade and the demonstration of the so called "home market effect": *ceteris paribus*, a larger country becomes a net exporter of the product for which it has higher domestic demand by merit of its size (Krugman [1980]). In such a case, the country size was proxied by its population only. Later, it was shown that geography also matters in the determination of the direction of trade and for the distribution of gains from trade (Tharakan and Thisse [2002]). Geographical size modified the trade pattern: trade went from a small country to a large one. The impact of trade also differed: a small country always benefited from free trade, but a large country lost contrary to Krugman's findings. The reason was that positive impact of differentiated product price reduction on consumers' welfare was not compensated for the loss of the part of the domestic market by national producers in the large country (Tharakan and Thisse [2002]). Krugman [1980] did not take into account that the production pattern also changed along with overall price index when countries began to trade.

Later works, however, paid closer attention to the determination of the production pattern that resulted from engaging in international trade

(Helpman [1981], Gabszewicz *et al* [1981], Lawrence and Spiller [1983], Markusen and Venables [1996]).

Partial equilibrium analysis by Gabszewicz *et al* [1981] showed that free trade lowers prices of "higher quality goods" to such a level that some "lower quality goods" were driven out of the market, thus reducing product variety. More complicated analysis by Lawrence and Spiller [1983] integrated the Hecksher-Ohlin approach with monopolistic competition in differentiated products, which was important for further development of international trade theory. By doing this, researchers avoided the result that firm's size "was fixed" by existing consumers' preferences and available technology and all adjustments in the output of a monopolistically competitive industry came from the changes in the number of firms (Neary [2000]). This was done by introducing "differences in factor proportions between fixed costs and variable costs" into the model (Neary [2000]) as well as reintroducing the conventional assumption about different national factor endowments. As a result, the capital-abundant country had larger firm size in monopolistically competitive sector whose production was considered as capital intensive relative to the goods produced in a perfectly competitive sector. The capital-abundant country also received a higher concentration in the monopolistically competitive sector compared with the same industry in its labour-abundant trading partner. In addition, price of differentiated products in the capital-abundant country was generally lower. It began to specialize in production of differentiated products becoming a net exporter of them. Its trading counterpart produced a greater volume of the homogeneous product in the competitive sector and therefore exported it to the capital-abundant country which would have faced a higher price for this product in the absence of trade. The latter result is not very surprising. The more important result is that the overall number of varieties produced in the monopolistically competitive sector by both countries remained unchanged but their production was reallocated due to free trade. Gains from international trade were positive for both countries, but the one that had smaller size benefited more (Lawrence and Spiller [1983]).

After that the developments in international trade proceeded in a number of directions:

- This setup was modified to analyze specific questions concerning international trade in intermediate products (Ethier [1982]), growing multinational corporation activity (Helpman [1984], Markusen and Venables [1996]), and trading block analysis (Frankel, Stein and Wei [1993]).

- Limitations of monopolistic competition framework were considered (Chao and Takayama [1990]) along with its empirical verification (Hummels and Levinson [1995]).

- Attempts were made to model oligopolistic markets under free trade (Roberts and Sonnenschein [1977], Brander [1981], Markusen [1981], Krugman [1981], Anderson, Donsimoni and Gabszewicz [1989], Bohm [1994], Dierker and Grodal [1999]);

- Efforts were made to relax some stringent assumptions of the conventional models with monopolistic competition (Neary [2000], Sebastien [2000], Montagna [2001], Melitz [2002], Yeaple [2005]).

Since the latter two themes are important to this thesis, they are considered in more detail below.

After introduction of the elements of the Hecksher-Ohlin approach in models with monopolistic competition, trade in products of the same industry was proved to have a factor endowment basis. Intra-industry trade in manufactured products is a complement to international factor movements, in contrast with inter-industry trade, which is essentially substitute for factor movements (Ethier [1982], Lawrence and Spiller [1983]).

However useful, the framework provided by Dixit and Stiglitz had many limitations (Neary [2000]). Extensive literature emerged attempting to relax some stringent assumptions. I would like to outline some of those. Benassy [1996] proposed to separate the parameter describing consumers' preferences for product variety from the one that measures the degree of substitutability between the differentiated products and firms' marker power. That enabled Montagna [2001] and Sebastien [2000] to consider the cases when consumers did not value wide variety of products available in the market *per se* but nevertheless perceived them to be close substitutes for each other.

Concerns were outlined briefly earlier with respect to the constant elasticity of substitution utility function, which leads to fixed size of firms,. The other problem was connected with the assumption of free entry that did not allow for sunk costs, learning-by-doing or restricted supply of entrepreneurial skills needed to enter the marker (Neary [2000]). Depending on the context, we still can neglect some of these issues as there are tradeoffs between the complexities of the model such that it captures real life phenomena and its tractability.

One of the most important limitations of conventional models of monopolistic competition and international trade is the assumptions of the homogeneity of the firms operating in the market. In reality, firms differ in their costs of production, market shares, and choices to export or not. Attempts to relax such stringent assumption started with Venables [1987]. He considered firms that were homogenous with respect to cost structure inside the home country, but that have different shares on the domestic and foreign markets. Thereafter many researchers have explored heterogeneity among firms in models of monopolistic competition (Van Long and Soubeyran [1997], Sebastien [2000], Montagna [2001], Melitz [2002], Yeaple [2005]). Under the assumption of heterogeneity, Van Long and Soubeyran [1997] supported a policy of setting an export subsidy or tax depending on concavity or convexity of the demand curve. Such a policy allows shifting domestic concentration in favour of the most efficient domestic firms and, as a result, a country implementing such policy gains from increased production efficiency. Thus, it was possible to achieve higher aggregate profits providing that the total revenue did not fall significantly.

Montagna [2001] considered the case when there was efficiency gap between the countries that traded with each other. After these countries opened to trade, the unification of the competitive conditions for both domestic and foreign firms took place lowering the efficiency requirements for enterprises operating in the more efficient country. Allowing less efficient firms to operate in the market of a more efficient country led to higher costs of production. Hence, notwithstanding that the consumers gained from increased product variety, the verdict was that "the results casts doubt on the efficiency of trade liberalization in generating welfare gains based on its rationalizing effects on industries." However, technological spillover effects that, once included in this theoretical framework, may lead to the opposite conclusion.

The investigation of the impact of trade on the dispersion of the knowledge among the different countries has received much attention in the modern literature. A detailed survey of the theoretical developments in that field is presented in Gong and Keller [2003]. These authors concluded that "productivity also increases due to learning through the interaction between foreign and domestic firms". "The greater importance of adoption of the technology from abroad – versus domestic technical change – in less developed compared to more developed countries suggests that learning through the international economic activity might be particularly important for less developed countries"(Gong and Keller [2003]).

International trade in final products is one of the main channels of technological spillovers along with foreign direct investment, imports of intermediate goods and purchases from multinational subsidiaries (Grossman and Helpman [1991], Coe and Helpman [1995], Eaton and Kortum [1996], Ben-David [1996], Keller [1998], Connoly [2003]). Nocco [2005][2] also emphasized that learning through knowledge spillovers was possible only if domestic firms in less productive countries had a significant opportunity to interact with their counterparts in more developed countries, which was possible if transportation costs and other natural and artificial barriers were not very high. Otherwise, instead of symmetric equilibria, agglomeration in the more productive country would occur. The latter result may persist even in the case of non-prohibitive trade obstacles if the technological gap between the countries is significant.

Returning to the question of incorporating heterogeneous firms into models of international trade, we proceed to consider the latest developments in the field of study. Melitz [2002] went further, than Montagna [2001], giving only some firms export status. Exporting firms incurred additional costs if they chose to export. As a result of partial equilibrium analysis, he showed how international trade brought on resource reallocations among the firms in the monopolistically competitive industry. Thus, one more channel through which international trade affected industry composition was described. A crucial result of the opening to trade or just further trade liberalization was that unambiguous welfare gains to the country arose.

Perhaps, the most important critique was that the Dixit-Stiglitz monopolistic competition model resembled to large extent models of perfect competition since it did not allow domestic firms to have different degrees of market power. In other words, it was assumed that individual firm's pricing decision had no impact on overall price index (Neary [2000]). However, attempts to create a general equilibrium model with oligopolistic market structure were hindered by influential work of Roberts and Sonnenschein [1977]. It was shown that introduction of the assumption that firms behave non-

[2] This article is being published currently and is available online at www.sciencedirect.com

competitively (i.e. they are able to influence the price level) into general equilibrium framework did not guarantee existence of equilibrium/equilibria (even if technology and preferences satisfy all standard assumptions of such models). This happened since the reaction curves representing the profit maximization conditions were not required to be continuous by standard assumptions about technology and preferences. Introduction of the special requirements to insure the continuity of firm's response functions would have been too restrictive for such models.

So it seemed as if theoretical results obtained by Roberts and Sonnenschein [1977] had proved "the fundamental weakness of the theory of monopolistic competition". However, the existence of intra-industry trade in case of monopolistic market structure was later analysed under assumptions of both homogeneous and differentiated products (Brander [1981], Markusen [1981], Anderson, Donsimoni and Gabszewicz [1989], Neven and Phillips [1985]). Partial equilibrium analysis showed that while consumers enjoyed lower prices as a result of free trade, oligopolists may have had lower profits and, thus, opposed the trade liberalization. (Anderson, Donsimoni and Gabszewicz [1989]).

Brander [1981], Markusen [1981], Neven and Phillips [1985] showed that "cross-hauling" trade (i.e. bilateral trade in identical products) reduced the monopoly power of the oligopolistic/monopolistic producers thus increasing welfare in both countries. But the conventional argument that free trade was beneficial because of increased competition on the markets was true only when it provoked expansion in the initially monopolized production (Markusen [1981]). Otherwise only one trading partner benefited from trade.

Several authors paid attention to the problem of price normalization that arose in a general equilibrium framework with oligopolistic market structure. The choice of numeraire (i.e. price normalization rule) affected the equilibrium by means of changing the oligopolists' objective function (Bohm [1994], Dierker and Grodal [1999]). Of course, such dependency of equilibrium on price normalization rule is not economically valid.

Having described the literature on the subject, it appears that many interesting controversies remain in the field of international trade. And it seems that, at least partially, this controversy stems from the existence of tradeoffs between simplicity and tractability of the models and their ability to reflect the complexity of the real situation.

Chapter 3

THE BASIC MODEL OF INTERNATIONAL TRADE WITH NON-PROHIBITIVE TRADE BARRIERS

The basic model allows a description of international trade with non-prohibitive trade barriers such as transportation costs. While the setup is similar to Montagna [2001], it differs in that non-prohibitive trade barriers are included and a classical constant elasticity utility function is used.

To start with, consider a hypothetical world that consists of only two countries – home and foreign – that are similar in all respects except the technical efficiency with which they produce the goods. In each country there are two sectors or industries – one is perfectly competitive and the other is monopolistically competitive. The perfectly competitive sector produces homogeneous products using a constant-returns-to-scale (CRS) technology and the monopolistically competitive sector is characterized by the production of differentiated product varieties and employs an increasing-returns-to-scale (IRS) technology. The only factor of production is labour, which is assumed to be perfectly mobile across industries in each country and whose price is normalized to 1. I assume that each firm in the monopolistically competitive sector produces with different costs such that no two firms produce varieties with the same marginal and average costs. As was mentioned above, countries are identical in all other respects including size, consumer preferences, the distributions of efficiency levels with which firms in the monopolistically competitive sector operate, and the degrees of firm heterogeneity. The only difference is the existence of a gap between the costs of the most efficient firms in the monopolistically competitive sector in each country. This difference is called the efficiency gap.

The countries can trade with each other. For simplicity, it is assumed that homogeneous goods can be transferred between the countries costlessly, but trade in the differentiated products is costly.

I start with the exposition of the demand side of the economy.

3.1. Demand side of the economy

To describe the consumer preferences I assume for simplicity that all consumers are identical in their preferences. Thus it is sufficient to consider the preferences of a representative consumer who has a CES sub-utility function for differentiated goods that is nested within an overarching Cobb-Douglas utility function:

$$U_t^j = \left(A_t^j\right)^{1-\mu}\left(D_t^j\right)^{\mu}, \mu \in (0;1) \tag{3.1}$$

where A_t^j is consumption of a homogeneous product of the competitive industry in country *j; j =h, f.* The upper subscript denotes the location of the consumers. With respect to lower subscripts, the first letter *t* denotes the costly trade case, index *i* shows the number of firm that produces the product and the last subscript shows in which country this firm operates.

$$A_t^j = A_{tj}^j + A_{t(-j)}^j \tag{3.2}$$

where for country *j* A_{tj}^j is the total consumption of the homogeneous good produced domestically and $A_{t(-j)}^j$ is consumption of the homogeneous good produced abroad.

D_t^j is the consumption of a composite differentiated product of the monopolistically competitive industry in country *j*, which is defined as follows:

$$D_t^h = \left(\int_1^{N_{th}+1} \left(D_{tih}^h \right)^{\frac{\sigma-1}{\sigma}} di + \int_1^{N_{tf}+1} \left(D_{tif}^h \right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{3.3}$$

where D_{tij}^j is a differentiated product variety produced by firm *i* in country *j* and consumed by consumers in the same country and $D_{ti(-j)}^j$ is the domestic consumption of imported varieties. Further, σ denotes the elasticity of substitution between the varieties and is assumed to be greater than 1. N_{tj} denotes the total number of firms producing monopolistically competitive product varieties in country *j*.

