THE IMPACT OF INCOME TAX RATE CHANGE ON THE ECONOMY OF UKRAINE

by

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Abstract

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This thesis studies the impact of change in income tax rate on the economic activity and, more precisely, the effect of recent decrease income tax from 20% to 13% at Ukraine. With this aim, the indivisible labor model was used in the context of shadow economy and tax evasion. As a result, the research has come with some interesting theoretical and empirical findings. First, it was shown that the behavior of an individual in the derived model is the same as in divisible labor model. Next, using Ukrainian data I simulated the transition dynamic and has found that probability to work officially increases from 15% under 20% tax rate to 24% under 13% rate. The transition of consumption and capital is the same under 20% and 13% rate. Nevertheless, the change in tax rate does not influence their steady-state values. Besides, life-time utility of individuals decreases with the considered change in tax rate because decrease in relative utility of social guarantees, which introduced in individual's utility function.

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Chapter 1

INTRODUCTION

During the last year substantial changes in fiscal policy happened in Ukraine, among them is the income tax rate reduction to 13%. Ukrainian income tax became one of the lowest in Europe. As a result, several interesting questions have arisen: Does this reduction have positive or negative effect on the Ukrainian economy? Does this change improve welfare? What is the influence of this income tax change on the share of shadow economy? Each of these questions is very important and requires detailed investigation.

The government fiscal policy has profound effect on different aspect of life. The income tax influences, at first, labor/leisure choice, tax revenue, and, as a result, public goods production (police, research, education, infrastructure, etc.) which affects the output. The income tax change affects the labor supply in both official economy and unofficial economy. Therefore, income tax rate has a substantial impact on economy as a whole. It is impossible to investigate all these phenomena simultaneously, but I will try to emphasize in my thesis the most important, from my point of view. It is the individual's choice between official and shadow sector of labor market.

However it is difficult to say something about the results of this policy in Ukraine because only one year has passed from the change. Moreover, Ukraine is a young country, which has not faced with such economic phenomenon as income tax cut before. Thus, the only way to predict results of the income tax reduction is to rely on theoretical approach.

Let's try to infer probable results of the income tax reduction for a country. As a result of income tax reduction, the quantity of hours that a worker wants to work can increase because wage increases due to tax rate reduction (income effect). Nevertheless on the other hand, the quantity of working hours can decrease because wage increase and people decide to work less (substitution effects). Thus we can not say exactly if people work more or less because it depends on magnitude of income and substitution effects. At the same time, income tax change provokes tax revenue change. If labor/leisure choice does not change significantly and tax reduces then the tax revenue decreases too. As a result, the provision of public goods has to decrease. This reduces output and economic growth. However this analysis has one very important drawback, namely it does not take into account shadow economy. This phenomenon is of prior importance in analyzing the impact of tax rate change on an economy because, as a result, of fiscal policy modification, the share of shadow economy may alter. This could lead to the substantial change in tax revenue.

The Ukrainian government decreased income tax and it hoped that part of the workers will prefer to declare their real income (work officially) and will pay taxes. Therefore the analysis of such policy for Ukraine should be held within the framework of shadow economy and tax evasion. Ukraine does not have strong government institutions that can provide implementation of Ukrainian laws. People may hide part of their income or work in the shadow and do not pay a tax on income. Tax evasion in Ukraine is not so expensive relative to the tax burden like in a developed country. Thus, it is a much encountered question for Ukrainian people: "Whether to declare their income and to pay taxes or not to declare and pay something for evading?" Actually it is the decision to work officially or unofficially. So, the tax evasion will play significant role in my research. The problem of tax evasion in Ukraine is connected closely with a problem of shadow economy. Ukrainian shadow labor market was investigated theoretically and empirically in many scientific works (Schneider, Enste (2000), Loayza (1996)). Based on these works shadow economy and tax evasion in Ukraine was estimated to be 40-50% (depending on the method of estimation).

The income tax cut can essentially decrease the share of the shadow economy and tax evasion. Moreover income tax is one of the most important factors in the people's choice of working officially or unofficially (people work unofficially in order to avoid the taxation). The income of the shadow economy is not taxed and lost for government; it leads to decrease of public goods production, the one of the important factor of economic growth. This change in the public goods production due to changes in income tax has to be a major concern of economists. Therefore one of the main purposes of my paper is to define theoretically the impact of income tax rate reduction on individual's choice to work officially or unofficially (to pay taxes or avoid).

In order to develop model I have to define the main possibility for income tax evasion. To avoid the tax people start to work unofficially. As Schneider, Enste (2000) stated, work in the shadow economy may have three forms:

1) Second job after (or even during) regular working hours;

2) Work by individuals who do not participate in the official labor market;

3) Work by people (e.g. clandestine, social fraud, or illegal immigrants) who are not allowed to work in the official economy.

The more frequent form of the tax evasion in Ukraine (and in the other post soviet countries) is unofficial work at the official sector of economy. People prefer not to register in the firm, as official workers. As a result, such worker is officially unemployed, but unofficially he/she receives wage and does not pay income tax because he/she does not declare the wage. Firm, where such individual works, does not pay payroll tax. In addition there is another possibility of tax evasion - to work officially (to be registered as official worker), but declare minimal wage. The individual pays income tax from minimal wage whereas the real (undeclared and paid in cash) wage exceed the minimal wage substantially.

However there is some risk for the individuals working in the shadow economy, they can be caught and in this case he/she has to pay penalty. In Ukraine there are no laws which determine the penalty for unofficial workers, but legislation for firms that hire workers unofficially and avoid the taxes exists. If firm is caught then the worker does not receive the wage promised. So, this reduction in wage can be considered as a penalty for unofficial worker.

The share of the labor force in the shadow economy and the choice of individuals whether to work in official sector or unofficial attract my attention. So, I decide to consider in my paper tax evasion as unofficial work in official sector of the economy. But in this economy the labor is indivisible and individuals choose to work officially or unofficially. Thus individual supplied the fixed amount of labor officially or unofficially. For the purpose of investigation I am going to use Hansen (1985) and Rogerson (1988) neoclassical growth model. In this model I will consider tax evasion in economy where labor supply in official sector is indivisible. It is new approach to the investigation of tax evasion. This approach is appropriate not only for Ukraine, but also to the other post-soviet countries.

In order to analyze the impact of income tax rate reduction on official economy I demonstrate the link between indivisible and divisible labor models.

The paper will be organized as follows. Chapter 2 will contain the review of the studies related to the topic. Thus in the first part of my literature review I will consider the papers that study the problem of tax evasion. In the second part one can find brief review of models with indivisible labor. Chapter 3 will include the detailed description and derivation of the dynamic general equilibrium model in context of tax evasion and indivisible labor. Chapter 4 will describe the parameters used in order to calibrate the model. Chapter 5 contains the methodology and the results of quantitative analysis (calibration, simulation). Chapter 6 concludes and summarizes the results.

Chapter 2

LITERATURE REVIEW

The first part of literature review will provide a concise survey of literature related to the topic of tax evasion. The second part will be review of the models with indivisible labor.

First of all, it is necessary to note, that the theoretical literature on tax evasion is summarized in Cowell (1990). The literature on tax evasion is closely related to literature on shadow economy; especially it applies to theoretical papers. Tax evasion covers only part of shadow economy, but many authors concentrate their attention on tax evasion and in their research do not take into "pure" household production, voluntary nonprofit services, and informal, irregular and criminal activities, in order to simplify the model or because of unavailability of the data. Thus, some studies on shadow economy will be considered in the literature review. Schneider and Enste (2000) review the theoretical studies on shadow economy.

Majority of studies on tax evasion consider labor/leisure choice that is affected by taxes and labor supply in shadow economy that is stimulated by high income tax. As Schneider and Enste (2000) stated "the bigger the difference between the total cost of labor in the official economy and after-tax earnings (from work), the greater the incentive to avoid this difference and to work in the shadow economy".

There exist different views on impact of tax evasion on economic growth. Some authors on a base of their research conclude that tax evasion has negative effect on economic growth because as a result of it tax revenue falls. But others had the opposite judgment.

Loayza (1996) studied relation of official and unofficial sectors of economy. He used the Multiple-Indicator-Multiple-Cause model. The main feature of this model is that the informal sector is defined as unobservable variable which has multiple causes and indicators. He uses endogenous growth and production function that depends on congestible public services. Loayza conducted empirical research for fourteen Latin American countries. He concluded that "in economies where (1) the statutory tax burden is lager than optimal, and where (2) enforcement of compliance is too weak, the increase in the relative size of the informal economy generate the reduction in economic growth". The cause for this phenomenon is reverse relation between the unofficial sector and public infrastructure, which influences significantly economic growth. So, Loayza postulates that reduction in shadow economy (or tax evasion) leads to increases in tax revenue and, as a result, increases in public goods production that stimulates economic growth. Loayza has opponents. For example, Schneider (2000) show that more than 66% of unofficial economy income is spent in official sector that has positive impact on economic growth. Adam and Ginsburgh (1985), Fichtenbaum (1989) drawn the same conclusion based on empirical analysis of Belgium and US economies.