The budget constraint of the representative consumer states that the total expenditures on consumption must be less than or equal to the sum of total wage income from selling labour to firms and total profits earned by firms including transportation revenues:

$$A_{tj}^j + A_{t(-j)}^j + P_t^j D_t^j \leq I_{tj} \tag{3.4}$$

where *I sub j* denotes the total income of consumer in country *j*. The price of the competitive sector product is normalized to 1 (the reason for such normalization will become clear in the next section of this chapter).

Following Krugman [1980], I assume iceberg type transportation costs where only a fraction of the good shipped by one country arrives in the other country. Let $\left(1 - \frac{1}{\tau}\right)$ be a fraction of the good that is lost in transit so that only fraction $\left(\frac{1}{\tau}\right)$ arrives. However, to avoid excessive complication, I assume that the homogeneous products can be traded costlessly and only trade in differentiated products is costly. Consumers in the home country, which is the more efficient one, face the following price index:

$$P_t^h = \left(\int_1^{N_{th}+1} (P_{tih})^{1-\sigma} di + \int_1^{N_{tf}+1} (\tau P_{tif})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}, \tau > 1 \tag{3.5}$$

Consequently, consumers in the foreign country, which is the less efficient, face the following price index:

$$P_t^f = \left(\int_1^{N_{th}+1} (\tau P_{tih})^{1-\sigma} di + \int_1^{N_{tf}+1} (P_{tif})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}, \tau > 1 \tag{3.6}$$

Let us consider the two-stage maximization problem of the representative consumer in the more efficient home country:

In the first stage a consumer solves:

$$\max_{A_{th}^h, A_{tf}^h, D_{th}} \left\{ \left(A_{th}^h + A_{tf}^h \right)^{1-\mu} \left(D_t^h \right)^{\mu} \right\}$$

$$s.t. A_{th}^h + A_{tf}^h + P_t^h D_t^h \leq I_{th} \tag{3.7}$$

where P_t^h is defined by formula (3.5) and the composite differentiated product is:

$$D_t^h = \left(\int_1^{N_{th}+1} \left(D_{tih}^h \right)^{\frac{o-1}{o}} di + \int_1^{N_{tf}+1} \left(D_{tif}^h \right)^{\frac{o-1}{o}} di \right)^{\frac{o}{o-1}} \tag{3.8}$$

Since the utility function is increasing in both homogeneous and composite differentiated products, I can replace the inequality sign in the above budget constraint with the equality one.

As a result we get the following solution:

$$A_t^{h*} = \left(A_{th}^h + A_{tf}^h\right)^* = (1-\mu)I_{th} \tag{3.9}$$

$$D_t^{h*} = \frac{\mu I_{th}}{P_t^h} \tag{3.10}$$

which is a standard result for Cobb-Douglas utility maximization problems. The consumer allocates fraction μ of total income to the consumption of the composite differentiated product variety and the rest is spent on the homogeneous product.

In the second stage of budgeting, we consider the problem:

$$\max_{\{D_{tih}^h, D_{tif}^h\}} \left\{ \left(\int_1^{N_{th}+1} \left(D_{tih}^h\right)^{\frac{\sigma-1}{\sigma}} di + \int_1^{N_{tf}+1} \left(D_{tif}^h\right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \right\}$$
$$s.t. \int_1^{N_{th}+1} P_{tih} D_{tih}^h di + \int_1^{N_{tf}+1} \tau P_{tif} D_{tif}^h di = P_t^h D_t^h = \mu I_{th} \tag{3.11}$$

In other words, the consumer solves the problem of the allocation of the fraction of income devoted to consumption of the composite differentiated product among the varieties of that product.

From this optimization problem we will get the demand for foreign and domestically produced differentiated product varieties. The demand of home country consumers for domestically produced differentiated product variety is

$$D_{tih}^h = D_t^h \left(\frac{P_{tih}}{P_t^h}\right)^{-\sigma} = \mu I_{th} \left(P_t^h\right)^{\sigma-1} P_{tih}^{-\sigma} \tag{3.12}$$

We obtain the conventional result that quantity demanded is inversely related to the price of the differentiated product variety and positively related to the fraction of income spent on the differentiated products. The demand for a differentiated product variety, however, depends positively on the total price index as well, because alterative varieties are substitutes.

The demand of home country consumers for a differentiated product variety produced in the less efficient foreign country is:

$$D_{tif}^{h} = D_{t}^{h}\left(\frac{\tau P_{tif}}{P_{t}^{h}}\right)^{-\sigma} = \mu I_{th}\left(P_{t}^{h}\right)^{\sigma-1}\tau^{-\sigma}P_{tif}^{-\sigma} \tag{3.13}$$

The same logic applies if we consider the two-stage maximization problem of a representative consumer in the less efficient foreign country.

As a result, we get the following solution:

$$A_{t}^{f*} = (1-\mu)I_{tf} \tag{3.14}$$

$$D_{t}^{f*} = \frac{\mu I_{tf}}{P_{t}^{f}} \tag{3.15}$$

In the second stage of budgeting, we obtain the demand of foreign country consumers for a differentiated product variety produced in their country:

$$D_{tif}^{f} = D_{t}^{f}\left(\frac{P_{tif}}{P_{t}^{f}}\right)^{-\sigma} = \mu I_{tf}\left(P_{t}^{f}\right)^{\sigma-1}P_{tif}^{-\sigma} \tag{3.16}$$

And the demand of foreign country consumers for a differentiated product variety produced in the more efficient home country is

$$D_{tih}^{f} = D_{tf}\left(\frac{\tau P_{tih}}{P_{t}^{f}}\right)^{-\sigma} = \mu I_{tf}\left(P_{t}^{f}\right)^{\sigma-1}\tau^{-\sigma}P_{tih}^{-\sigma} \tag{3.17}$$

Now, we will turn to the supply side of the economy.

3.2. Supply side of the economy

Having considered the demand side of the hypothetical economy, I proceed to the description of the supply side. As described above, each country consists of two sectors. The competitive sector produces the homogeneous product A_{tj} whose price is normalized to 1. Competitive equilibrium implies that price of homogeneous product is equal to the marginal cost of production. If we combine this fact with the assumption of CRS technology in the competitive sector in both countries, we get the equality between average cost of production and prices. I also assume that the only factor of production is labour. The production function in the competitive sector in both countries is the following:

$$A_{tj} = wL_{A_{tj}} \tag{3.18}$$

where $L_{A_{tj}}$ the demand for labour in the competitive sector in corresponding country.

$$C_{tij} = w(\alpha + \beta_{tij} Q_{tij}) \tag{3.19}$$

where C_{tij} denotes the unit costs of production for differentiated product variety *i* in country *j*; α represents fixed costs, which are same for each firm inside the country and equal across the two countries; and β_{tij} represents the firm-specific marginal cost of production. Further, *w* denotes wage. Given perfect labor mobility between competitive and monopolistically competitive sectors within each country (but not between countries), the wage is equalized across sectors and remains normalized to 1.

For simplicity I also assume that each firm will produce at least a single variety and, therefore, the number of firms in each country will be equal to the number of varieties produced.

Each firm in the monopolistically competitive sector is assumed to choose its price, P_{tij}, so as to solve its profit maximization problem. In equations (3.5) and (3.6) above we have defined the price indexes for the consumers in the two countries. Now we define the producers' price index for each country as:

$$P_{th} = \left(\int_{1}^{N_{th}+1} \left(P_{tih} \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{3.20}$$

and $$P_{tf} = \left(\int_{1}^{N_{tf}+1} \left(P_{tif} \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{3.21}$$

Therefore, the consumers' price index is a function of producers' price indexes:

$$P_t^h = \left(\left(P_{th} \right)^{1-\sigma} + \left(\tau P_{tf} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{3.22}$$

and $$P_t^f = \left(\left(P_{tf} \right)^{1-\sigma} + \left(\tau P_{th} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{3.23}$$

Correspondingly, the demand for differentiated product variety produced by firm *i* in the more efficient home country will be the sum of demand that comes from the representative domestic and foreign consumers:

$$D_{tih} = D_{tih}^h + \tau D_{tih}^f = \mu P_{tih}^{-\sigma} \left(I_{th} \left(P_t^h \right)^{\sigma-1} + I_{tf} \left(P_t^f \right)^{\sigma-1} \tau^{1-\sigma} \right) \tag{3.24}$$

Similarly, the demand for product variety *i* produced in the less efficient foreign country is:

$$D_{tif} = \tau D_{tif}^h + D_{tif}^f = \mu P_{tif}^{-\sigma} \left(I_{tf} \left(P_t^f \right)^{\sigma-1} + I_{th} \left(P_t^h \right)^{\sigma-1} \tau^{1-\sigma} \right) \tag{3.25}$$

Each firm in both countries solves the problem of profit maximization by choosing its price:

$$\max_{P_{tij}} \left\{ P_{tij} D_{tij} - C_{tij} D_{tij} \right\} \tag{3.26}$$

It can be easily shown that it implies the mark-up pricing rule:

$$P_{tih} = \frac{\sigma}{\sigma - 1} \beta_{tih} = \varpi \beta_{tih} \tag{3.27}$$

Consequently, we get the result that the price a producer in the monopolistically competitive sector charges is equal to marginal cost multiplied by constant mark-up that is greater than zero. I define constant mark-up as $\varpi \equiv \frac{\sigma}{\sigma - 1}$.

To go further it is necessary to introduce additional assumptions concerning both the distribution of firm-specific marginal costs and the free entry and zero-profit condition. These assumptions will make it possible to define the equilibrium number of firms on monopolistically competitive market in each country and aggregate firm data on output produced and profits.

Concerning firm-specific marginal costs, I follow Montagna [2001] and assume that:

- No two firms in the same country have identical marginal costs of production;

- All firms in each country can be ranked in ascending order in accordance with their marginal costs from the most efficient at i=1 to the least efficient at i=N sub tj +1 if it is assumed that marginal costs are given by:

$$\beta_{tij} = \phi_j i^{\delta} \tag{3.28}$$

where β_{tij} is the marginal cost of firm *i*, and δ describes the degree of heterogeneity. Under the above assumptions ϕ_j stands for the costs of the most efficient firm in each country, $(\phi_f - \phi_h)$>0 constitutes the efficiency gap between two countries. Assuming free entry we now have the situation then the last $(N_j + 1)$th firm has zero profits and the rest of the firms on the monopolistically competitive market earn non-negative profits.

$$\pi_{t(N_{tj}+1)j} = 0 \wedge P_{t(N_{tj}+1)j} = \varpi\phi_j(N_{tj}+1)^{\delta} \tag{3.29}$$

3.3. The equilibrium in monopolistically competitive market

Substitution of the equilibrium price from (3.27) into the overall demand function (3.24) gives us the equilibrium quantity of differentiated product variety produced by firm *i* in the home country:

$$Q_{tih} = \mu\varpi^{-\sigma}\beta_{tih}^{-\sigma}\left(I_{th}\left(P_t^h\right)^{\sigma-1} + I_{tf}\left(P_t^f\right)^{\sigma-1}\tau^{1-\sigma}\right) \tag{3.30}$$

In order to get the equilibrium total quantity of differentiated product variety produced in home country we aggregate equation (3.30) and receive the following result:

$$\begin{aligned} Q_{th} &= \left(\int_1^{N_{th}+1}\left(\mu\varpi^{-\sigma}\beta_{tih}^{-\sigma}\left(I_{th}\left(P_t^h\right)^{\sigma-1} + I_{tf}\left(P_t^f\right)^{\sigma-1}\tau^{1-\sigma}\right)^{\frac{\sigma-1}{\sigma}}\right)di\right)^{\frac{\sigma}{\sigma-1}} = \\ &= \mu\varpi^{-\sigma}\left(I_{th}\left(P_t^h\right)^{\sigma-1} + I_{tf}\left(P_t^f\right)^{\sigma-1}\tau^{1-\sigma}\right)\left(\int_1^{N_{th}+1}\left(\beta_{tih}^{-\sigma}\right)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{3.31}$$

Correspondingly, the equilibrium profit of firm *i* in the home country will be:

$$\pi_{tih} = \frac{1}{\sigma - 1} \beta_{tih}^{1-\sigma} \varpi^{-\sigma} \mu \left(I_{th} \left(P_t^h \right)^{\sigma-1} + I_{tf} \left(P_t^f \right)^{\sigma-1} \tau^{1-\sigma} \right) - \alpha \qquad (3.32)$$

And, the aggregate profit of all firms in the more efficient home country is:

$$\begin{aligned} \pi_{th} &= \int_1^{N_{th}+1} \left(\frac{1}{\sigma - 1} \beta_{tih}^{1-\sigma} \varpi^{-\sigma} \mu \left(I_{th} \left(P_t^h \right)^{\sigma-1} + I_{tf} \left(P_t^f \right)^{\sigma-1} \tau^{1-\sigma} \right) - \alpha \right) di = \\ &= \frac{1}{\sigma - 1} \varpi^{-\sigma} \mu \left(I_{th} \left(P_t^h \right)^{\sigma-1} + I_{tf} \left(P_t^f \right)^{\sigma-1} \tau^{1-\sigma} \right) \int_1^{N_{th}+1} \beta_{tih}^{1-\sigma} di - \alpha N_{th} \end{aligned} \qquad (3.33)$$

Aggregation of the producers' prices yields

$$P_{th} = \left(\int_1^{N_{th}+1} \left(\varpi \beta_{tih} \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \qquad (3.34)$$

which must be equal the producers' price index defined in (3.20).

The least efficient firm in the more efficient home country will be indexed by $i = N_{th} + 1$. Assuming that $\beta_{tih} = \varphi_h i^{\delta}$, we have stated zero-profit condition in (3.29).