Lemieux, Fortin and Frechette (1994) studied the effect of taxes on labor supply in the shadow economy. They use additive-separable utility function and two-stage decision process for the consumers. The very important idea of this research is that the labor earnings in the shadow sector are a concave function of hours of work. By contrast, the wage rate of a worker in the regular sector does not vary with the number of hours worked. The result this study suggest that hours worked in the shadow sector are quite responsive to changes in the net wage in regular sector. But tax revenue is not specified in this study. Only utility maximization problem is considered there and public good (tax revenue) is not included in utility function. Production part of the economy is not studied in this paper. On the other hand, the aim of this paper is only estimation of tax impact on work incentive, but not on economic growth.

One of the interesting questions on this topic is the spiral nature of tax evasion (increase of tax evasion as a result of tax evasion). The tax evasion leads to decrease in tax revenue. The consequence of it is reduction in quality and quantity of public goods production. This can provoke rise in tax rate, combined with deterioration of quantity and quality of public goods. The effect of this situation is stronger incentives for tax evasion. Johnson, Kaufmann and Zoido-Lobaton (1998b) investigate this relationship. They conclude that the level of tax evasion is lower in countries which have high tax revenue, if it is achieved from low tax rates. Their overall conclusion is "wealthier countries of the OECD, as well as some in Eastern Europe, find themselves in the "good equilibrium" of relatively low tax and regulatory burdens, sizable revenue mobilization, good rule of law and corruption control, and (relatively) small unofficial economy. By contrast, a number of countries of Latin America and the former Soviet Union exhibit characteristics consistent with a "bad equilibrium": tax and regulatory discretion and burden on the firm are high incidence of bribery and a relatively high share of activities in the unofficial economy" (Johnson, Kaufmann, Zoido-Lobaton (1998b)).

There are a lot of others papers that have explored the tax evasion in different contexts: Slemrod (1985), Bruce and Turnovsky (1999); Clotfelter (1983), Gordon (1992), Stuart (1981) and others.

We begin the review of the papers associated with indivisible labor with Hansen (1985) and Rogerson (1988) model because they were the first who considered the neoclassical growth model with indivisible labor. In such a model agents do not choose the hours of work. Individual can work only fixed number of hours or does not work at all. So, instead of hours worked he/she choose the probability of working (the probability of being employed). This probability is determined each period of time by lottery. Individual engages in such employment lottery. If he/she wins this lottery then he/she is employed in this period of time, otherwise – he/she is not employed. So, there exist two states of the nature and depending on which state is happened, agents receive the expected labor income. Hansen assumes logarithmic utility function additively separable in consumption and leisure.

Hansen considers the model where the labor is indivisible and agent can buy unemployment insurance each period (because he/she does not know, before lottery takes place, if he/she will be employed or unemployed). Agent can choose any amount of this insurance. If individual does not work he/she is paid by unemployment compensation. So, in this problem individual choose the probability of working, amount of insurance he/she purchase, consumption and investment, depending on he/she works or not.

Basu and Renstrom (2002) studied optimal dynamic taxation using the Hansen's model with indivisible labor. They found that the optimal income tax is zero, when leisure is neutral good. If leisure is normal good then the optimal income tax is greater than zero and if the leisure is inferior good then the labor should be subsidized. Besides, they analyze the second-best labor tax and they conclude that it depends also on the degree of complementarily between consumption and leisure. The distinctive feature of Basu's and Renstrom's model is absence of assumption of additive separable leisure in utility function because this assumption implies that leisure is a normal good. For purposes of their investigation they derive a range of preferences including non-separable leisure and belong to the Hyperbolic Absolute Risk Aversion class of preferences.

Maliar and Maliar (2004) study implications of the dynamic general equilibrium model with indivisible labor. The major difference of this model is the heterogeneity of agents. They differ in initial wealth and labor productivities that are subject to idiosyncratic shocks. They show that "if there is a continuum of identical agents who have additive utility functions, if labor is indivisible (i.e. if agents can work either a fixed number of hours or not at all), and if agents choose employment probabilities by trading lotteries, then the indivisible labor economy behaves like a representative-agent divisible-labor economy with a linear disutility of labor." The individual behavior in the indivisible-labor economy and divisible-labor quasi-linear economy is equivalent if and only if the agent from indivisible labor economy, who works with probability one or zero, corresponds to the agent from divisible labor quasi-linear economy, which works a maximum number of hours or does not work at all. But when the agents are heterogeneous this equivalence is only at the individual level (not on aggregate level).

The work that investigates the link between one-consumer and multi-consumer economies under the quasi-linear utility function (also assumed by the Hansen (1985)) is Maliar and Maliar (2003). They use the divisible-labor model with infinitely lived heterogeneous agents who differ in initial endowments, preferences and productivities. These agents derive the utility from consumption and leisure. The agents' utility functions are quasi-linear: additive in consumption and leisure and linear in leisure. In the beginning authors consider the same model, but with homogenous agents and they show that such economy has an indeterminacy of equilibrium: "If an inferior equilibrium in the homogenous agents economy exists, then an infinite number of equilibrium allocations exists for individual hours worked". These results apply to the Hansen indivisible-labor model – the optimal choice of the employment probability is not uniquely determined. Maliar and Maliar propose to introduce heterogeneity in labor in order to get rid of indeterminacy. Authors consider the case when labor productivities are subject to shocks. They show that if the markets are complete then the optimal allocation of individual working hours is at the corner: individual works maximum hours if his/her productivity is high and not to work at all if his/her productivity is low. In this case the working hours are uniquely determined. It is important that we can use such heterogeneity in the indivisible-labor model with quasi-linear preferences to get rid of indeterminacy in optimal choice of employment probability. In this case individual will choose high probability of working if productivity of labor is high.

I also want to pay attention to the exploration of this topic by EERC students. Khmilevska (2002) studied household behavior in terms of shadow economy. She analyses labor supply. Khmilevska used the two sector model with official and unofficial sectors. She considered consumer part of the economy. This work is empirical and the interesting results have been obtained: the shadow economy occupies the 35.5% of country's GDP, and approximately 71% of respondents (data supplied by Tacis UEPLAC project) are involved in the shadow economy.

Another thesis associated with tax evasion is thesis of Trehub (2004). He studied the pattern of tax evasion in Russia. He uses survey method (the same technique as applied by Clotfelter in 1983) for empirical estimation of tax evasion in Russia and their changes due to PIT reform (2001) which is associated with income tax rate cuts and 13% flat tax rate imposition. He found that more than 60% of households spend more than they earn. He made conclusion that the tax evasion is very common practice fore Russia.

My paper will be different from these two, first of all, because I am going to consider tax evasion using indivisible-labor framework. My work will be theoretical, but it will be calibrated with parameters that accord with Ukraine experience. Then I will use simulation in order to define the outcome of the tax rate variation.

Based on this brief literature review we can conclude that there exist many studies connected with problems of tax evasion. These works are theoretical and empirical. However the results of research differ significantly. According to some studies shadow economy and tax evasion have a negative effect on official economy and its growth, but some studies show opposite effect. Majority of authors study only one part of economy – consumers' part of economy or producing part of economy. Some authors consider economy with two sectors: official and unofficial.

The main feature of my work is using of the indivisible labor framework for tax evasion analysis. This approach is new and it is not used before. In my model hours worked officially is indivisible and agent can not choose hours he/she work officially (agent can work only fixed number of hours either officially or unofficially). So, individual can only choose the probability of working officially. Moreover, individual faces with exogenous probability to be caught if he/she works unofficially and if he/she is caught, he/she pay penalty. Before playing the lottery agent can buy insurance.

I think that this model reflects the Ukrainian reality because it describes situation that is typical for Ukrainian labor market. People choose to work either officially or unofficially all time i.e. labor is indivisible.

C h a p t e r   3

DESCRIPTION OF THE MODEL

Following the Hansen (1985) and Rogerson (1988) neoclassical growth model, we consider tax evasion in economy where labor supply in official sector is indivisible.

The indivisible labor model

Planner's problem

In this model individual can work fixed number of hours $\bar{n}$ either officially or unofficially. In each period of time individual engages in employment lottery, choosing the probability of working officially α_t, and the probability of working unofficially, $(1 - \alpha_t)$. So, rather than deciding whether to work or not, the individual would randomize his/her decision, engaging in this lottery. Individual receive different labor income ex post and, as a result, extrinsic labor-income uncertainty arises. If agent works unofficially he faces with exogenous probability $\beta(\gamma_t)$ to be caught or probability $(1 - \beta(\gamma_t))$ not to be caught, where γ_t is cost of avoiders detection and penalty collection. The household's consumption $(c^{i,u}_t)$ is thus conditional on whether the agent works officially, unofficially and he is not caught or unofficially and he is caught. If individual works officially then he pays income tax τ_t. If individual works unofficially then he does not pay the tax. If individual works unofficially and he is caught then he must pay penalty ρ_t. Individual's utility depends on consumption in this state and on S – parameter which define social guarantees, giving to official workers by the firm. Assume that agent has these social guaranties only if he works officially.

Time is discrete and the horizon is infinite: t∈T, where T={0,1,…,∞}. The economy consists of a continuum of infinitely-lived homogeneous agents, the output producing firm, government and two insurance companies.

The agents own the capital stock and rent it to the firm. Capital depreciates at the rate d∈(0,1]. Before playing the lottery, the agent buys security (insurance) from two insurance companies. Security issued by the first insurance company pays if agent works unofficially and is not caught, otherwise this security does not pay. Second security pays if agent works unofficially and he is caught, otherwise it pays zero. If agent works officially none of the securities pay to the agent

The problem solved by agent is follows:

$$\max_{\{x_t\}_{t\in T}} E_0\left[\sum_{t=0}^{\infty}\delta^t\left\{\alpha_t u(c_t^o,S)+(1-\beta(\gamma_t))(1-\alpha_t)u(c_t^{u,n},0)+\beta(\gamma_t)(1-\alpha_t)u(c_t^{u,c},0)\right\}\right] \quad (1)$$

subject to

$$c_t^o + k_{t+1}^o + \sum_{j=1}^{2} p_t^j q_t^j = (1-d+r_t)k_t + (1-\tau_t)\bar{n}w_t, \quad (2)$$

$$c_t^{u,n} + k_{t+1}^{u,n} + \sum_{j=1}^{2} p_t^j q_t^j = (1-d+r_t)k_t + \bar{n}w_t + q_t^1, \quad (3)$$

$$c_t^{u,c} + k_{t+1}^{u,c} + \sum_{j=1}^{2} p_t^j q_t^j = (1-d+r_t)k_t + (1-\rho_t)\bar{n}w_t + q_t^2, \quad (4)$$

where $\{x_t\}_{t\in T} = \{\alpha_t, c_t^{i,u}, k_{t+1}^{i,u}, q_t^j\}_{t\in T}^{i\in\{o,u\},u\in\{n,c\}}$, and initial endowment (k_0) is given. The subscript i∈{o,u} refers to officially and unofficially working states, in the same time u∈{c,n} – to be caught and not to be caught states if agent work unofficially. Variables $c^{i,u}_{t}$ and $k^{i,u}_{k+1}$ denote consumption and capital in state i, u; r_t – the price of capital, w_t the wage.

The momentary utility function, u, is continuously differentiable, strictly increasing in its arguments and strictly concave.

The production firm's problem

The production firm rents capital k_t, and hires labor:

$$\max \pi_t^F = f(k_t, h_t) - r_t k_t - h_t w_t \tag{5}$$

where $h_t = N\bar{n}$, N – number of individuals in the economy. The production function has CRS, is strictly concave, continuously differentiable, strictly increasing with respect to both arguments and satisfies the Inada conditions.

Government's constraint

Government collects the tax and penalty from individuals. γ_t is the cost of the penalty collection.

$$G = \tau_t \bar{n}\alpha_t w_t + \beta(\gamma_t)(1-\alpha_t)\bar{n} w_t \rho_t - \gamma_t = \bar{G} \tag{6}$$

The purpose of my paper is investigation of the tax policy impact on the economy. Thus, in order to isolate effects of taxation from effects of government expenditures, we assume that the tax revenue is constant and it is used to finance lump-sum transfer payments.

Insurance companies' problem

There are two insurance companies in the economy. Assume, like in Hansen (1985), that the insurance company maximizes the period by period expected profit.

First insurance company sells insurance q_t^1 to individual (before realization of employment lottery), with price p_t^1, and pays q_t^1 if individual in period t works unofficially and not being caught and zero otherwise.

$$\max_{\{q_t^1\}} \pi_t^{IC,1} = p_t^1 q_t^1 - (1-\beta(\gamma_t))(1-\alpha_t)q_t^1 \tag{7}$$

Second insurance company sells insurance q_t^2 to individual (before realization of employment lottery), with price p_t^2, and pays q_t^2 if individual in period t works unofficially and being caught and zero otherwise.

$$\max_{\{q_t^s\}} \pi_t^{IC,2} = p_t^2 q_t^2 - \beta(\gamma_t)(1-\alpha_t)q_t^2 \tag{8}$$

where $\beta(\gamma_t)(1-\alpha_t^s)$ is probability to work unofficially and to be caught; $(1-\beta(\gamma_t))(1-\alpha_t^s)$ is probability to work unofficially and not to be caught.

Definition. A competitive equilibrium of the economy described by this model is the sequence of consumer's allocation, $\{x_t\}_{t\in T} = \{\alpha_t, c_t^{i,u}, k_{t+1}^{i,u}, q_t^j\}_{t\in T}^{i\in\{o,u\},u\in\{n,c\},j\in\{1,2\}}$, the producer firm's allocation $\{k_t, h_t\}_{t\in T}$, the insurance companies' allocation $\{q_t^j\}_{t\in T}^{j\in\{1,2\}}$, prices and government's constraint, such that given prices

(a) the consumer's allocation solves the utility maximization problem;

(b) the producer's allocation solves the profit maximization problem of firm;

(c) the insurance companies' allocation solves the profit maximization problem of these companies;

(d) all markets clear.

The divisible labor model

Assume additive utility function:

$u(c_t, S) = v(c_t) + \varpi(S)$, where $\varpi(S) = \dfrac{S}{w_t(1-\tau)} = A$

where $v' > 0, v'' < 0$ and $\varpi' > 0, \varpi'' < 0$

Parameter A illustrates utility derived by agent from social guarantees, if he works officially. This parameter changes with the level of net wage because parameter is relative. If individual works officially and receive the net wage w, then he receives the defined amount of social guarantees. If the net wage change (for example increase, as a result of income tax decreases), the amount of social guarantees does not change, but with higher wage these guarantees are not so important for individual because he can use his higher wage to satisfy his social needs.

Agent solves the following utility-maximization problem:

$$\max_{\{b_t\}_{t\in T}} E_0\left[\sum_{t=0}^{\infty} \delta^t \{v(c_t) + A\alpha_t\}\right] \quad (9)$$

subject to

$$c_t + k_{t+1} = (1-d+r_t)k_t + \alpha_t(1-\tau_t)w_t\bar{n} + (1-\alpha_t)(1-\beta(\gamma_t))w_t\bar{n} + \beta(\gamma_t)(1-\alpha_t)(1-\rho_t)w_t\bar{n} \quad (10)$$

where $\{b_t\}_{t\in T} = \{\alpha_t, c_t, k_{t+1}\}_{t\in T}$, c_t - consumption, k_{t+1} – capital, α_t – share of time that agent works officially. If agent works unofficially he faces with exogenous probability $\beta(\gamma_t)$ to be caught or probability (1 - $\beta(\gamma_t)$) not to be caught, where γ_t is cost of avoiders detection and penalty collection. If individual works officially then he pays income tax τ_t. If individual works unofficially then he does not pay the tax. If individual works unofficially and he is caught then he must pay penalty ρ_t.

Production firm's problem is the same as in the indivisible labor economy. It is described by Eq. (5).

Proposition 1. Agent's behavior in the indivisible labor economy Eqs. (1) – (8) is identical to his behavior in divisible labor economy Eqs. (9) – (10).

Proof. See Appendix A.

Thus, agent's behavior in this indivisible labor model (where agents are identical, except the realization of lotteries) is the same as in the divisible labor model, where agents choose the share of time that they work officially and unofficially. We can interpret α_t as share of time that individual officially and (1-α_t) – share of time that agent works unofficially. Consumption and savings of the agent do not depend on the agent's status (work officially or unofficially and not being caught, unofficially and being caught). It is result of perfect risk-sharing which is possible due to introduction two insurance companies to the model (Arrow-Debreu securities).

Chapter 4

DATA DESCRIPTION

To study the effect of taxation in the neoclassical model, we calibrate it with parameters that accord with Ukraine experience. In such a case the model will explain the consequences of the income tax rate change and change of the penalty paying by individual if he/she is caught working unofficially.

For the purposes of calibration we choose a one year as a model's period.

We do not need time series for the calibration. We need only ration of the average values of the variables. Consumption to output ratio is c/y=0.7261, a yearly capital to output ratio is k/y=3.1663 will be used in the process of calibration. The value of the interest rate chosen is 0.1446. These statistics are taken from Boyarchuk, Maliar and Maliar (2004).