Now I take the expression for the equilibrium profit (3.32) and rewrite it for the least efficient firm:

$$\pi_{t(N_{th}+1)h} = \frac{1}{\sigma - 1} \left(\phi_h \left(N_{th} + 1 \right)^{\delta} \right)^{1-\sigma} \varpi^{-\sigma} \mu \left(I_{th} \left(P_t^h \right)^{\sigma-1} + I_{tf} \left(P_t^f \right)^{\sigma-1} \tau^{1-\sigma} \right) - \alpha = 0$$

(3.35)

Similarly, for the least efficient foreign country the corresponding zero-profit condition will be:

$$\pi_{t(N_{tf}+1)f} = \frac{1}{\sigma - 1}\left(\phi_f\left(N_{tf}+1\right)^{\delta}\right)^{1-\sigma}\varpi^{-\sigma}\mu\left(I_{th}\left(P_t^h\right)^{\sigma-1}\tau^{1-\sigma} + I_{tf}\left(P_t^f\right)^{\sigma-1}\right) - \alpha = 0$$

(3.36)

Both of these expressions can be rewritten in terms of producers' price indexes:

$$\frac{1}{\sigma-1}\left(\phi_h\left(N_{th}+1\right)^{\delta}\right)^{1-\sigma}\varpi^{-\sigma}\mu\left(\frac{I_{th}}{\left(P_{th}\right)^{1-\sigma}+\left(\tau P_{tf}\right)^{1-\sigma}}+\frac{\tau^{1-\sigma}I_{tf}}{\left(P_{tf}\right)^{1-\sigma}+\left(\tau P_{th}\right)^{1-\sigma}}\right)-\alpha=0$$

(3.37)

$$\frac{1}{\sigma-1}\left(\phi_f\left(N_{tf}+1\right)^{\delta}\right)^{1-\sigma}\varpi^{-\sigma}\mu\left(\frac{I_{tf}}{\left(P_{tf}\right)^{1-\sigma}+\left(\tau P_{th}\right)^{1-\sigma}}+\frac{\tau^{1-\sigma}I_{th}}{\left(P_{th}\right)^{1-\sigma}+\left(\tau P_{tf}\right)^{1-\sigma}}\right)-\alpha=0$$

(3.38)

To these two equations we add the two aggregated producers' price indexes given by (3.20) and (3.21):

$$P_{th} = \left(\int_1^{N_{th}+1}\left(P_{tih}\right)^{1-\sigma}di\right)^{\frac{1}{1-\sigma}} = \left(\int_1^{N_{th}+1}\left(\varpi\phi_h i^{\delta}\right)^{1-\sigma}di\right)^{\frac{1}{1-\sigma}} = \varpi\phi_h\left(\frac{\left(N_{th}+1\right)^{\theta}-1}{\theta}\right)^{\frac{1}{1-\sigma}}$$

(3.39)

$$P_{tf} = \left(\int_1^{N_{tf}+1}\left(P_{tif}\right)^{1-\sigma}di\right)^{\frac{1}{1-\sigma}} = \varpi\phi_f\left(\frac{\left(N_{tf}+1\right)^{\theta}-1}{\theta}\right)^{\frac{1}{1-\sigma}} \qquad (3.40)$$

where $\theta = \delta(1-\sigma)+1$.

The system of equations (3.37)-(3.40) allows us to find unique solution for N_{th}, N_{tf}, P_{th}, and P_{tf}, which describe the equilibrium in case of a trade regime with the transportation costs.

Now, having equilibrium values for price indexes and equilibrium number of firms in the monopolistically competitive sector in each country, I am able to aggregate quantities of the differentiated product as well as profits across firms. Afterwards, it will be possible to examine the equilibrium in the competitive sector of the economy.

The total quantity of differentiated product offered in the monopolistically competitive sector in the home (more efficient) country is:

$$
\begin{aligned}
Q_{th} &= \mu\varpi^{-\sigma}\left(I_{th}\left(P_t^h\right)^{\sigma-1} + I_{tf}\left(P_t^f\right)^{\sigma-1}\tau^{1-\sigma}\right)\left(\int_1^{N_{th}+1}\left(\beta_{tih}^{-\sigma}\right)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}} = \\
&= \mu\varpi^{-\sigma}\left(I_{th}\left(P_t^h\right)^{\sigma-1} + I_{tf}\left(P_t^f\right)^{\sigma-1}\tau^{1-\sigma}\right)\left(\left(\frac{P_{th}}{\varpi}\right)^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}} = \\
&= \mu P_{th}^{-\sigma}\left(\frac{I_{th}}{\left(P_{th}\right)^{1-\sigma} + \left(\tau P_{tf}\right)^{1-\sigma}} + \frac{\tau^{1-\sigma}I_{tf}}{\left(P_{tf}\right)^{1-\sigma} + \left(\tau P_{th}\right)^{1-\sigma}}\right)
\end{aligned}
\tag{3.41}
$$

Aggregation of the firms' profits in the home country yields:

$$
\begin{aligned}
\pi_{th} &= \frac{1}{\sigma-1}\mu\varpi^{-\sigma}\left(I_{th}\left(P_t^h\right)^{\sigma-1} + I_{tf}\left(P_t^f\right)^{\sigma-1}\tau^{1-\sigma}\right)\int_1^{N_{th}+1}\beta_{tih}^{1-\sigma}di - \alpha N_{th} = \\
&= \frac{1}{\sigma-1}\mu\varpi^{-\sigma}\left(I_{th}\left(P_t^h\right)^{\sigma-1} + I_{tf}\left(P_t^f\right)^{\sigma-1}\tau^{1-\sigma}\right)\left(\frac{P_{th}}{\varpi}\right)^{1-\sigma} - \alpha N_{th} = \\
&= \frac{1}{\sigma}\mu P_{th}^{1-\sigma}\left(\frac{I_{th}}{\left(P_{th}\right)^{1-\sigma} + \left(\tau P_{tf}\right)^{1-\sigma}} + \frac{\tau^{1-\sigma}I_{tf}}{\left(P_{tf}\right)^{1-\sigma} + \left(\tau P_{th}\right)^{1-\sigma}}\right) - \alpha N_{th}
\end{aligned}
\tag{3.42}
$$

It is quite useful to derive the overall expenditures that a country makes on the differentiated product varieties that are produced in each country. The expenditure of home country consumers on domestically produced differentiated products is:

$$E_{th}^{h} \equiv \int_{1}^{N_{th}+1} D_{tih}^{h} P_{tih} di = \mu I_{th} \frac{\left(P_{th}\right)^{1-\sigma}}{\left(P_{th}\right)^{1-\sigma} + \left(\tau P_{tf}\right)^{1-\sigma}} \tag{3.43}$$

Similarly, given that home country consumers face $\tau P_{tif} = \tau \varpi \beta_{tif}$ and the demand of home country consumers for the differentiated product varieties produced abroad is described by equation (3.13), we get the following expression for the home country's overall expenditure on foreign varieties:

$$E_{tf}^{h} \equiv \int_{1}^{N_{th}+1} D_{tif}^{h} \tau P_{tif} di = \mu I_{th} \frac{\left(\tau P_{tf}\right)^{1-\sigma}}{\left(P_{th}\right)^{1-\sigma} + \left(\tau P_{tf}\right)^{1-\sigma}} \tag{3.44}$$

Naturally the sum of these two expenditures is equal to the part of income dedicated to the spending on differentiated products. Taking into account the results obtained above in equations (3.43) and (3.44), the aggregate outputs and profits arising in the home country are as follows:

$$\begin{aligned} Q_{th} &= \mu P_{th}^{-\sigma} \left(\frac{I_{th}}{\left(P_{th}\right)^{1-\sigma} + \left(\tau P_{tf}\right)^{1-\sigma}} + \frac{\tau^{1-\sigma} I_{tf}}{\left(P_{tf}\right)^{1-\sigma} + \left(\tau P_{th}\right)^{1-\sigma}} \right) = \\ &= \frac{1}{P_{th}} \left(E_{th}^{h} + E_{th}^{f} \right) \end{aligned} \tag{3.45}$$

$$\begin{aligned} \pi_{th} &= \frac{1}{\sigma} \mu P_{th}^{1-\sigma} \left(\frac{I_{th}}{\left(P_{th}\right)^{1-\sigma} + \left(\tau P_{tf}\right)^{1-\sigma}} + \frac{\tau^{1-\sigma} I_{tf}}{\left(P_{tf}\right)^{1-\sigma} + \left(\tau P_{th}\right)^{1-\sigma}} \right) - \alpha N_{th} = \\ &= \frac{1}{\sigma} \left(E_{th}^{h} + E_{th}^{f} \right) - \alpha N_{th} \end{aligned} \tag{3.46}$$

Analogously, we could derive the corresponding expressions for the foreign country.

3.4. The equilibrium in the labour market and market for homogeneous product

I now proceed to the description of the equilibrium in the competitive sector of the more efficient home country. The demand for the homogeneous product of the competitive sector is given in (3.9), and the supply of that production is:

$$A_{th}^{s} = L_{A_{th}}^{d} \tag{3.47}$$

The equilibrium on the competitive market requires that:

$$(1-\mu)I_{th} \equiv A_t^{h^d} = A_{th}^{h} + A_{tf}^{h} \equiv A_t^{h^s} \tag{3.48}$$

where $A_t^{h^s}$ and $A_t^{h^d}$ denote total supply of and demand for homogeneous product to the home country.

And on the labour market we have:

$$\overline{L}^{s} = L_{D_{th}}^{d} + L_{A_{th}}^{d} \tag{3.49}$$

where

$$L_{D_{th}}^{d} = \int_{1}^{N_{th}+1} C_{tih}\,di =$$

$$= \frac{1}{\varpi}\mu P_{th}^{1-\sigma}\left(\frac{I_{th}}{(P_{th})^{1-\sigma} + (\tau P_{tf})^{1-\sigma}} + \frac{\tau^{1-\sigma} I_{tf}}{(P_{tf})^{1-\sigma} + (\tau P_{th})^{1-\sigma}}\right) + \alpha N_{th} =$$

$$= \alpha N_{th} + \frac{1}{\varpi}\left(E_{th}^{h} + E_{th}^{f}\right) = L_{D_{th}}^{d} \tag{3.50}$$

is the total labour demand in the monopolistically competitive sector.

Finally, overall income earned in home country is defined as sum of all profits earned by firms and wages received by the worker:

$$I_{th} = \overline{L}^{s} + \pi_{th} = \overline{L}^{s} + \frac{1}{\sigma}\left(E_{th}^{h} + E_{th}^{f}\right) - \alpha N_{th} \tag{3.51}$$

Similarly, we could derive the corresponding aggregate values for the less efficient foreign country

This completes the description of the trading equilibrium. The next chapter we will move on to analyze the effects of trade barriers.

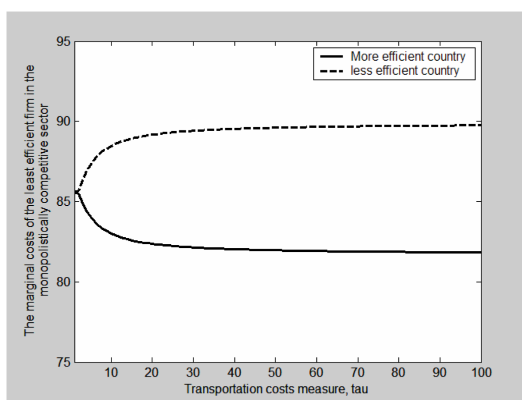
Chapter 4

THE ANALYSIS OF TRADE WITH NON-PROHIBITIVE BARRIERS EQUILIBRIUM

To present the results of the modelling I have made simulations (see Appendix 3) for the case of standard CES preferences for variety, which were described in the previous chapter. The predictions of the model are consistent with the theoretical findings of Montagna [2001] in the limiting cases of autarky and pure free trade without barriers, as shown in the Appendix 1 and 2.

In her free trade equilibrium analysis Montagna [2001] defined the relationship among the efficiency cut-off points from the fact that, in the case of free trade without any trade barriers, the prices of the least efficient firms in both countries were equal. The presence of the barriers to trade changes the situation.

Figure 4.1. *The results of the simulations of the basic model: the impact on the efficiency cut-off points during the transition from autarky to free trade*



The first important result is that in Montagna [2001] pure free trade leads to the complete unification of competitive conditions such that the marginal costs are equal across countries. More generally, openness to trade only results in the decrease in the efficiency gap but not in its elimination, under assumption of the costly trade, of course. That is ***efficiency deterioration will not be as large as Montagna [2001] predicts but will also depend on how it is costly to trade (i.e. on τ).*** The simulations suggest that as trade barriers fall we indeed have a decrease in the efficiency gap, which is shown as the difference between the marginal costs of the least efficient firms in monopolistically competitive sector in each country (Figure 4.1).

Further, in the case of costly trade, any firm in either country faces demand from domestic consumers that is less than it would be under autarky since now foreign firms also operate on the domestic market. Further, consumers now allocate the fraction of their income devoted to the consumption of the differentiated products among spending on the varieties produced both domestically and abroad. ***Even when trade barriers exist opening to trade from autarky can generally can be considered as an expansion of the market for differentiated products for more efficient country and for the homogeneous product of the less efficient country***. Consequently, ***the patterns of production and trade specialization have the same directions as in the case of free trade with no barriers described by Montagna [2001]; however, the extent of specialization relative to autarky under trade with barriers is now lower*** (see Figure 4.2). This can be shown easily. From the equations (3.37) and (3.39) we find that the overall expenditure on differentiated products produced in the more efficient country is higher than that for the less efficient country (see also Figure 4.3):

$$\frac{E_{th}^{h}+E_{th}^{f}}{E_{tf}^{f}+E_{tf}^{h}}=\left(\frac{(N_{th}+1)^{\theta}-1}{(N_{th}+1)^{\theta-1}}\Big/\frac{(N_{tf}+1)^{\theta}-1}{(N_{tf}+1)^{\theta-1}}\right)>1 \qquad (4.1)$$

Consequently, the output of the monopolistically competitive sector of more efficient country would be higher than in the less efficient country.