Also, we need government spending to output ratio (G/y) and "cost of tax, penalty collection and avoiders detection" to output (γ/y). In our model government spending consists mainly of tax and penalty receipts. Thus, we use receipts from taxation to GDP ratio as G/y. We choose costs of Tax Administration activity as proxy for "cost of tax, penalty collection and avoiders detection". The numerical values of these ratios are 0.017 and 0.0017 approximately.

We can normalize the number of hours worked to 1.

In addition we have to fix from the data probability to be caught if agent works unofficially. For this purpose we use the data on the number of criminal proceedings instituted and the data on number of taxpayer registered for a last 4 years for Ukraine (the data used is from the official site of the Tax Administration of Ukraine). To determine this variable we divide the average number of criminal proceedings instituted by the average number of taxpayer registered multiplied by the share of shadow economy. The numerical value of this parameter is 0.015.

To sum up, we use the following values of the parameters:

Table 4.1: The Values of Parameters, which characterize Economy of Ukraine

c/y	k/y	r	γ/y	G/y	n	τ	β(γ)
7261	3.1663	0.1446	0.0017	0.017	1	0.2	0.015

Chapter 5

QUANTITATIVE ANALYSIS

Calibration and solution for steady state

For the purposes of analysis assume additive separable utility function and Cobb-Douglas production function: $u(c_t, S) = v(c_t) + \varpi(S) = \dfrac{c^{1-v} - 1}{1-v} + A\alpha_t$, $f(k_t, h_t) = k_t^{\mu} h_t^{1-\mu}$ and the function for probability of being caught: $\beta(\gamma) = 1 - \dfrac{1}{1 + \varsigma\gamma}$. The production function has CRS, is strictly concave, continuously differentiable, strictly increasing with respect to both arguments and satisfies the Inada conditions. The function for probability has the properties of probability function: $\beta(0) = 0$, $\beta'(\gamma) > 0$, $\lim_{\gamma \to \infty} \beta(\gamma) = 1$.

Capital share, μ

$$f(k_t, h_t) = k_t^{\mu} h_t^{1-\mu}$$

$$f(k_t, h_t) = \frac{\partial f(k_t, h_t)}{\partial k_t} k + \frac{\partial f(k_t, h_t)}{\partial h_t} h_t$$

$$\mu f(k_t, h_t) = \frac{\partial f(k_t, h_t)}{\partial k_t} k_h$$

$$\mu = \frac{\frac{\partial f(k_t, h_t)}{\partial k_t} k_h}{f(k_t, h_t)} = \frac{r_t k_t}{y_t} = 0.4578$$

Depreciation rate, d

Capital accumulation equation $k_{t+1} = k_t(1-d) + i_t$. Suppose we are in the steady state,

$$k_{t+1} = k_t = k \Rightarrow k = k(1-d) + i \Rightarrow$$

$$d = \frac{i}{k} = \frac{i/y}{k/y} = \frac{1-c/y}{k/y} = \frac{1-0.7261}{3.1663} = 0.0865$$

Discount factor, δ

Using Euler equation (26) evaluated in steady state we receive:

$$1 = \delta(1-d+r) = \delta(1-d+\mu k^{\mu-1} h^{1-\mu}) = \delta(1-d+\mu \frac{y}{k})$$

$$\delta = \frac{1}{1-d+\mu \frac{1}{k/y}} = \frac{1}{1-0.0865+0.4578 \frac{1}{3.1663}} = 0.9451$$

The probability function parameter, ζ

$$\beta(\gamma) = 1 - \frac{1}{1+\varsigma\gamma} \Rightarrow \varsigma = \frac{\beta(\gamma)}{\gamma(1-\beta(\gamma))}$$

$$\varsigma = \frac{\beta(\gamma)}{y \frac{\gamma}{y}(1-\beta(\gamma))} = \frac{\beta(\gamma)}{\frac{w\bar{n}}{1-\mu} \frac{\gamma}{y}(1-\beta(\gamma))} = \frac{\beta(\gamma)}{\frac{w\bar{n}}{1-\mu} \frac{\gamma}{y}(1-\beta(\gamma))}$$

$$w = (1-\mu)\left(\frac{k}{\bar{n}}\right)^{\mu} = (1-\mu)\left(\frac{\bar{n}\left(\frac{r}{\mu}\right)^{\frac{1}{\mu-1}}}{\bar{n}}\right)^{\mu} = (1-\mu)\left(\frac{r}{\mu}\right)^{\frac{\mu}{\mu-1}}$$

$$\varsigma = \frac{\beta(\gamma)}{\left(\frac{r}{\mu}\right)^{\frac{\mu}{\mu-1}}\frac{\gamma}{y}(1-\beta(\gamma))} = 3.3854$$

We did not calibrate coefficient of risk-aversion in the utility function. We will consider three alternative values for this coefficient $\nu \in \{0.5,1,5\}$ and then we will perform sensitivity analysis with respect to this parameter. We take these values because they reflect the different types of individual's risk-aversion. The value of this coefficient equal to 0.5 corresponds to risk-lower person. Coefficient of risk-aversion equal to 1 relates to risk-neutral person and logarithmic utility function. The last value of this coefficient is for risk-avers person. If variation in coefficient of risk-aversion does not affect equilibrium considerably, we can say that our results are robust. Thus, we have to compute all parameters (or variables in steady state) for all of three values of risk-aversion.

Let's summarize the values of the calibrated parameters in the table:

Table 5.1: Calibrated Values of the Parameters

μ - capital share	d - depreciation	δ - discount factor	ζ - the probability parameter
0.4578	0.0865	0.9451	3.3845

Now, using the calibrated values of parameters, we can compute steady states for our model.

Steady state value of capital is evaluated from (31), using calibrated parameters:

$$k_t = \bar{n}\left(\frac{r_t}{\mu}\right)^{\frac{1}{\mu-1}} = 8.37731$$

Steady state values of consumption, probability to work officially, and penalty we can compute from the system of equations which characterize the competitive equilibrium of the problem. First equation of the system is budget constraint is steady state: $c + k = (1 - d + r)k + w(1 - \alpha\tau - \beta(\gamma)(1-\alpha)\rho)$. Then, FOC: $A\alpha + c^{-v}\left[\beta(\gamma)\rho w\bar{n} - \tau w\bar{n}\right] = 0$. Next two equations are Government constraint (6) expression for optimal level of costs of tax, penalty collection and avoiders' detection:

$$\frac{\partial G}{\partial \gamma_t} = 0 \Rightarrow \frac{\varsigma}{(1+\varsigma\gamma_t)^2}(1-\alpha_t)\bar{n}(1-\mu)\left(\frac{r}{\mu}\right)^{\frac{\mu}{\mu-1}}\rho_t = 1$$

From this system of four equations we can find A - parameter in utility function and steady state levels of consumption, probability to work officially, and penalty for each value of coefficient of risk aversion. Results of computations are summarized in the following table.

Table 5.2: The Steady State Values of Consumption, Penalty, Utility Parameter and Probability to Work Officially, t=20%.

v - coefficient of risk-aversion	A –utility parameter	α - probability to work officially	c - consumption	ρ - penalty
v=0.5	1.314533979	0.1565299859	1.871927338	0.2515893605

v=1	0.9607874535	0.1565299859	1.871927338	0.2515893605
v=5	0.07824766301	0.1565299859	1.871927338	0.2515893605

These values are computed for tax rate 20%. We can consider this tax rate as an average income tax rate in Ukraine for last years, taking into account progressiveness of Ukrainian tax system.

We also can draw important consideration from the table: for every coefficient of risk aversion the values of variables in steady state are same. The only parameter which changes is utility parameter A. So, we can conclude that results do not depend on risk aversion of agent.

Now, let us analyze the results obtained. Probability to work officially is approximately 15%. It means that approximately 85% of people participate in unofficial labor market. This result is some higher than estimation results of Shneider and Enste (2000), but not significantly. They find that the share of shadow economy in Ukraine is 28-43% of GDP and share of participation in unofficial economy is approximately 70%. Such a result is possible if people participated in unofficial economy produce a very small part of official GDP.

Steady state level of consumption and capital is 1.8716 and 8.37731 correspondingly. We can interpret these results only relative to each other or to some variables that characterize the steady state of the economy because during the calibration we use ratios instead of the values of variables.

The steady state value of penalty is 25% approximately. Also, we have the interesting results for utility parameter A, which illustrate utility derived by agent from social guarantees, if he works officially. This parameter changes with the level of net wage because parameter is relative. If individual works officially and receive the net wage w, then he receives the defined amount of social guarantees.

If the net wage change (for example increase, as a result of income tax decreases), the amount of social guarantees does not change, but with higher wage these guarantees are not so important for individual because he can use his higher wage to satisfy his social needs. It is only parameter that changes with risk-aversion. So, I expect that this parameter decline if tax rate decrease and if risk aversion increase

Now, let us carry out a policy experiment. This experiment reflects Ukrainian fiscal policy modification. Suppose that income tax rate is reduced to 13%. Using the calibrated parameters of the model, we recomputed steady states for tax rate equal to 13% for every assumed coefficient of risk aversion. For calculations we use the same system of for equations as in the previous case. Results are summed up in the table.

Table 5.3: The Steady State Values of Consumption, Penalty, Utility Parameter and Probability to Work Officially, t=13%.

v - coefficient of risk-aversion	A –utility parameter	α - probability to work officially	c - consumption	ρ - penalty
v=0.5	0.5478147376	0.240815363	1.871927338	0.2795210428
v=1	0.400395528	0.240815363	1.871927338	0.2795210428
v=5	0.03260868388	0.240815363	1.871927338	0.2795210428

In this case we also can infer that steady state level of variables does not depend on the risk-aversion of individual and only parameter which changes is utility parameter.

We can see from the table that the change in income tax does not influence consumption, but probability to work officially increase significantly. Penalty also increases but not much. Utility parameter A, which reflects relative utility derived by agent from social guarantees, decrease, as I expect.

Thus, the analysis suggests that tax policy (reduction of income tax rate) conducting in Ukraine has quite significant effect on the size of unofficial labor market. This phenomena is confirmed by theory (Shneider, Enste (2000), Stuart (1981))

Dynamic solution of the problem

To solve this model dynamically we have to log-linearize the FOCs, taking a first order Taylor approximation around the steady state to replace the equations with approximations, which are linear in the log-deviations of the variables $\tilde{k}_t$, $\tilde{c}_t$, and $\tilde{\alpha}_t$. The process of log-linearization is described in the Appendix B. As a result of log-linearization we receive three equations that depend on three variables $\tilde{k}_t$, $\tilde{c}_t$, and $\tilde{\alpha}_t$. I postulate that that the decision functions have the following form:

$$\tilde{k}_{t+1} = \Phi_{KK}\tilde{k}_t \qquad (11)$$

$$\tilde{c}_t = \Phi_{KC}\tilde{k}_t \qquad (12)$$

$$\tilde{\alpha}_t = \Phi_{KA}\tilde{k}_t \qquad (13)$$

Where the coefficients Φ_{KK}, Φ_{KC}, and Φ_{KA} are unknown constant, which we can find from the system of tree linearized FOCs. This method is Method of Undetermined Coefficients. Thus, we compute these coefficients for two levels tax rates and for each coefficient of risk-aversion. The results of calculation are presented in the table.

Table 5.4: The Values of Decision Function's Coefficients (Method of Undetermined Coefficients)

Tax, τ	v	Φ_{KK}	Φ_{KC}	Φ_{KA}
20%	0.5	1.2285	-0.7965	0.8694
	1	1.1661	-0.5202	1.0042
	5	1.088	-0.1832	1.5341
13%	0.5	1.2291	-0.7951	1.3660
	1	1.1666	-0.51895	1.6006
	5	1.0876	-0.1839	2.8092

Now, recall that $\tilde{x}_t = \log x_t - \log x$. So, we can rewrite (11) as $\log k_{t+1} - \log k = \Phi_{KK}(\log k_t - \log k)$ or $\log \frac{k_{t+1}}{k} = \Phi_{KK} \log \frac{k_t}{k}$. If we get rid of logarithms we will receive recursive dependence of capital in period t on capital in period t+1.

$$k_{t+1} = k\left(\frac{k_t}{k}\right)^{\Phi_{KK}} \qquad (14)$$

We can use (14) to simulate the series for capital, which reflect the convergence of this variable to steady state. Then we have to do the same transformation with (12) and (13). As a result, we have two equations: for consumption and probability to work officially.

$$c_t = c\left(\frac{k_t}{k}\right)^{\Phi_{KC}} \tag{15}$$

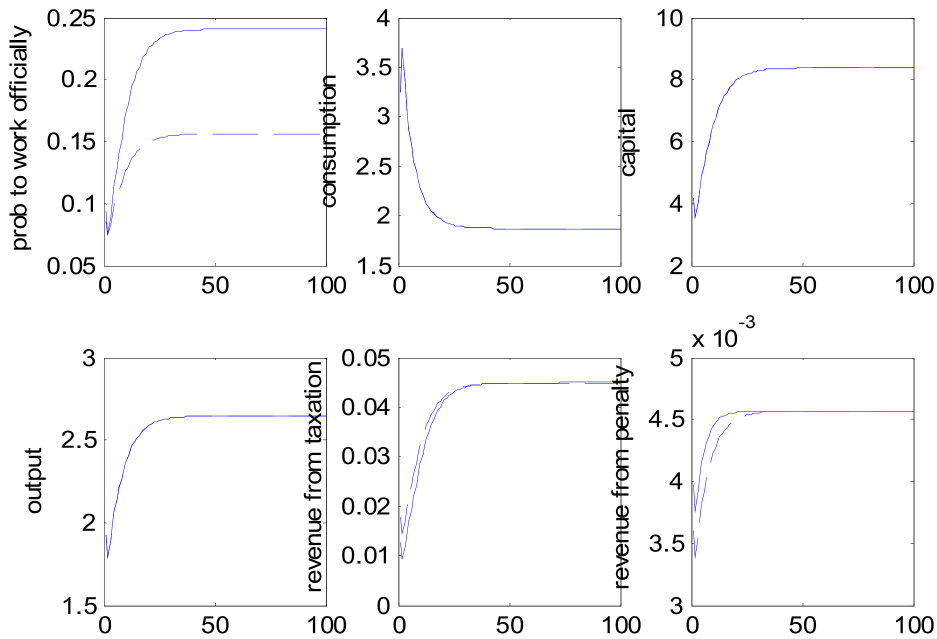
$$\alpha_t = \alpha\left(\frac{k_t}{k}\right)^{\Phi_{KA}} \tag{16}$$

Using (14), (15) and (16), we can simulate series for capital, consumption and probability to work officially. We simulate series for two different tax rates: 20% and 13% and for each of coefficient of risk-aversion. In order to simulate the series we assume that k_0 is equal to one half of capital value in steady state, 4.1887.

Let us consider the results of simulations. On the Figures 1-5 transitional dynamics for Probability to Work Officially, Consumption, Capital, Output, Revenue from Taxation and revenue from Penalty Collection for tree different coefficients of risk-aversion are simulated. The tendencies for different risk-aversion are the very similar. So, we can conclude that the result and the model are robust to the changes in models parameters. The main difference in these figures is curvature of the graphs. The graphs for lower coefficient of risk-aversion are steeper. The intuition for this fact is follows: the more risk-averse individual, the lower his willingness to substitute present consumption for future consumption. As a result, transaction path of capital and probability to work officially are slow relatively to those for less risk-averse person. The consequence of this is gradual transaction dynamics of other.

Let us consider the behavior of consumption in the light of our model. First of all it is necessary to admit that the transaction dynamics for both tax levels is identical. The level of consumption decreases during transition period because of higher value of capital accumulation, as a result of fall in tax rate.

Figure 5.1: Simulated Transaction Path for Probability to Work Officially, Consumption, Capital, Output, Revenue from Taxation and revenue from Penalty Collection, for Tax Level of 20% (dotted line) and 13% (solid line), v=0.5



According to assumed Cobb-Douglas production function, the essential input in our production is capital since we have normalized the labor input to one. Thus, the direction of change in output is the same as the change in capital.

Next, let us examine the dynamic of tax revenue and revenue from penalty collection. Before, it is essentially to note that in our model the government revenue is constant with purpose to isolate the influence of this factor on official economy from impact of tax rate change. On the value of the revenue both decrease in tax rate and increase in probability to work officially (rise in official sector of labor market) influence. Since in our model tax revenue is affected positively by the tax rate and by probability to work officially, the tax rate cut decreases the tax revenue and higher probability to work officially increases tax revenue. The effect of the tax rate fall is greater and, as a result, tax revenue transaction path is slower for tax rate equal to 13%. In order to maintain constant level of government revenue the revenue from penalty collection for 13% tax rate get to steady state rapidly than for 20%.