Figure 4.2. *The results of the simulations of the basic model: the impact on total output in the monopolistically competitive sector caused by the movement from autarky to free trade*:

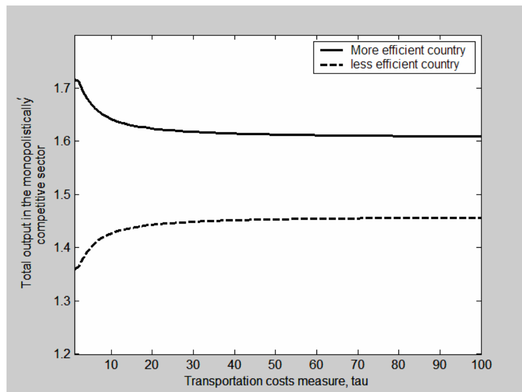
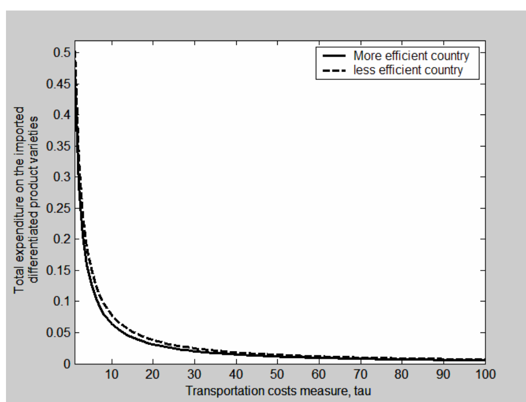


Figure 4.3. *The results of the simulations of the basic model: the impact on expenditures for imported differentiated products over the transition from autarky to free trade*



Formally from equation (3.45) and its analogue for the less efficient foreign country we obtain:

$$Q_{th} > Q_{tf} \wedge Q_{nth} > Q_{th} \quad ^{3} \tag{4.2}$$

as for more efficient country in free trade we have higher expenditure on differentiated product varieties if there is no need to pay costs of their transportation.

And from equations (3.47), (3.49) and (3.50):

$$L^{d}_{D_{th}} > L^{d}_{D_{tf}} \Rightarrow A^{s}_{th} < A^{s}_{tf} \tag{4.3}$$

In other words, since the more efficient country specializes in the production of differentiated products, its firms operating in the monopolistically competitive sector demand more labour. Given that both countries have the same labour endowment and that this factor is not mobile across the countries, the amount of labour allocated to the perfectly competitive sector is lower in the more efficient country than in the less efficient country. The natural implication is lower production of the homogeneous product in the competitive sector of the more efficient country than in less efficient country (see Figure 4.4) and specialization of the less efficient country in the production of the homogeneous product.

Further, the more efficient home country producers of differentiated goods have, on average, lower prices because they produce varieties with lower marginal costs. If other things were equal, the efficiency deterioration that the more efficient country experiences as a result of further openness to trade and the expansion of the monopolistically competitive market would lead to an increase in the overall producers' price index. This would happen due to the fact that new firms with higher marginal and average costs have entered the market. However, at the same time incumbent firms also experience increases in the scale of production and realize the economies of scale, which leads to a

[3] The first lower subscript t denotes the case of costly trade and alternatively, nt, denotes free trade.

decrease in average and marginal costs. So generally we have two opposite effects on the producers' price index in the more efficient country. Figure 4.5 suggests that the efficiency deterioration effect is milder.

Figure 4.4. *The results of the simulations of the basic model: the impact on the total output of the homogeneous product in the competitive sector caused by the movement from autarky to free trade*:

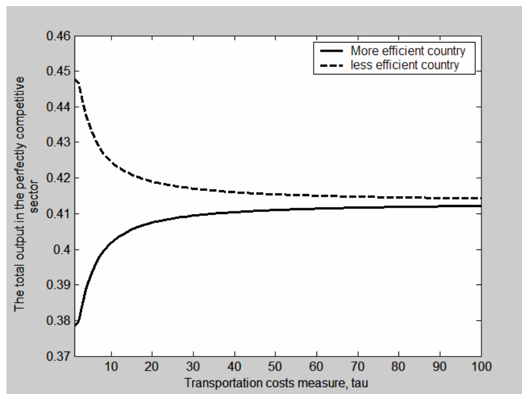
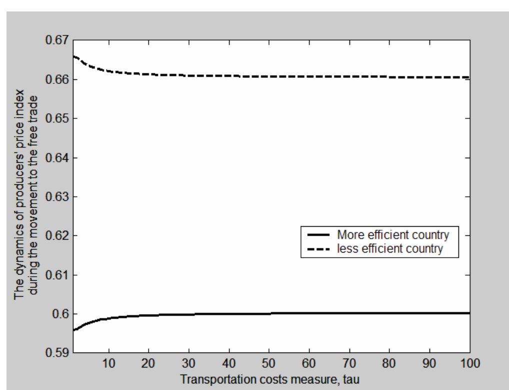


Figure 4.5. *The results of the simulations of the basic model: the impact on the producer price indexes for differentiated products over the transition from autarky to free trade*



In the case of less efficient country, the efficiency improvement due to the increased competition faced by domestic producers drives the less efficient producers out of he monopolistically competitive market and the remaining firms do not realize economies of scale since the demand for product varieties is lower compared to the autarky level. This happens due to the fall in domestic demand as consumers facing non-prohibitive trade barriers get the access to cheaper imported varieties. This decline in domestic demand is not fully offset by increased demand from the more efficient country because of the high prices charged by the inefficient foreign country producers.

Naturally we expect to have greater number of firms in the monopolistically competitive sector in the more efficient country than in the less efficient one in case of costly trade. The equality between the efficiency cut-off points in the two countries allows to claim that the number of firms in the more efficient country exceed the one in the less efficient country. However, as $\tau \to \infty$ (so that a fraction of the good that arrives at the other country becomes equal to 0 and we return to the autarky case) domestic consumers' demand for domestically produced varieties increases. In the efficient country, this does not fully offset the loss in demand from the foreign country. Consequently, the number of the firms operating on the monopolistically competitive market decreases in the more efficient country. Meanwhile, in the less efficient country the rise in domestic demand exceeds the decline in demand from the more efficient country and we observe an increase in the number of varieties produced as we move toward autarky.

At the outset, I assumed the same fixed costs for all firms in both countries. Due to the fact that all firms use a constant mark-up pricing rule, the constant fraction of the price that a producer receives for his product is used to cover the fixed costs incurred as a result of production. This means that the higher the price of the variety, the lower the output that is needed to cover fixed costs and to break even. The above fact explains why the overall number of firms operating on the monopolistically competitive market (and the number of the differentiated product varieties) under autarky is equal in the two countries notwithstanding the efficiency gap exists between them. In the less efficient country, the least efficient firm simply produces a smaller quantity of the differentiated product variety to break even, and, therefore, the total quantity of differentiated products produced in the two countries does differ under autarky.

Figure 4.6. *The results of the simulation of the basic model: the changes in the overall number of firms operating in the monopolistically competitive industries of both countriescaused by the movement from autarky to free trade.*

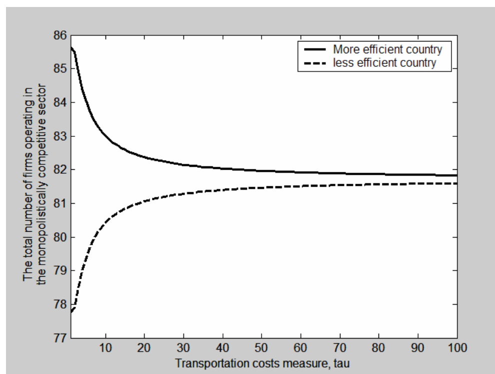
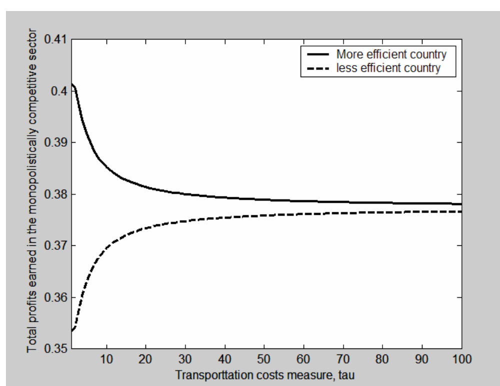


Figure 4.7. *The results of the simulations of the basic model: the impact on total profits in the monopolistically competitive sector caused by the movement from autarky to free trade:*



The total profits earned in the monopolistically competitive sector are higher in the more efficient country since it specializes in the production of the differentiated product varieties. However, in autarky we will have the equality in profits earned in the monopolistically competitive sector (see Figure 4.7).

In general, we are also very interested in the welfare changes arising from reductions in trade barriers. This topic is examined in the next chapter.

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[6] Again, the lower subscript nt denotes free trade case, while t indicates trade with some barriers in the form of transportation costs.

Chapter 5

THE WELFARE ANALYSIS OF COSTLY TRADE EQUILIBRIUM

Generally, we are interested not in the income or overall quantity of the differentiated products consumed but in the utility or welfare that our representative consumer derives. Of course, we start from the derivation of the indirect utility function. The latter can be found if we consider the dual minimization problem of choosing the commodity bundle that ensures the minimal expenditures but at the same time allows the representative consumer to achieve some minimal level of utility.

So for equation (3.1) and budget constraint (3.4), I can state the following dual minimization problem:

$$\min_{A_{tj}, D_{tj}} E = \left\{ A_t^j + P_t^j D_t^j \right\}$$

$$s.t. \overline{U} \le \left(A_t^j \right)^{1-\mu} \left(D_t^j \right)^{\mu}, \overline{U} = const \qquad (5.1)$$

The solution to such maximization problem are the quantities of homogeneous good, A_t^j, and composite differentiated product, D_t^j, that are demanded as functions of the consumers' price level, P_t^j, and the reservation level of utility, $\overline{U}$:

$$A_t^{j*} = \left(\frac{1-\mu}{\mu} \right)^{\mu} \left(P_t^j \right)^{\mu} \overline{U} \qquad (5.2)$$

$$D_t^{j*} = \left(\frac{1-\mu}{\mu}\right)^{\mu-1} \left(P_t^j\right)^{\mu-1} \overline{U} \tag{5.3}$$

And, the equilibrium value of expenditures is:

$$E^* = \frac{(1-\mu)^{\mu-1}}{\mu^{\mu}} \left(P_t^j\right)^{\mu} \overline{U} \tag{5.4}$$

Consequently, the indirect utility function has the following form:

$$V_t^j = \frac{(1-\mu)^{1-\mu}}{\mu^{-\mu}} \left(P_t^j\right)^{-\mu} I_{tj} \tag{5.5}$$

The next task is to compare the welfare levels of the two countries at different levels of transportation costs.

In the previous chapter, we showed by means of simulations that the more efficient country earns higher profit in the monopolistically competitive sector than the less efficient country in all costly trade equilibria. Therefore,

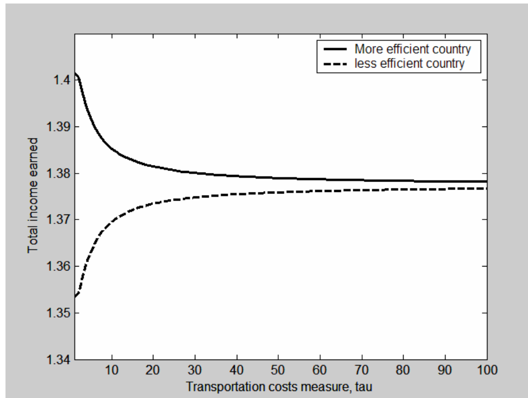
$$I_{tf} < I_{th}. \tag{5.6}$$

Recall that equation (3.51) says:

$$I_{tj} = \overline{L}^s + \frac{1}{\sigma}\left(E_{tj}^j + E_{tj}^{-j}\right) - \alpha N_{tj} \text{ , } j = h, f \tag{5.7}$$

Home income is higher than foreign income because the labour endowment is the same in both counties, but the more efficient country earns higher profit in the monopolistically competitive sector than the less efficient country. The simulations for the basic model (see Appendix 1) show that this in fact is true.

Figure 5.1. *The results of the simulation of the basic model: the changes in total income for both countries caused by the movement from autarky to free trade.*



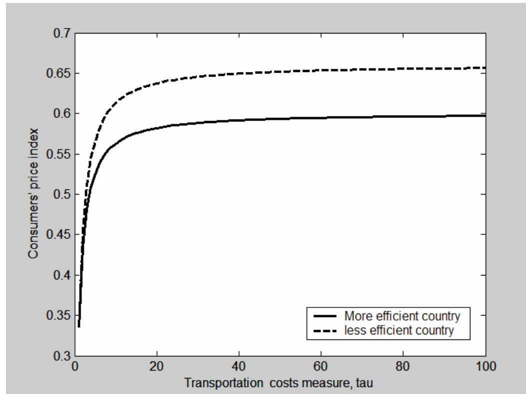
The consumers' price index in the more efficient country is lower than in the less efficient country. In the absence of love of variety, the only channels through which trade affects welfare are changes in the price index and income. The former effect Montagna [2001] calls "the efficiency effect of trade". However, it is the consumers' price index not producers' one that enters the welfare function (5.5). So effect of efficiency deterioration is not direct. The breadth of product choice is the other channel through which trade affects welfare: the CES utility has the property that if we decrease the total quantity of each variety by 50% and then double the number of varieties we will get an increase in utility. And consumers not interested just in the number of varieties of differentiated product that they consume. They are flexible and able to substitute towards cheaper varieties.