Figure 5.2: Simulated Transaction Path for Probability to Work Officially, Consumption, Capital, Output, Revenue from Taxation and revenue from Penalty Collection, for Tax Level of 20% (dotted line) and 13% (solid line), v=1

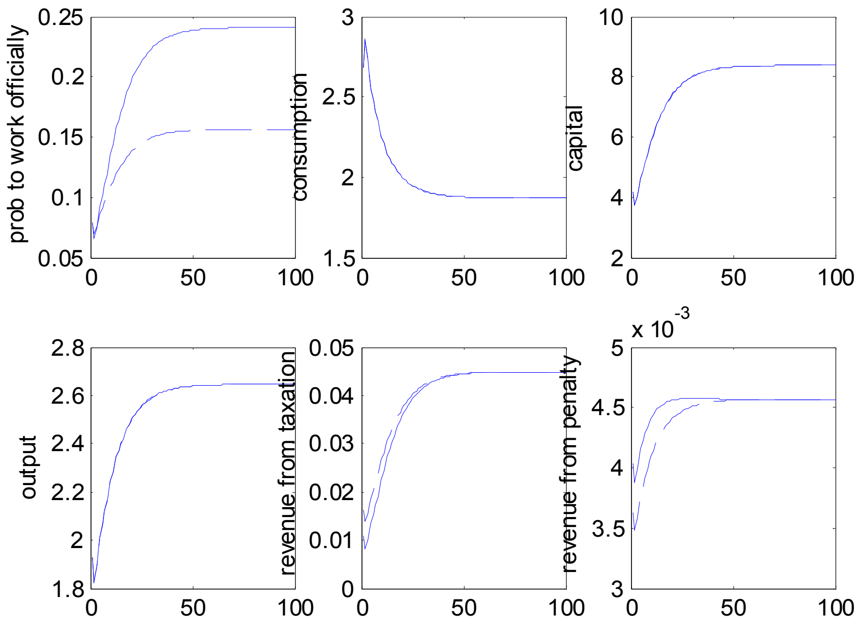
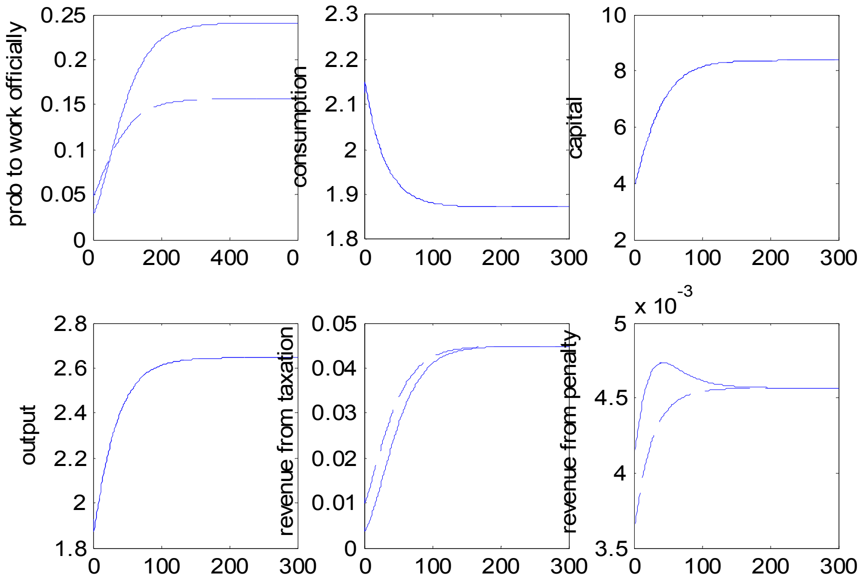


Figure 5.3: Simulated Transaction Path for Probability to Work Officially, Consumption, Capital, Output, Revenue from Taxation and

revenue from Penalty Collection, for Tax Level of 20% (dotted line) and 13% (solid line), v=5



Now, let us analyze the impact of tax rate change on welfare. Recall that the representative agent has the additive separable utility function of the following form:

$u(c_t, S) = v(c_t) + \varpi(\alpha_t) = \dfrac{c^{1-v} - 1}{1-v} + A\alpha_t$, where c is consumption, v – coefficient of risk-aversion, α - probability to work officially, A – parameter that reflects relative utility derived by agent from social guarantees.

Agent maximizes life-time utility: $U = \sum_{t=1}^{\infty} \delta_t \left(\dfrac{c_t^{1-v} - 1}{1-v} + A\alpha_t\right)$. From our steady state computations and from simulations we know that the steady state value of consumption does not change when tax rate decrease and it has the same transition dynamics for same coefficient of risk-aversion for different tax rates.

So, if we want to compare the value of the sum of utilities discounted to period 0, we have to compare only the second terms in utility function because the first terms are equal, as a result, the sums of its discounted values are also equal, and discount factor is the same.

Let us compute the second term of utility function for each of the tree coefficients of risk aversion and for two level of the income tax. The results are presented in the table.

Tax rate	A*α		
	V=0.5	V=1	V=5
20%	0.2057	0.15036	0.01225
13%	0.1319	0.09664	0.00785

From these computations we can conclude that individual derives more utility when tax rate was 20%. This result is obvious because utility parameter A decrease more sharply than the probability to work officially increase. This is true for each level of risk-aversion. Thus, the utility of individual is greater if tax 20% than if it is 13%. But at the same time the utility from consumption does not change. So, we can conclude that in economy where individual receive utility not only from consumption, but also from social guaranties utility of which he evaluates relative to net wage, the overall utility in each period of time decrease with decrease in tax.

So, the income tax reduction results in significant rise of official sector of labor market, but to decrease in life-time, as a result of sacrifice in utility from social guarantees.

Chapter 6

SUMMARY AND CONCLUSIONS

In my thesis I have investigated the impact of income tax rate reduction on the economy of Ukraine. My analysis is turned within the frame of shadow economy and tax evasion. The model with indivisible official labor was used because, it is the model which reflects real situation in the Ukrainian labor market. People choose to work fixed number of time unofficially (in order not to pay the income tax) and officially. I consider the job to be official if the income (wage) earned by individual working here is declared, and as unofficial otherwise. The situation described can be implemented only within the context of indivisible labor model.

My first result is mainly theoretical: an agent behavior in my indivisible labor model, where he chooses probability to work officially, is the same as in the divisible labor model, where agents choose the share of time that they work officially. We can interpret probability to work officially as share of time that individual work according to divisible labor model. Consumption and savings of an agent do not depend on the agent's status (work officially, or unofficially and not being caught, or unofficially and being caught). Obtained result shows the situation of perfect risk-sharing which is possible due to introduction two insurance companies to the model (Arrow-Debreu securities). This result is rather important for further consideration and could be used in developing such kind of labor models.

Next, my model parameters were calibrated with Ukrainian data, in order to reflect Ukrainian reality. To get effect of tax reduction, I have solved the model for steady state and dynamically for tax level of 20%. Then the policy experiment was conducted: decrease in income tax rate to 13%.

The main results can be summarized as follows:

First, the probability to work officially is approximately 15% for the tax level of 20%. We can interpret this probability as participation in official labor market. This result is not differ significantly from those obtained in other research including empirical (Shneider and Enste (2000) is one of them). As a result of income tax rate reduction probability to work officially increases to 24%. This tells us that the unofficial shadow sector of labor market decreases. Such kind of dependence could be found in numerous empirical and theoretical papers developed before.

Second, the steady state level of consumption does not change as a result of new tax policy. The level of consumption decreases during transition period because of higher value of capital accumulation, as a result of fall in tax rate.

Third, the life-time utility of agents decline, as a result of income tax rate reduction. Since we have additive-separable utility function, we can consider the life-time utility from consumption and life-time utility from the parameter (that reflects relative utility derived by agent from social guarantees) separately. The life-time utility from consumption does not change with decrease in income tax rate, but in the same time relative utility derived by agent from social guarantees decreases, because this parameter falls more sharply than the probability to work officially increases. Results are the same for tree different coefficient of risk aversion. Mentioned decrease is sharper because the net wage increase (as a result of tax cut), the amount of social guarantees does not change, but with higher wage these guarantees are not so important for individual and he can use his higher wage to satisfy his social needs. So, we can conclude that in economy where an individual receives his utility not only from consumption, but also from social guaranties that he evaluates relative to net wage, the life-time utility decreases with decrease in tax.

In addition, it is important to note that the results obtained are the same for different coefficients of risk-aversion. It tells us that the model is robust to the change in the parameters. So, using this model, it is possible to test the different type of policies, not only tax change. It will be very interesting, from my point of view, to introduce in the model the parameter that defines efficiency of tax system or the factor identifying the complexity of taxation. It allows us to investigate the impact of these phenomena on shadow sector of labor market. The latter one will be helpful in sense of the recent changes in Ukrainian income tax system from progressive type to constant.