Naturally, price level in both countries decreases with the costs of transporting goods. So generally, the results suggest that:

$$P_t^f > P_{nt}^f \quad (5.8)$$

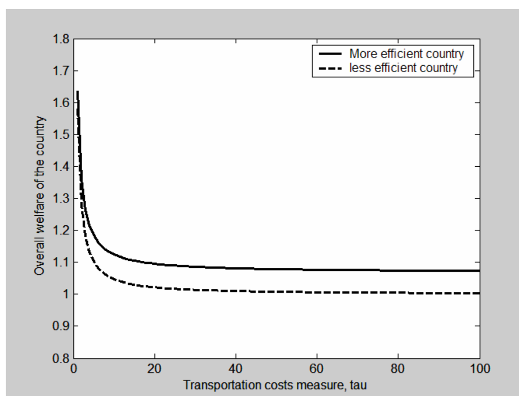
[6]

Figure 5.2. *The results of the simulations of the basic model: the impact on the consumer price indexes for differentiated products over the transition from autarky to free trade*



In general, our simulations suggest that free trade remains first-best equilibrium as shown on Figure 5.3., but welfare unambiguously increases with the reduction of trade barriers.

Figure 5.3. *The results of the simulation of the basic model: the changes in welfare in both countries caused by the movement from autarky to free trade.*



Therefore, ***the partial removal of the barriers to trade increases consumer welfare in the more efficient country through both the price and income channels. While income in the less efficient country declines due to lower profits earned in the monopolistically competitive sector, this is more than offset by the price effect. As we have seen, any movement from an equilibrium with high trade costs to one with lower***

trade costs causes a decline in the overall consumer price index in both countries leading to a positive price effect on welfare. Owners of the firms producing differentiated product varieties in the less efficient country suffer from trade liberalization, but consumers gain from the lower prices and wider choice.

Chapter 6

CONCLUSIONS

Generally, opening to trade provides a further expansion of the market for differentiated products for the more efficient country, but this is hindered to some extent by any remaining barriers to trade. Even in a free trade equilibrium without trade barriers, producers in the less efficient foreign country still sell at higher prices due to high costs of production but consumers located in the foreign country now are able to substitute cheaper varieties produced in the more efficient home country for these more expensive varieties. On the other hand, in the presence of trade barriers, the price of imported goods goes up in the less efficient country resulting in diminished substitution compared with the case of a pure free trade equilibrium. As trade barriers increases, the number of the firms operating on the monopolistically competitive market in the efficient country decreases progressively until autarky is reached, while in the less efficient country there is a continuous increase in the number of varieties due to diminished international competition.

The total number of firms affects the efficiency cut-off points for monopolistically competitive industries. Whenever trade is possible, either with trade barriers or without them, the more efficient home country specializes more in the production of the differentiated product. Consequently, the more efficient country experiences a deterioration in the efficiency of its marginal firms. This deterioration is actually lower in the case where trade remains costly. Consequently, the assumption of the presence of transportation costs does reduces the efficiency deterioration in the marginal firms of the more efficient country that has been identified by Montgna [2001]. The latter conclusion stems from the fact that we basically will have a narrowing of the gap between the countries instead of a unification of competitive conditions.

The efficiency of the composition of the monopolistically competitive industry in the less efficient foreign country is also important. As barriers to trade are reduced, the less efficient country experiences decreases in the total number of firms operating in the monopolistically competitive sector compared to the case of autarky and, more important, only the most efficient firms stay on the market.

The efficiency of the composition of the monopolistically competitive industry affects the price index that a representative consumer faces. However, here the effect of trade obstacles and economies of scale are also present. Our findings suggest that consumer price indexes decrease in the both countries as we move to equilibria with lower transportation costs.

With respect to economic welfare, lowering barriers to trade causes an increase in income and reduction in consumer prices for more efficient country, and leads to a decrease in both income and prices in the less efficient country. Notwithstanding the presence of the efficiency deterioration in the more efficient country and the losses experienced by owners of firms in the monopolistically competitive sector in the less efficient one, the maximum possible level of welfare in both countries arises in case of pure free trade where barriers are totally absent. This conclusion is highly relevant for a country such as Ukraine with a higher cost structure. Important fact is that efficient countries such as the EU would also gain conditional on reimposing the conventional assumption of love for variety. The latter result appears to arise because preferences for variety and realization of the economies of scale overcome the efficiency problems identified by Montagna [2001].

Taking into account the fact that the policy component of trade barriers is rather artificial (e.g., tariff protection), the implication of this thesis for the trade policy is straightforward. Welfare in both countries will increase from a balanced reduction in tariff protection. However, if group of owners of firms in the monopolistically competitive industry of less efficient country are well organized to protect their interests, they may oppose trade liberalization by arguing that it is necessary to protect the domestic industry from foreign competition. The latter vested interests complicate the movement to the free trade to a large extent.

Propositions for further research: From a theoretical perspective, it would be interesting to look what happens if we introduce transportation costs or tariff protection in the opposite case where "love for variety" is absent and the elasticity of substitution between the differentiated product varieties is low. Montagna [2001] claims that it is possible for more efficient country to experience net welfare loss from going from autarky to free trade. The question is whether a generalized model of the type introduced in this thesis confirms that result when barriers to trade are present.

BIBLIOGRAPHY

Anderson, S. P., Donsimoni, M. –P. Gabszewicz, J.J. Is *International Trade Profitable to Oligopolistic Industries?* International Economic Review, Vol. 30, No. 4 (Nov., 1989), 725-733

Benassy, Jean-Pascal *Taste for Variety and Optimum Production Patterns in Monopolistic Competition*. Economic Letters 52 (1996), 41-47.

Ben-David, Dan. *Trade and Convergence Among Countries*. Journal of International Economics, Vol. 40, No. 3-4, (May, 1996), 279-298

Bhagwati, Jagdish. *The Pure Theory of Intermational Trade: A Survey*. The Economic Journal, Vol. 74, No. 293(Mar., 1964), 1-84.

Bohm, Volker. *The Foundation of the Theory of Monopolistic Competition Revisited*. Journal of Economic Theory, Vol. 63 (1994), 208-218

Brander, James A. *Intra-industry Trade in Identical Commodities*. Journal of international Economics, Vol. 11(1981)

Chao, Chi-Chur, Takayama, Akira. *Monopolistic Competition, Nonhomoteticity, and the Stability of the Chamberlinean Tangency Solution*, International Economic Review, Vol. 31, No. 1 (Feb., 1990), pp. 73-86

Chatterjee, S. and R.W. Cooper,. *Entry and exit, product variety and the business cycle*, NBER Working Paper No. 4562, Dec., 1993, 1-45.

Coe, David T., Helpman, Elhanan. *International R&D Spillovers*. European Economic Review, Vol. 39 (1995), 859-887

Connoly, Mishelle. *The Dual Nature of Trade: Measuring its Impact on Imitation and Growth*. Journal of Development Economics, Vol. 72 (2003), 31-55

Dierker, Egbert, Grodal, Birgit. *The Price Normalization Problem in Imperfect Competition and the Objective of the Firm*. Economic Theory, Vol. 14 (1999), 257-284.

Dixit, Avinash K., Stiglitz, Joseph.E. *Monopolistic Competition and Optimal Product Diversity*. The American Economic Review, Vol. 67, No. 3 (Jun., 1977), 297-308.

Eaton, Jonathan, Kortum, Samuel. *Trade in Ideas: Patenting and Productivity in the OECD*. Journal of International Economics, Vol. 40, No. 3-4 (May., 1996) 251-278

Ethier, Wilfred J. *National and International Returns to Scale in the Modern Theory of* International *Trade*. The American Economic Review, Vol. 72, No. 3 (Jun., 1982), 389-405.

Frankel, Jeffrey, Stein, Ernesto and Wei, Shang-jin. *Continental Trade Blocks: Are They Natural or Super-Natural*. NBER Working Paper No. 4588, December, 1993.

Gabszewicz *et al*. *International Trade in Differentiated Products*. International Economic Review, Vol. 22, No.3 (Oct., 1981), 527-534.

Gong, Guan, Keller, Wolfgang. *Convergence and Polarization in Global Income Levels: A Review of Recent Results on the Role of International Technology Diffusion*. Research Policy, Vol. 32 (2003), 1055-1079

Grossman, Gene M., Helpman, Elhanan. Trade, Knowledge Spillover and Growth. European Economic Review, Vol. 35 (1991), 517-526

Grubel, Herbert G. *Intra-Industry Specialization and the Pattern of Trade*. The Canadian Journal of Economics and Political Science, Vol. 35, No. 3 (Aug., 1967), 374-388.

Helpman, Elhanan. *International trade in the presence of product differentiation, economies of scale and monopolistic competition : A Chamberlin-Heckscher-Ohlin approach,* The Journal of International Economics, Vol. 11 (Aug., 1981), 305-446

Helpman, Elhanan. *A Simple theory of International Trade with Multinational Corporations,* The Journal of Political Economy, Vol. 91, No. 3 (Jun., 1984), 451-471

Hummels, David. Levinhson, James , *Monopolistic Competition and International Trade: Reconsidering the Evidence*, The Quarterly Journal of Economics, Vol. 110, No 3 (Aug., 1995), pp. 799-836.

Keller, Wolfgang. *Are International R&D Spillovers Trade-related?: Analyzing Spillovers Among Randomly Matched Trade Partners*. European Economic Review, Vol. 42, No. 8, (Sep., 1998), 1469-1481

Krugman, Paul. *Scale Economies, Product Differentiation, and the Pattern of Trade.* The American Economic Review, Vol. 70, No. 5 (Dec., 1980), 950-959

Krugman, Paul. *Intra-industry Specialization and the Gains from Trade*. The Journal of Political Economy, Vol. 89, No.5 (Oct., 1981), 959-973

Lancaster, Kelvin. *Intra-industry trade under perfect monopolistic competition***.** The Journal of International Economics, Vol. 10 (May, 1980), 151-175.

Lawrence, Colin, Spiller, Pablo T. *Product Diversity, Economies of Scale, and International Trade*. The Quarterly Journal of Economics, Vol.98, No. 1 (Feb., 1983), 63-83.

Markusen, James. *Trade and the Gains from Trade with Imperfect Competition*. Journal of International Economics, Vol. 11 (1981), 531-551

Markusen, James, Venables, Anthony. *The Theory of Endowment, Intra-Industry, and Multinational Trade*. NBER Working Paper No. 5529, April 1996

Melitz, Mark J. *The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity.* NBER Working Paper No. 8881, April 2002

Montagna, Catia . *Efficiency Gaps, Love of Variety and International Trade*, Economica. February 2001; 68(269), 27-44

Neary, Peter J. *Monopolistic Competition and International Trade Theory*. University College Dublin and CEPR, October, 2000

Neven, Damien, Phillips, Louis. *Discriminating Oligopolies and Common Maukers.* The Journal of Industrial Economics, Vol. 34, No. 2 (Dec., 1985), 133-149

Nocco, Antonella. *The Rise and Fall of Regional Inequalities with Technological Differences and Knowledge Spillovers*. Regional Science and Urban Economics xx (2005), xxx-xxx

Roberts, John, Sonnenschein, Hugo. *On the Foundations of the Theory of Monopolistic Competition*. Econometrica, Vol. 45, No. 1(Jan., 1977), 101-113

Romer, P.M.*: Growth based on increasing returns due to specialisation*, American Economic Review (Papers and Proceedings), Vol. 77(1987), 56-72.

Sebastien, Jean. *International Trade and Firms' heterogeneity Under Monopolistic Competition*, CEPII Working Paper #00-13. September, 2000

Tharakan, Joe , Thisse, Jacques-François. *The Importance of Being Small. Or When Countries are Areas and Not Points*. Regional Science and Urban Economics, Vol. 32 (2002), 381-408

Van Long, Ngo, Soubeyran, Antoine. *Cost Heterogeneity, Industry Concentration and Strategic Trade Policy*, Journal of International Economics, Vol. 43 (1997), 207-220.

Venables, Antony J. *Trade and Trade Policy with Differentiated Products: A Chamberlinean-Ricardian Model.* The Economic Journal, Vol. 97, No. 387 (Sep., 1987), 700-717.

Xu, Bin, Wang, Jianmao. *Capital Goods Trade and R&D Spillovers in the OECD*. Canadian Journal of Economics. Vol. 32, No. 5 (Nov., 1999), 1258–1274

Yeaple, Stephen Ross. *A Simple Model of Firm Heterogeneity, International Trade, and Wages*. Journal of International Economics Vol, 65 (2005), 1-20

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Appendix 1

AUTARKY

A1.1. The demand side of the economy

The main purpose of this section is to demonstrate the limiting case of the autarky that can be inferred from the basic model of Chapter 3.

Recalling that in the model of international trade with non-prohibitive trade barriers on the demand side we have:

$$P_t^h = \left(\int_1^{N_{th}+1} (P_{tih})^{1-\sigma} di + \int_1^{N_{tf}+1} (\tau P_{tif})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}, \tau > 1 \quad \text{(A1.1)}$$

$$D_t^h = \left(\int_1^{N_{th}+1} (D_{tih}^h)^{\frac{\sigma-1}{\sigma}} di + \int_1^{N_{tf}+1} (D_{tif}^h)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \quad \text{(A1.2)}$$

representing the price index and the demand for composite differentiated product of the monopolistically competitive sectors that consumers in the more efficient country face. In the limiting case of the autarky we have that $\tau \to \infty$. Since I have previously assumed that $\sigma > 1$, it follows that $(\tau)^{\sigma-1} \to 0$ and the consumer price index becomes equal to the domestic producers' price index:

$$P_a^h = \left(\int_1^{N_{ah}+1} (P_{aih})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} = P_{ah} \quad \text{(A1.3)}$$

This happens due to the fact that consumers do not buy any foreign differentiated product variety[7]:

$$D_{aif}^{h} = \mu I_{ah} \left(P_{a}^{h}\right)^{\sigma-1} \tau^{-\sigma} P_{aif}^{-\sigma} = 0 \forall i \quad \text{(A1.4)}$$

And, similarly, the demand of foreign country consumers for the differentiated product variety produced in the home (more efficient) country is

$$D_{aih}^{f} = \mu I_{af} \left(P_{a}^{f}\right)^{\sigma-1} \tau^{-\sigma} P_{aih}^{-\sigma} = 0 \quad \text{(A1.5)}$$

Therefore, the overall composite commodity index in autarky consists of only domestically produced varieties:

$$D_{a}^{h} = \left(\int_{1}^{N_{ah}+1} \left(D_{aih}\right)^{\frac{o-1}{o}} di \right)^{\frac{o}{o-1}} \quad \text{(A1.6)}$$

Correspondingly, the domestic producers face only demand from the domestic consumers only:

$$D_{aij}^{j} = \mu I_{aj} \left(P_{a}^{j}\right)^{\sigma-1} P_{aij}^{-\sigma},\ j = h, f \quad \text{(A1.7)}$$

A careful reader can easily verify that the same result may be obtained if we consider the two-stage maximization of the representative consumer's utility function:

$$U_{a}^{j} = \left(A_{a}^{j}\right)^{1-\mu} \left(D_{a}^{j}\right)^{\mu},\ \mu \in (0;1) \quad \text{(A1.8)}$$

Budget constraint of the representative consumer in the case of autarky becomes:

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[7] Lower subscript *a* denotes case of autarky

$$A_a^j + P_a^j D_a^j = A_a^j + \int_1^{N_{aj}+1} P_{aij} D_{aij} di \le I_{aj} \tag{A1.9}$$

Price of the competitive sector product is still normalized to 1.

On the first stage of budgeting a representative consumer solves the problem:

$$\max_{A_a^j, D_a^j} \left\{ \left(A_a^j\right)^{1-\mu} \left(D_a^j\right)^{\mu} \right\}$$
$$s.t. A_a^j + P_a^j D_a^j \le I_{aj} \tag{A1.10}$$

and gets his income divided according to such rule:

$$A_a^{j^*} = (1-\mu) I_{aj} \tag{A1.11}$$

$$D_a^{j^*} = \frac{\mu I_{aj}}{P_a^j} \tag{A1.12}$$

On the second stage of the utility maximization he solves:

$$\max_{\{D_{aij}\}_{i=1}^{N_{aj}}} \left\{ \left(\int_1^{N_{aj}+1} \left(D_{aij}\right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \right\}$$
$$s.t. \int_1^{N_{aj}+1} P_{aij} D_{aij} di = P_a^j D_a^j = \mu I_{aj} \tag{A1.13}$$

Note that I wrote equality sign in the budget constraint in both maximization problems as in the equilibrium we will get all total income spent on the products since the utility is increasing function in the latter ones. As a solution to the above problem we will get the demand functions for each differentiated product variety in each country:

$$D_{aij}^{j\ *} = \mu I_{aj}\left(P_a^j\right)^{\sigma-1} P_{aij}^{-\sigma} \tag{A1.14}$$

which are exactly the equation (A1.7) that was obtained earlier

A1.2. The supply side of rhe economy and the equilibrium in the monopolistically competitive market

Having considered the demand side of the hypothetical economy I proceed to the description of the supply side of it. Again, in the competitive sector we have the supply of the homogeneous product is equal to supply of labour in that sector:

$$A_{aj} = L_{A_{aj}} \tag{A1.15}$$

and the demand we have in the equation (A1.11).

The other sector that is characterized by monopolistic competition and IRS technology is still has the cost function

$$C_{aij} = w(\alpha + \beta_{aij} Q_{aij}) \tag{A1.16}$$

where α fixed costs, same for each firm inside the country and equal across the two countries and wage w is normalized to 1.

Decreasing average costs of production in monopolistically competitive sector ensure that each firm will produce a single variety and, therefore, the number of firms in each country will be equal to the number of varieties produced.

In the labour market the following constraint should hold: the total exogenously determined labour supply must be equal to labour demand that comes from both competitive and monopolistically competitive sector:

$$\overline{L}_{aj} = L_{A_{aj}} + L_{D_{aj}} \tag{A1.17}$$

where $L_{D_{aj}} = \alpha N_{aj} + \int_{1}^{N_{aj}+1} C_{aij} di$ (A1.18)

It is worth noting that so far the analysis was exactly the same as one done by Montagna [2001]. However, I do not follow her rather artificial assumption about the "love for variety" since the CES utility form of preferences automatically ensures the existence of that "love for variety".

Now we will proceed to the description of the monopolistically competitive sector. Each firm in the monopolistically competitive sector is assumed to choose price P_{aij} so as to solve the following maximization problem:

$$\max_{P_{aij}} \left\{ P_{aij} D^{j}_{aij} - C_{aij} D^{j}_{aij} \right\} = \max_{P_{aij}} \left\{ \mu I_{aj} \left(P^{j}_{a} \right)^{\sigma-1} P^{1-\sigma}_{aij} - \alpha - \beta_{aij} \mu I_{aj} \left(P^{j}_{a} \right)^{\sigma-1} P^{-\sigma}_{aij} \right\}$$

(A1.19)

From the first-order condition we get the same price rule:

$$P^{*}_{aij} = \beta_{aij} \left(\frac{\sigma}{\sigma - 1} \right) \quad \text{(A1.20)}$$

The total quantity of a differentiated product variety produced is defined by substitution of the equilibrium price (A1.20) into the demand function (A1.14):

$$Q_{aij} = \varpi^{-\sigma} \mu I_{aj} P^{\sigma-1}_{aj} \beta^{-\sigma}_{aij} \quad \text{(A1.21)}$$

and from now on we will use the fact that consumers' and producers' price indexes are the same:

$$P^{h}_{a} = P_{ah} \quad \text{(A1.22)}$$

Consequently, firm's revenue and profits are:

$$R_{aij} = P_{aij} Q_{aij} = \varpi^{1-\sigma} \mu I_{aj} P_{aj}^{\sigma-1} \beta_{aij}^{1-\sigma} \quad \text{(A1.23)}$$

$$\pi_{aij} = \frac{1}{\sigma - 1} \varpi^{-\sigma} \beta_{aij}^{1-\sigma} \mu I_{aj} P_{aj}^{\sigma-1} - \alpha \quad \text{(A1.24)}$$

Couple of the quite interesting results can be already shown. Let us define the ratio of the quantities of differentiated product varieties and the ratio of revenues of the two different firms in the same country:

$$\frac{Q_{aij}}{Q_{akj}} = \frac{\varpi^{-\sigma} \mu I_{aj} P_{aj}^{\sigma-1} \beta_{aij}^{-\sigma}}{\varpi^{-\sigma} \mu I_{aj} P_{aj}^{\sigma-1} \beta_{akj}^{-\sigma}} = \frac{\beta_{aij}^{-\sigma}}{\beta_{akj}^{-\sigma}} = \left(\frac{\beta_{aij}}{\beta_{akj}} \right)^{-\sigma} \quad \text{(A1.25)}$$

$$\frac{R_{aij}}{R_{akj}} = \frac{\varpi^{1-\sigma} \mu I_{aj} P_{aj}^{\sigma-1} \beta_{aij}^{1-\sigma}}{\varpi^{1-\sigma} \mu I_{aj} P_{aj}^{\sigma-1} \beta_{akj}^{1-\sigma}} = \frac{\beta_{aij}^{1-\sigma}}{\beta_{akj}^{1-\sigma}} = \left(\frac{\beta_{aij}}{\beta_{akj}} \right)^{1-\sigma} \quad \text{(A1.26)}$$

I can conclude with certainty that ***in the following setup of the model more efficient firms in each country will have higher market share in monopolistically competitive sector, notwithstanding that their individual price decisions do not affect overall price index***.

As before we assume the following distribution for firms' marginal costs:

$$\beta_{ij} = \phi_j i^{\delta} \quad \text{(A1.27)}$$

Under the above assumptions ϕ_j denotes the costs of the most efficient firm in each country, and $(\phi_f - \phi_h)$ constitutes the efficiency gap between two countries. Assuming the free entry we now have the situation then the last $(N_{aj} + 1)$ firm has zero profit and the rest firms on the market earn non-negative one.

$$\pi_{a(N_{aj}+1)j} = 0 \wedge P_{a(N_{aj}+1)j} = \varpi\phi_j\left(N_{aj}+1\right)^{\delta} \Rightarrow$$

$$\Rightarrow \pi_{a(N_{aj}+1)j} = \frac{\varpi^{-\sigma}}{\sigma-1}\left(\phi_j\left(N_{aj}+1\right)^{\delta}\right)^{1-\sigma}\mu I_{aj}P_{aj}^{\sigma-1} = \alpha \tag{A1.28}$$

that is exactly the equation (3.37) with $\tau \to \infty$.

After some rearrangement of the last condition we get the following:

$$P_{aj} = \left(\frac{\mu I_{aj}}{\sigma\alpha}\right)^{\frac{1}{1-\sigma}}\varpi\phi_j\left(N_{aj}+1\right)^{\delta} \tag{A1.29}$$

which means that price index depends on the number of firms operating on the monopolistically competitive market.

On the other hand from the definition of the price index we know that

$$P_{aj} = \left(\int_1^{N_{aj}+1}\left(\phi_j\varpi i^{\delta}\right)^{1-\sigma}di\right)^{\frac{1}{1-\sigma}} = \phi_j\varpi\left(\frac{\left(N_{aj}+1\right)^{\theta}-1}{\theta}\right)^{\frac{1}{1-\sigma}}$$

$$\theta = \delta(1-\sigma)+1 \tag{A1.30}$$

Substitution of the above expression in the equation (A1.29) yields:

$$\left(\frac{\mu I_{aj}}{\sigma\alpha}\right)^{\frac{1}{1-\sigma}}\left(N_{aj}+1\right)^{\delta} = \left(\frac{\left(N_{aj}+1\right)^{\theta}-1}{\theta}\right)^{\frac{1}{1-\sigma}} \Rightarrow$$

$$\Rightarrow N_{aj}^{*} = N_{aj}\left(\mu,\theta,I_{aj},\sigma,\alpha,\delta\right) \tag{A1.31}$$

And we clearly see that in autarky the equilibrium number of firma in each country depends on the overall income of that country and on a

set of the parameters among which we do not have the parameter describing the efficiency of the most productive firm in the country.

Now I aggregate the firms' profits and quantities produced and then compare them with the results of the basic model of Chapter 3.

Aggregation of firms' outputs in the monopolistically competitive sector yields the following:

$$Q_{aj} = \left(\int_{1}^{N_{aj}+1} \left(\varpi^{-\sigma} \mu I_{aj} P_{aj}^{\sigma-1} \beta_{aij}^{-\sigma} \right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} = \frac{\mu I_{aj}}{P_{aj}} \qquad \text{(A1.32)}$$

Clearly, ***more efficient country will produce more differentiated product varieties than less efficient one in autarky, other things being equal.***

Then the ***total revenue obtained by all firms in each country*** is:

$$R_{aj} = P_{aj} Q_{aj} = \mu I_{aj} \left(\frac{\left(N_{aj} + 1 \right)^{\theta} - 1}{\theta} \right)^{\frac{1}{1-\sigma}} \qquad \text{(A1.33)}$$

and is ***the same for both countries in autarky if the income is the same*** as was assumed in Montagna [2001].

Aggregate profits are:

$$\pi_{aj} = \int_{1}^{N_{aj}+1} \pi_{aij} di = \frac{\mu I_{aj}}{\sigma} - \alpha N_{aj} \qquad \text{(A1.34)}$$

Again we conclude that in the autarkic equilibrium we have aggregate profits are the same in both countries if they have the same level of income.

Having derived all necessary equations for aggregate variables we are now able to check if the model of the Chapter 3 satisfies them.