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APPENDIX A (Proof of Proposition 1)

Insurance company 1 problem:

$$\frac{\partial \pi_t^{IC1}}{\partial q_t^1} = p_t^1 - (1-\beta(\gamma_t))(1-\alpha_t) = 0$$

$$p_t^1 = (1-\beta(\gamma_t))(1-\alpha_t)$$

(A1)

Insurance company 2 problem:

$$\frac{\partial \pi_t^{IC2}}{\partial q_t^2} = p_t^2 - \beta(\gamma_t)(1-\alpha_t) = 0$$

$$p_t^2 = \beta(\gamma_t)(1-\alpha_t)$$

(A2)

Utility maximization problem:

Bellman equation:

$$V_t = \max_{\{x_t\}_{t\in T}} \{\alpha_t[u(c_t^o, S) + \delta E_t V_{t+1}^o] + (1-\beta(\gamma_t))(1-\alpha_t)[u(c_t^{un}, 0) + \delta E_t V_{t+1}^{un}] + (1-\beta(\gamma_t))(1-\alpha_t)[u(c_t^{uc}, 0) + \delta E_t V_{t+1}^{uc}]\} \quad \text{(A3)}$$

V_t is a value function in period t. It depends on state variable k_t. $V_{t+1}^o, V_{t+1}^{un}, V_{t+1}^{uc}$ are value functions in period t+1 for individuals which work officially, unofficially and not caught, unofficially and caught.

Than we express $c_t^o, c_t^{un}, c_t^{uc}$ from constraints and substitute them into the Bellman equation. Also we substitute profit-maximization conditions for the insurance companies 1 and 2 - (A1) and (A2), respectively, into the Bellman equation.

$$
\begin{aligned}
V_t = \max_{\{x_t\}_{t\in T}} \{ & \alpha_t [u((1-d+r_t)k_t + (1-\tau_t)\overline{n}w_t - k_{t+1}^o - (1-\beta(\gamma_t))(1-\alpha_t)q_t^1 - \\
& -\beta(\gamma_t)(1-\alpha_t)q_t^2, S) + \delta E_t V_{t+1}^o] + (1-\beta(\gamma_t))(1-\alpha_t)[u((1-d+r_t)k_t + \overline{n}w_t + \\
& + q_t^1 - k_{t+1}^{u,n} - (1-\beta(\gamma_t))(1-\alpha_t)q_t^1 - \beta(\gamma_t)(1-\alpha_t)q_t^2, 0) + \delta E_t V_{t+1}^{un}] + \\
& + (1-\beta(\gamma_t))(1-\alpha_t)[u((1-d+r_t)k_t + (1-\rho_t)\overline{n}w_t + q_t^2 - \\
& - k_{t+1}^{u,c} - (1-\beta(\gamma_t))(1-\alpha_t)q_t^1 - \beta(\gamma_t)(1-\alpha_t)q_t^2, 0) + \delta E_t V_{t+1}^{uc}]\}
\end{aligned} \tag{A4}
$$

FOCs:

$$
\begin{aligned}
\frac{\partial V_t}{\partial q_t^1} = & \alpha_t u_1(c_t^0, S)(-(1-\beta(\gamma_t))(1-\alpha_t)) + (1-\beta(\gamma_t))(1-\alpha_t)u_1(c_t^{un}, 0)(1- \\
& -(1-\beta(\gamma_t))(1-\alpha_t)) + \beta(\gamma_t)(1-\alpha_t)u_1(c_t^{uc}, 0)(-(1-\beta(\gamma_t))(1-\alpha_t)) = 0
\end{aligned}
$$

$$
\alpha_t u_1(c_t^0, S) + (1-\beta(\gamma_t))(1-\alpha_t)u_1(c_t^{un}, 0) + \beta(\gamma_t)(1-\alpha_t)u_1(c_t^{uc}, 0) = u_1(c_t^{un}, 0) \tag{A5}
$$

$$
\begin{aligned}
\frac{\partial V_t}{\partial q_t^2} = & \alpha_t u_1(c_t^0, S)(-\beta(\gamma_t)(1-\alpha_t)) + (1-\beta(\gamma_t))(1-\alpha_t)u_1(c_t^{un}, 0)(-\beta(\gamma_t)(1-\alpha_t)) + \\
& + \beta(\gamma_t)(1-\alpha_t)u_1(c_t^{uc}, 0)(1-\beta(\gamma_t)(1-\alpha_t)) = 0
\end{aligned}
$$

$$
\alpha_t u_1(c_t^0, S) + (1-\beta(\gamma_t))(1-\alpha_t)u_1(c_t^{un}, 0) + \beta(\gamma_t)(1-\alpha_t)u_1(c_t^{uc}, 0) = u_1(c_t^{uc}, 0) \tag{A6}
$$

From (A5) and (A6) we can conclude that $u_1(c_t^{un}, 0) = u_1(c_t^{uc}, 0)$. Using this equality, rearrange (A5):

$$\alpha_t u_1(c_t^0,S)+(1-\beta(\gamma_t))(1-\alpha_t)u_1(c_t^{un},0)+\beta(\gamma_t)(1-\alpha_t)u_1(c_t^{uc},0)-$$
$$-u_1(c_t^{un},0)=\alpha_t u_1(c_t^0,S)+(1-\alpha_t)u_1(c_t^{un},0)-\beta(\gamma_t)(1-\alpha_t)u_1(c_t^{un},0)+$$
$$+\beta(\gamma_t)(1-\alpha_t)u_1(c_t^{uc},0)-u_1(c_t^{un},0)=\alpha_t u_1(c_t^0,S)+(1-\alpha_t)u_1(c_t^{un},0)-$$
$$-u_1(c_t^{un},0)=\alpha_t u_1(c_t^0,S)+u_1(c_t^{un},0)-\alpha_t u_1(c_t^{un},0)-u_1(c_t^{un},0)=$$
$$=\alpha_t u_1(c_t^0,S)-\alpha_t u_1(c_t^{un},0)=0$$

$\alpha_t u_1(c_t^0,S)=\alpha_t u_1(c_t^{un},0)\Rightarrow u_1(c_t^{un},0)=u_1(c_t^{uc},0)=u_1(c_t^o,S)$ and $c_t^{un}=c_t^{uc}$

$$\frac{\partial V_t}{\partial k_{t+1}^o}=\alpha_t\{u_1(c_t^o,S)(-1)+\delta E_t[\frac{\partial V_{t+1}^o}{\partial k_{t+1}^o}]\}=0\Rightarrow u_1(c_t^o,S)=\delta E_t[\frac{\partial V_{t+1}^o}{\partial k_{t+1}^o}] \quad \text{(A7)}$$

$$\frac{\partial V_t}{\partial k_{t+1}^{un}}=\alpha_t\{u_1(c_t^{un},0)(-1)+\delta E_t[\frac{\partial V_{t+1}^{un}}{\partial k_{t+1}^{un}}]\}=0\Rightarrow u_1(c_t^{un},0)=\delta E_t[\frac{\partial V_{t+1}^{un}}{\partial k_{t+1}^{un}}] \quad \text{(A8)}$$

$$\frac{\partial V_t}{\partial k_{t+1}^{uc}}=\alpha_t\{u_1(c_t^{uc},0)(-1)+\delta E_t[\frac{\partial V_{t+1}^{uc}}{\partial k_{t+1}^{uc}}]\}=0\Rightarrow u_1(c_t^{uc},0)=\delta E_t[\frac{\partial V_{t+1}^{uc}}{\partial k_{t+1}^{uc}}] \quad \text{(A9)}$$

From (A8) and (A9) and using $u_1(c_t^{un},0)=u_1(c_t^{uc},0)=u_1(c_t^o,S)\Rightarrow$ $k_{t+1}^{un}=k_{t+1}^{uc}=k_{t+1}^o=k_{t+1}$

$$\frac{\partial V_t}{\partial \alpha_t}=[u(c_t^o,S)+\delta E_t V_{t+1}^o]+\alpha_t u_1(c_t^o,S)[q_t^1(1-\beta(\gamma_t))+q_t^2\beta(\gamma_t)]-(1-\beta(\gamma_t))$$
$$[u(c_t^{un},0)+\delta E_t V_{t+1}^{un}]+(1-\beta(\gamma_t))(1-\alpha_t)u_1(c_t^{un},0)[q_t^1(1-\beta(\gamma_t))+q_t^2\beta(\gamma_t)]-$$
$$-\beta(\gamma_t)[u(c_t^{uc},0)+\delta E_t V_{t+1}^{uc}]+\beta(\gamma_t)(1-\alpha_t)u_1(c_t^{uc},0)[q_t^1(1-\beta(\gamma_t))+q_t^2\beta(\gamma_t)]=0$$