Total quantity of differentiated product offered in the monopolistically competitive sector in the home (more efficient) country is:

$$Q_{th} = \mu P_{th}^{-\sigma}\left(\frac{I_{th}}{(P_{th})^{1-\sigma}+(\tau P_{tf})^{1-\sigma}}+\frac{\tau^{1-\sigma}I_{tf}}{(P_{tf})^{1-\sigma}+(\tau P_{th})^{1-\sigma}}\right)\overset{\tau\to\infty}{=}$$
$$= \mu P_{ah}^{-\sigma}\left(\frac{I_{ah}}{(P_{ah})^{1-\sigma}}\right)=\frac{\mu I_{ah}}{P_{ah}} \tag{A1.35}$$

Aggregation of the firms' profits in the home country yields:

$$\pi_{th} = \frac{1}{\sigma}\mu P_{th}^{1-\sigma}\left(\frac{I_{th}}{(P_{th})^{1-\sigma}+(\tau P_{tf})^{1-\sigma}}+\frac{\tau^{1-\sigma}I_{tf}}{(P_{tf})^{1-\sigma}+(\tau P_{th})^{1-\sigma}}\right)-\alpha N_{th}\overset{\tau\to\infty}{=}$$
$$= \frac{1}{\sigma}\mu P_{ah}^{1-\sigma}\left(\frac{I_{ah}}{(P_{ah})^{1-\sigma}}\right)-\alpha N_{ah}=\frac{1}{\sigma}\mu I_{ah}-\alpha N_{ah} \tag{A1.36}$$

It is quite useful for verifying our results to derive the overall expenditures on the differentiated product varieties for a country. I define the expenditures of home country consumers on the differentiated product varieties produced domestically as:

$$E_{th}^{h} \equiv \int_{1}^{N_{th}+1} D_{tih}^{h}P_{tih}\,di = \mu I_{th}\frac{(P_{th})^{1-\sigma}}{(P_{th})^{1-\sigma}+(\tau P_{tf})^{1-\sigma}}\overset{\tau\to\infty}{=}\mu I_{ah} \tag{A1.37}$$

$$E_{tf}^{h} \equiv \int_{1}^{N_{th}+1} D_{tif}^{h}\tau P_{tif}\,di = \mu I_{th}\frac{(\tau P_{tf})^{1-\sigma}}{(P_{th})^{1-\sigma}+(\tau P_{tf})^{1-\sigma}}\overset{\tau\to\infty}{=}0 \tag{A1.38}$$

so indeed in the case of autarky we get the results that overall expenditures on the domestically produced varieties equal to the share of income spent on differentiated product, and total expenditures on the imported varieties are 0.

Analogically we derive the corresponding expressions for the foreign country.

A1.3. The equilibrium in the labour market and market for homogeneous product

From now on I proceed to the description of the equilibrium in the competitive sector in the home (more efficient) country.

$$A_{ah}^{s} = L_{A_{ah}}^{d} \tag{A1.39}$$

The equilibrium on the competitive market requires that

$$(1-\mu)I_{ah} \equiv A_{a}^{h^{d}} = A_{ah}^{s} = L_{A_{ah}}^{d} \tag{A1.40}$$

And on the labour market we have:

$$\overline{L}^{s} = L_{D_{ah}}^{d} + L_{A_{ah}}^{d} \tag{A1.41}$$

$$L_{D_{ah}}^{d} = \alpha N_{ah} + \int_{1}^{N_{ah}+1} C_{aih} di = \alpha N_{ah} + \frac{1}{\varpi}\left(E_{ah}^{h}\right) = \alpha N_{ah} + \frac{\mu I_{ah}}{\varpi} \tag{A1.42}$$

which is exactly the same as in Montagna [2001].

Solving the system of equations determining the labour market equilibrium we get:

$$I_{aj} = \frac{\sigma}{\sigma-\mu}\left(\overline{L}_{aj} - \alpha N_{aj}\right) \tag{A1.43}$$

and we are done with the description of rhe autarkic equilibrium.

THE CASE OF FREE TRADE WITHOUT TRADE BARRIERS

In this chapter I consider the opposite extreme case of free trade without any trade barriers (i.e. now we will set $\tau = 1$).

A2.1. Demand side of the economy

As before, the total demand for homogeneous product is[8]:

$$A^j_{nt} = A^j_{ntj} + A^j_{nt(-j)} \tag{A2.1}$$

where for country j A^j_{ntj} is the consumption of the homogeneous good produced domestically and $A^j_{nt(-j)}$ is consumption of the homogeneous good produced abroad.

D^j_{nt}, consumption of a composite differentiated product of monopolistically competitive industry in country *j*, is defined as follows:

$$D^j_{nt} = \left(\int_1^{N_{ntj}+1} \left(D^j_{ntij}\right)^{\frac{\sigma-1}{\sigma}} di + \int_1^{N_{nt(-j)}+1} \left(D^j_{nti(-j)}\right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{A2.2}$$

Budget constraint of the representative consumer implies that sum of all expenditures on the differentiated product variety and homogeneous product is less or equal to the income of a representative consumer:

$$A^j_{ntj} + A^j_{nt(-j)} + P^j_{nt} D^j_{nt} \le I_{ntj} \tag{A2.3}$$

Price of the competitive sector product is still normalized to 1.

Consumers in the home country (more efficient one) now face the following price index:

$$P_{nt}^{h} = P_{t}^{h}(\tau = 1) = \left(\int_{1}^{N_{th}+1} (P_{tih})^{1-\sigma} di + \int_{1}^{N_{tf}+1} (\tau P_{tif})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} =$$

$$= \left(\int_{1}^{N_{nth}+1} (P_{ntih})^{1-\sigma} di + \int_{1}^{N_{ntf}+1} (P_{ntif})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{A2.4}$$

Consequently, consumers in the foreign country (less efficient one) in the case of free trade face the same price index:

$$P_{nt}^{f} = P_{t}^{f}(\tau = 1) = \left(\int_{1}^{N_{th}+1} (\tau P_{tih})^{1-\sigma} di + \int_{1}^{N_{tf}+1} (P_{tif})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} =$$

$$\left(\int_{1}^{N_{nth}+1} (\tau P_{ntih})^{1-\sigma} di + \int_{1}^{N_{ntf}+1} (P_{ntif})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} = P_{nt}^{h} \tag{A2.5}$$

Next as a result of two-stage utility maximization we get the following solution:

$$A_{nt}^{j*} = \left(A_{ntj}^{j} + A_{nt(-j)}^{j}\right)^{*} = (1-\mu) I_{ntj} \tag{A2.6}$$

$$D_{nt}^{j*} = \frac{\mu I_{ntj}}{P_{nt}^{j}} \tag{A2.7}$$

And the demand of country *j* consumers for domestically produced differentiated product variety is

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[8] The lower subscript *nt* denotes the case of trade without any barriers

$$D^{j}_{ntij} = D^{j}_{nt}\left(\frac{P_{ntij}}{P^{j}_{nt}}\right)^{-\sigma} = \mu I_{ntj}\left(P^{j}_{nt}\right)^{\sigma-1}\left(P_{ntij}\right)^{-\sigma} \quad \text{(A2.8)}$$

On the other hand, the demand of a country *j*'s consumers for the differentiated product variety produced in the foreign country is

$$D^{j}_{nti(-j)} = D^{j}_{nt}\left(\frac{P_{nti(-j)}}{P^{j}_{nt}}\right)^{-\sigma} = \mu I_{ntj}\left(P^{j}_{nt}\right)^{\sigma-1}\left(P_{nti(-j)}\right)^{-\sigma} \quad \text{(A2.9)}$$

Now we will turn to the supply side of the economy.

A2.2. Supply side of the economy

The production function in the competitive sector in both countries is:

$$A_{ntj} = L_{A_{ntj}} \quad \text{(A2.10)}$$

where $L_{A_{ntj}}$ the demand for labour in the competitive sector in corresponding country.

The other sector is still characterized by monopolistic competition and IRS technology. The cost function is the following:

$$C_{ntij} = w(\alpha + \beta_{ntij} Q_{ntij}) \quad \text{(A2.11)}$$

where $L_{D_{ntj}}$ is a total labour demand of the monopolistically competitive sector, α fixed costs, same for each firm inside the country and equal across the two countries; C_{ntij} are the costs of the production of the differentiated product variety *i* in country *j*; β_{ntij} are firm specific marginal costs of production. As a result of assumed perfect labour mobility between competitive and monopolistically competitive sectors in each country (but not across the countries) we get the fact that wage w is normalized to 1.

Decreasing average costs of production in monopolistically competitive sector ensure that each firm will produce a single variety and, therefore, the number of firms in each country will be equal to the number of varieties produced.

Each firm in the monopolistically competitive sector is assumed to choose price P_{ntij} so as to solve the profit maximization problem. We define the producers' price index for each country as:

$$P_{ntj} = \left(\int_{1}^{N_{ntj}+1} \left(P_{ntij}\right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{A2.12}$$

Therefore, the consumers' price index is a function of producers' price indexes:

$$P_{nt}^{j} = \left(\left(P_{ntj}\right)^{1-\sigma} + \left(P_{nt(-j)}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{A2.13}$$

Consider for simplicity the home (more efficient) country. The demand for differentiated product variety produced by firm i in the home country will be the following:

$$D_{ntih} = D_{ntih}^{h} + D_{ntih}^{f} = \mu\left(P_{ntih}\right)^{-\sigma}\left(P_{nth}\right)^{\sigma-1}\left(I_{nth} + I_{ntf}\right) \tag{A2.14}$$

where $\left(I_{nth} + I_{ntf}\right)$ represents the world income.

Each firm in both countries maximizes its profit and choses prices according to the following rule:

$$P_{ntih} = \frac{\sigma}{\sigma - 1}\beta_{ntih} = \varpi\beta_{ntih} \tag{A2.15}$$

and I define constant mark-up by $\varpi \equiv \frac{\sigma}{\sigma - 1}$

3.3. The equilibrium in monopolistically competitive market

Consequently, the equilibrium quantity of differentiated product variety produced by firm *i* in the home country:

$$Q_{ntih} = \mu\varpi^{-\sigma}\beta_{ntih}^{-\sigma}\left(P_{nth}\right)^{\sigma-1}\left(I_{nth} + I_{ntf}\right) \tag{A2.16}$$

Correspondingly, the equilibrium profit of the firm *i* in the home country will be:

$$\pi_{ntih} = \frac{1}{\sigma - 1}\beta_{ntih}^{1-\sigma}\varpi^{-\sigma}\mu\left(P_{nth}\right)^{\sigma-1}\left(I_{nth} + I_{ntf}\right) - \alpha \tag{A2.17}$$

Aggregation of the producers' prices yields:

$$P_{nth} = \left(\int_{1}^{N_{nth}+1}\left(\varpi\beta_{ntih}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \tag{A2.18}$$

The least efficient firm in the home (more efficient) country will be indexed by $i = N_{nth} + 1$. Assuming that $\beta_{ntih} = \varphi_h i^{\delta}$ we state zero-profit condition as follows:

$$\pi_{nt(N_{nth}+1)h}\left(\beta_{nt(N_{nth}+1)h}\right) = 0 \tag{A2.19}$$

and $\beta_{nt(N_{nth}+1)h} = \phi_h\left(N_{nth} + 1\right)^{\delta}$ (A2.20)

So

$$\pi_{nt(N_{nth}+1)h} = \frac{1}{\sigma - 1}\left(\phi_h \left(N_{nth} + 1\right)^{\delta}\right)^{1-\sigma} \varpi^{-\sigma} \mu \left(P_{nth}\right)^{\sigma-1} \left(I_{nth} + I_{ntf}\right) - \alpha = 0 \quad \text{(A2.21)}$$

Similarly, for the foreign (less efficient) country the corresponding zero profit condition will look like:

$$\pi_{nt(N_{nth}+1)f} = \frac{1}{\sigma - 1}\left(\phi_f \left(N_{ntf} + 1\right)^{\delta}\right)^{1-\sigma} \varpi^{-\sigma} \mu \left(P_{ntf}\right)^{\sigma-1} \left(I_{nth} + I_{ntf}\right) - \alpha = 0$$

(A2.22)

The above equations imply that the prices of the least efficient firms are equal and give us the result of Montagna [2001] that ***in the case of free trade without any barriers we have the unification of the competitive condition on the monopolistically competitive market***.

Two equations (A2.18) and (A2.21) allow us to find unique solution for N_{nth}, and P_{nth}, to describe the equilibrium in case of free trade without the transportation costs.

It is quite useful to derive the overall expenditures on the differentiated product varieties for a country (as a sum of all expenditures on the domestic varieties):

$$E_{nth}^{h} = \mu I_{nth} \frac{\left(P_{nth}\right)^{1-\sigma}}{\left(P_{nth}\right)^{1-\sigma} + \left(P_{ntf}\right)^{1-\sigma}} = \mu I_{nth} \frac{\left(P_{nth}\right)^{1-\sigma}}{\left(P_{nt}^{h}\right)^{1-\sigma}} \quad \text{(A2.23)}$$

Similarly, we get the following expression for the overall expenditures on the foreign varieties:

$$E_{nth}^{h} = \mu I_{nth} \frac{\left(P_{ntf}\right)^{1-\sigma}}{\left(P_{nth}\right)^{1-\sigma} + \left(P_{ntf}\right)^{1-\sigma}} = \mu I_{nth} \frac{\left(P_{ntf}\right)^{1-\sigma}}{\left(P_{nt}^{h}\right)^{1-\sigma}} \quad \text{(A2.24)}$$

Naturally, the sum of these two expenditures is equal to the part of income dedicated to the spending on differentiated products. Taking into account results obtained above, I can present aggregated variables as follows:

$$Q_{nth} = \varpi\left(E^{h}_{nth} + E^{f}_{nth}\right) \qquad \text{(A2.25)}$$

$$\pi_{nth} = \frac{1}{\sigma}\left(E^{h}_{nth} + E^{f}_{nth}\right) - \alpha N_{nth} \qquad \text{(A2.26)}$$

Similarly we derive the corresponding expressions for the foreign country.

3.4. The equilibrium in the labour market and market for homogeneous product

From now on I proceed to the description of the equilibrium in the competitive sector in the home:

$$A^{s}_{nth} = L^{d}_{A_{nth}} \qquad \text{(A2.27)}$$

The equilibrium on the competitive market requires that

$$\left(1-\mu\right)I_{nth} \equiv A^{h\,d}_{nt} = A^{h}_{nth} + A^{h}_{ntf} \equiv A^{h\,s}_{nt} \qquad \text{(A2.28)}$$

And on the labour market we have:

$$\overline{L}^{s} = L^{d}_{D_{nth}} + L^{d}_{A_{nth}} \qquad \text{(A2.29)}$$

$$L^{d}_{D_{nth}} = \alpha N_{nth} + \int_{1}^{N_{nth}+1} C_{ntih}\,di == \alpha N_{nth} + \frac{1}{\varpi}\left(E^{h}_{nth} + E^{f}_{nth}\right) \qquad \text{(A2.30)}$$

Finally, overall income earned in home country is defined as:

$$I_{nth} = \overline{L}^s + \pi_{nth} = \overline{L}^s + \frac{1}{\sigma}\left(E^h_{nth} + E^f_{nth}\right) - \alpha N_{nth} \qquad \text{(A2.31)}$$

Similarly, we derive the corresponding aggregate values for foreign (less efficient) country. Again the results of the model are exactly as in free trade case in Montagma [2001].

Appendix 3

SIMULATIONS FOR BASIC MODEL OF CHAPTER 3

To make the simulation I took 22 equations that fully describe the equilibrium in case of international trade with trade barriers. These are:

$$\frac{1}{\sigma-1}\left(\phi_h\left(N_{th}+1\right)^{\delta}\right)^{1-\sigma}\varpi^{-\sigma}\mu\left(\frac{I_{th}}{\left(P_{th}\right)^{1-\sigma}+\left(\tau P_{tf}\right)^{1-\sigma}}+\frac{\tau^{1-\sigma}I_{tf}}{\left(P_{tf}\right)^{1-\sigma}+\left(\tau P_{th}\right)^{1-\sigma}}\right)-\alpha=0 \qquad \text{(A3.1)}$$

$$\frac{1}{\sigma-1}\left(\phi_f\left(N_{tf}+1\right)^{\delta}\right)^{1-\sigma}\varpi^{-\sigma}\mu\left(\frac{I_{tf}}{\left(P_{tf}\right)^{1-\sigma}+\left(\tau P_{th}\right)^{1-\sigma}}+\frac{\tau^{1-\sigma}I_{th}}{\left(P_{th}\right)^{1-\sigma}+\left(\tau P_{tf}\right)^{1-\sigma}}\right)-\alpha=0 \qquad \text{(A3.2)}$$

$$P_{th}=\varpi\phi_h\left(\frac{\left(N_{th}+1\right)^{\theta}-1}{\theta}\right)^{\frac{1}{1-\sigma}} \qquad \text{(A3.3)}$$

$$P_{tf}=\varpi\phi_f\left(\frac{\left(N_{tf}+1\right)^{\theta}-1}{\theta}\right)^{\frac{1}{1-\sigma}} \qquad \text{(A3.4)}$$

$$P_t^h=\left(\left(P_{th}\right)^{1-\sigma}+\left(\tau P_{tf}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \qquad \text{(A3.5)}$$

$$P_t^f=\left(\left(P_{tf}\right)^{1-\sigma}+\left(\tau P_{th}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \qquad \text{(A3.6)}$$

$$Q_{th} = \mu P_{th}^{-\sigma}\left(\frac{I_{th}}{(P_{th})^{1-\sigma} + (\tau P_{tf})^{1-\sigma}} + \frac{\tau^{1-\sigma} I_{tf}}{(P_{tf})^{1-\sigma} + (\tau P_{th})^{1-\sigma}}\right) \tag{A3.7}$$

$$\pi_{th} = \frac{1}{\sigma}\mu P_{th}^{1-\sigma}\left(\frac{I_{th}}{(P_{th})^{1-\sigma} + (\tau P_{tf})^{1-\sigma}} + \frac{\tau^{1-\sigma} I_{tf}}{(P_{tf})^{1-\sigma} + (\tau P_{th})^{1-\sigma}}\right) - \alpha N_{th} \tag{A3.8}$$

$$Q_{tf} = \mu P_{tf}^{-\sigma}\left(\frac{I_{tf}}{(P_{tf})^{1-\sigma} + (\tau P_{th})^{1-\sigma}} + \frac{\tau^{1-\sigma} I_{th}}{(P_{th})^{1-\sigma} + (\tau P_{tf})^{1-\sigma}}\right) \tag{A3.9}$$

$$\pi_{tf} = \frac{1}{\sigma}\mu P_{tf}^{1-\sigma}\left(\frac{I_{tf}}{(P_{tf})^{1-\sigma} + (\tau P_{th})^{1-\sigma}} + \frac{\tau^{1-\sigma} I_{th}}{(P_{th})^{1-\sigma} + (\tau P_{tf})^{1-\sigma}}\right) - \alpha N_{tf} \tag{A3.10}$$

$$E_{tf}^{h} = \mu I_{th}\frac{(\tau P_{tf})^{1-\sigma}}{(P_{th})^{1-\sigma} + (\tau P_{tf})^{1-\sigma}} \tag{A3.11}$$

$$E_{th}^{f} = \mu I_{tf}\frac{(\tau P_{th})^{1-\sigma}}{(P_{tf})^{1-\sigma} + (\tau P_{th})^{1-\sigma}} \tag{A3.12}$$

$$A_{th}^{s} = L_{A_{th}}^{d} \tag{A3.13}$$

$$A_{tf}^{s} = L_{A_{tf}}^{d} \tag{A3.14}$$

$$\overline{L}^{s} = L_{D_{th}}^{d} + L_{A_{th}}^{d} \tag{A3.15}$$

$$\overline{L}^{s} = L_{D_{tf}}^{d} + L_{A_{tf}}^{d} \tag{A3.16}$$

$$L_{D_{th}}^{d} = \frac{1}{\varpi}\mu P_{th}^{1-\sigma}\left(\frac{I_{th}}{(P_{th})^{1-\sigma} + (\tau P_{tf})^{1-\sigma}} + \frac{\tau^{1-\sigma} I_{tf}}{(P_{tf})^{1-\sigma} + (\tau P_{th})^{1-\sigma}}\right) + \alpha N_{th} \tag{A3.17}$$

$$L^{d}_{D_{tf}} = \frac{1}{\varpi}\mu P_{tf}^{1-\sigma}\left(\frac{I_{tf}}{(P_{tf})^{1-\sigma}+(\tau P_{th})^{1-\sigma}}+\frac{\tau^{1-\sigma}I_{th}}{(P_{th})^{1-\sigma}+(\tau P_{tf})^{1-\sigma}}\right)+\alpha N_{tf} \quad \text{(A3.18)}$$

$$I_{th} = \overline{L}^{s} + \pi_{th} \quad \text{(A3.19)}$$

$$I_{tf} = \overline{L}^{s} + \pi_{tf} \quad \text{(A3.20)}$$

$$V_{th} = \frac{(1-\mu)^{1-\mu}}{\mu^{-\mu}}\left(P_{t}^{h}\right)^{-\mu} I_{th} \quad \text{(A3.21)}$$

$$V_{tf} = \frac{(1-\mu)^{1-\mu}}{\mu^{-\mu}}\left(P_{t}^{f}\right)^{-\mu} I_{tf} \quad \text{(A3.22)}$$

For needs of programming I denote the variables and parameters in the following way:

N_{th} =N, the total number of firms in the monopolistically competitive sector in the home more efficient country;

N_{tf} =n, the total number of firms in the monopolistically competitive sector in the foreign less efficient country;

P_{th} =p, the producers' price index in the home more efficient country;

P_{tf} =q, the producers' price index in the foreign less efficient country;

I_{th} =U, total income of the home more efficient country;

I_{tf} =u, total income of the foreign less efficient country;

P_{t}^{h} =P, consumers' price index in home more efficient country;

P_t^f =Q, consumers' price index in foreign less efficient country;

Q_{th} =D, the total output in the monopolistically competitive sector of the home more efficient country;

Q_{tf} =d, the total output in the monopolistically competitive sector of the foreign less efficient country;

π_{th} =Z, total profits earned in the monopolistically competitive sector of the home more efficient country;

π_{tf} =z, total profits earned in the monopolistically competitive sector of the foreign less efficient country;

E_{tf}^h =G, total expenditures of the home country consumers on the imported differentiated product varieties;

E_{th}^f =g, total expenditures of the foreign less efficient country consumers on the imported differentiated product varieties

A_{th}^s =S, the supply of homogeneous product varieties in the perfectly competitive sector in the home more efficient country;

A_{tf}^s =s, the supply of homogeneous product varieties in the perfectly competitive sector in the foreign less efficient country;

$L_{D_{th}}^d$ =K, the demand for labour in the monopolistically competitive sector of the home more efficient country;

$L_{D_{tf}}^d$ =k, the demand for labour in the monopolistically competitive sector of the foreign less efficient country;

V_{th} =V, the total welfare of the home more efficient country;

V_{tf} =v, the total welfare of the foreign less efficient country;

$\hat{\beta}_h$ =R, the efficiency cutoff point for the home more efficient country;

$\hat{\beta}_f$ =r, the efficiency cutoff point for the foreign less efficient country;

The parameters are denoted in the following way:

ϕ_h =h, the minimal marginal costs of the most efficient firm in the home more efficient country;

ϕ_f =f, the minimal marginal costs of the most efficient firm in the foreign less efficient country;

$\overline{L}^s$ =L for more efficient country and l for less efficient one, the labour endowment, assumed to be the same for both countries;

σ=a, the elasticity of substitution between the differentiated product varieties;

τ=t, the trade costs;

μ=m, the fraction of income spend on the differentiated product varieties;

ω=w, constant mark-up;

δ =b, the degree of firms' heterogeneity;

α=c, fixed costs.

Following Montagna [2001] I have assumed:

ϕ_h =h=1; ϕ_f =f=1.1; $\overline{L}^S$ =L=l=1; σ=a=2.1; μ=m=0.7; ω=w= $\frac{a}{a-1}$; δ=b=1; α=c=0.001.

Then the system of equations described above was solved for the $\tau \in (1;100)$ in Maple by running the following set of commands:

```
eqn1:=P=((p)^(1-a)+(t*q)^(1-a))^(1/(1-a));

eqn2:=Q=((q)^(1-a)+(t*p)^(1-a))^(1/(1-a));

eqn3:=p=w*h*((-1+(N+1)^(b*(1-a)+1))/(b*(1-
a)+1))^(1/(1-a));

eqn4:=q=w*f*((-1+(n+1)^(b*(1-a)+1))/(b*(1-
a)+1))^(1/(1-a));

eqn5:=c=((U)/((t*q)^(1-a)+p^(1-a))+(u*t^(1-
a))/((t*p)^(1-a)+q^(1-a)))*m*(w^(-a))*(1/(a-
1))*(h*(N+1)^b)^(1-a);

eqn6:=c=((u)/((t*p)^(1-a)+q^(1-a))+(U*t^(1-
a))/((t* q)^(1-a)+p^(1-a)))*m*(w^(-a))*(1/(a-
1))*(f*(n+1)^b)^(1-a);

eqn7:=Z=((U)/((t*q)^(1-a)+p^(1-a))+(u*t^(1-
a))/((t*p)^(1-a)+q^(1-a)))*m*(1/a)*p^(1-a)-c*N;

eqn8:=z=((u)/((t*p)^(1-a)+q^(1-a))+(U*t^(1-
a))/((t*q)^(1-a)+p^(1-a)))*m*(1/a)*q^(1-a)-c*n;

eqn9:=U=L+Z;

eqn10:=u=l+z;
```

```
eqn11:=K=((U)/((t*q)^(1-a)+p^(1-a))+(u*t^(1-
a))/((t*p)^(1-a)+q^(1-a)))*m*(w^(-1))*p^(1-
a)+c*N; eqn12:=k=((u)/((t*p)^(1-a)+q^(1-
a))+(U*t^(1-a))/((t*q)^(1-a)+p^(1-a)))*m*(w^(-
1))*q^(1-a)+c*n;
eqn13:=S=L-K;
eqn14:=s=l-k;
eqn15:=D=((U)/((t*q)^(1-a)+p^(1-a))+(u*t^(1-
a))/((t*p)^(1-a)+q^(1-a)))*m*p^(-a);
eqn16:=d=((u)/((t*p)^(1-a)+q^(1-a))+(U*t^(1-
a))/((t*q)^(1-a)+p^(1-a)))*m*q^(-a);
eqn17:=R=h*(N)^b;
eqn18:=r=f*n^b;
eqn19:=V=((1-m)^(1-m)/m^(-m))*U*P^(-m);
eqn20:=v=((1-m)^(1-m)/m^(-m))*u*Q^(-m);
eqn21:=G=(m*U*(t*q)^(1-a))/(((t*q)^(1-a))+(p^(1-
a)));
eqn22:=g=(m*u*(t*p)^(1-a))/(((t*p)^(1-a))+(q^(1-
a)));
a:=2.1;
w:=a/(a-1);
f:=1.1;
h:=1;
```

```
b:=1;
c:=0.001;
m:=0.7;
L:=1;
l:=1;
for t from 1 to 100 do
fsolve({eqn1,eqn2,eqn3,eqn4,eqn5,eqn6,eqn7,eqn8,e
qn9,eqn10,eqn11,eqn12,eqn13,eqn14,eqn15,eqn16,eqn
17,eqn18,eqn19,eqn20,eqn21,eqn22},{P,p,Q,q,N,n,U,
u,Z,z,D,d,S,s,R,r,K,k,V,v,G,g},{P=0..50,p=0..50,Q
=0..10,q=0..10,N=0..300,n=0..300,U=0..50,u=0..50,
Z=0..50,z=0..50,D=0..50,d=0..50,S=0..50,s=0..50,R
=0..300,r=0..300,K=0..50,k=0..50,V=0..50,v=0..50,
G=0..10,g=0..10}) end do;
```