$$u(c_t^o,S)+\delta E_t V_{t+1}^o-(1-\beta(\gamma_t))[u(c_t^{un},0)+\delta E_t V_{t+1}^{un}]-\beta(\gamma_t)[u(c_t^{uc},0)+\delta E_t V_{t+1}^{uc}]+$$
$$+[q_t^1(1-\beta(\gamma_t))+q_t^2\beta(\gamma_t)][\alpha_t u_1(c_t^o,S)+(1-\beta(\gamma_t))(1-\alpha_t)u_1(c_t^{un},0)+$$
$$+\beta(\gamma_t)(1-\alpha_t)u_1(c_t^{uc},0)]=u(c_t^o,S)+\delta E_t V_{t+1}^o-(1-\beta(\gamma_t))[u(c_t^{un},0)+\delta E_t V_{t+1}^{un}]-$$
$$-\beta(\gamma_t)[u(c_t^{uc},0)+\delta E_t V_{t+1}^{uc}]+[q_t^1(1-\beta(\gamma_t))+q_t^2\beta(\gamma_t)]u_1(c_t^{un},0)=u(c_t^o,S)+$$
$$+\delta E_t V_{t+1}^o-[u(c_t^{un},0)+\delta E_t V_{t+1}^{un}]+\beta(\gamma_t)[u(c_t^{un},0)+\delta E_t V_{t+1}^{un}]-\beta(\gamma_t)[u(c_t^{uc},0)+$$
$$+\delta E_t V_{t+1}^{uc}]+[q_t^1(1-\beta(\gamma_t))+q_t^2\beta(\gamma_t)]u_1(c_t^{un},0)=u(c_t^o,S)-u(c_t^{un},0)+[q_t^1(1-\beta(\gamma_t))+$$
$$+q_t^2\beta(\gamma_t)]u_1(c_t^{un},0)=0$$

$$u(c_t^o,S)-u(c_t^{un},0)+[q_t^1(1-\beta(\gamma_t))+q_t^2\beta(\gamma_t)]u_1(c_t^{un},0)=0 \qquad \text{(A10)}$$

The envelope condition for the problem (A4):

$$\frac{\partial V_t}{\partial k_t}=u_1(c_t^o,S)(1-d+r_t)=u_1(c_t^{un},S)(1-d+r_t)=u_1(c_t^{uc},S)(1-d+r_t)= \qquad \text{(A11)}$$
$$=u_1(c_t^{i,u},0)(1-d+r_t)$$

Updating this expression and putting it into (A7) - (A9), we receive following Euler equations:

$$u_1(c_t^o,S)=\delta E_t[u_1(c_{t+1}^o,S)(1-d+r_{t+1})]=\delta E_t[u_1(c_{t+1}^{i,u},0)(1-d+r_{t+1})]$$

$$u_1(c_t^{i,u},0)=\delta E_t[u_1(c_{t+1}^o,S)(1-d+r_{t+1})]=\delta E_t[u_1(c_{t+1}^{i,u},0)(1-d+r_{t+1})]$$

where $i\in\{o,u\}$ and $u\in\{n,c\}$

Using the fact that $k_{t+1}^{un}=k_{t+1}^{uc}=k_{t+1}^o=k_{t+1}$ we subtract constraint (3) from constraint (2) and constraint (4) from constraint (2):

$$c_t^o-c_t^{un}=(1-\tau_t)w_t\bar{n}-w_t\bar{n}-q_t^1 \;\Rightarrow\; q_t^1=c_t^{un}-c_t^o-\tau_t w_t\bar{n} \qquad \text{(A12)}$$

$$c_t^o-c_t^{uc}=(1-\tau_t)w_t\bar{n}-(1-\rho_t)w_t\bar{n}-q_t^2 \Rightarrow q_t^2=c_t^{uc}-c_t^o-\tau_t w_t\bar{n}+\rho_t w_t\bar{n} \qquad \text{(A13)}$$

Then, using equation (A12) and (A13), profit-maximization conditions for the insurance companies - (A1) and (A2), and $k_{t+1}^{un} = k_{t+1}^{uc} = k_{t+1}^{o} = k_{t+1}$ we can replace constraints (2) – (4) by one constraint:

$$\alpha_t c_t^o + (1-\alpha_t)(1-\beta(\gamma_t))c_t^{un} + \beta(\gamma_t)(1-\alpha_t)c_t^{uc} + k_{t+1} + (1-\alpha_t)(1-\beta(\gamma_t))(c_t^{un} - c_t^o - \tau_t w_t \bar{n}) + \beta(\gamma_t)(1-\alpha_t)(c_t^{uc} - c_t^o - \tau_t w_t \bar{n} + \rho_t w_t \bar{n}) = (1-d+r_t)k_t + \alpha_t(1-\tau_t)w_t \bar{n} + (1-\alpha_t)(1-\beta(\gamma_t))w_t \bar{n} + \beta(\gamma_t)(1-\alpha_t)(1-\rho_t)w_t \bar{n} + (1-\alpha_t)(1-\beta(\gamma_t))(c_t^{un} - c_t^o - \tau_t w_t \bar{n}) + \beta(\gamma_t)(1-\alpha_t)(c_t^{uc} - c_t^o - \tau_t w_t \bar{n} + \rho_t w_t \bar{n})$$

$$\alpha_t c_t^o + (1-\alpha_t)(1-\beta(\gamma_t))c_t^{un} + \beta(\gamma_t)(1-\alpha_t)c_t^{uc} + k_{t+1} = (1-d+r_t)k_t + \alpha_t(1-\tau_t)w_t \bar{n} + (1-\alpha_t)(1-\beta(\gamma_t))w_t \bar{n} + +\beta(\gamma_t)(1-\alpha_t)(1-\rho_t)w_t \bar{n} \tag{A14}$$

Assume that $u(c_t, S) = v(c_t) + \varpi(S)$ - additive utility function $\Rightarrow$ $c_t^o = c_t^{un} = c_t^{uc} = c_t$ from condition $u_1(c_t^{un}, 0) = u_1(c_t^{uc}, 0) = u_1(c_t^o, S)$. As a result we can rewrite budget constraint (26) as:

$$c_t + k_{t+1} = (1-d+r_t)k_t - \alpha_t \tau_t w_t \bar{n} + w_t \bar{n} - \beta(\gamma_t)(1-\alpha_t)\rho_t w_t \bar{n} \quad \text{or}$$

$$c_t + k_{t+1} = (1-d+r_t)k_t + w_t \bar{n}(1 - \alpha_t \tau_t - \beta(\gamma_t)(1-\alpha_t)\rho_t) \tag{A15}$$

This constraint is identical to the constraint for divisible labor problem where agent chooses the share of time that he works officially and unofficially.

We can rewrite Euler equation that is also identical to the Euler equation for divisible-labor problem:

$$v'(c_t) = \delta E_t[v'(c_{t+1})(1-d+r_{t+1})] \tag{A16}$$

Also, we can rewrite (A10) (using (A12) and (A13)):

$$v(c_t^o) + \widetilde{w}(S) - v(c_t^{un}) - \widetilde{w}(0) + [(c_t^{un} - c_t^o - \tau_t w_t \overline{n})(1 - \beta(\gamma_t)) + (c_t^{uc} - c_t^o - \tau_t w_t \overline{n} + $$
$$+ \rho_t w_t \overline{n})\beta(\gamma_t)]v_1(c_t^{un}) = 0$$

$$\widetilde{w}'(S) + v_1(c_t^{un})[\beta(\gamma_t)\rho_t w_t \overline{n} - \tau_t w_t \overline{n}] = 0 \qquad \text{(A17)}$$

Recursive formulation of the agent's problem in the divisible-labor economy has the following form:

$$V_t = \max_{\{b_t\}_{t\in T}} \{v(c_t) + A\alpha_t + \delta E_t V_{t+1}\} \qquad \text{(A18)}$$

subject to

$$c_t + k_{t+1} = (1 - d + r_t)k_t + \alpha_t(1 - \tau_t)w_t \overline{n} + (1 - \alpha_t)(1 - \beta(\gamma_t))w_t \overline{n} + \beta(\gamma_t)(1 - \alpha_t)(1 - \rho_t)w_t \overline{n}$$

Solution of problem (A18) is FOCs Eqs. (A16) – (A17).

APPENDIX B (Log-linearization)

Let's log-linearize the FOCs of the problem (9)-(10) and constraint (27)

1) Log-linearization of the Euler equation:

$$v'(c_t) = \delta E_t[v'(c_{t+1})(1-d+r_{t+1})]$$

$$c_t^{-v} = \delta E_t\left[c_{t+1}^{-v}\left(1-d+\mu\left(\frac{k_t}{n_t}\right)^{\mu-1}\right)\right]$$

$$v'(c_t) \approx v'(c) + v''(c)dc_t = v'(c) + v''(c)c\frac{dc_t}{c} = v'(c) + v''(c)c\tilde{c}_t = -vc^{-v} - vc\tilde{c}_t$$

$$r_{t+1} = f_1(k_{t+1}, \overline{n}) \approx f_1(k,n) + f_{11}(k,n)dk_{t+1} = f_1(k,n) + f_{11}(k,n)k\tilde{k}_{t+1} = \mu\left(\frac{k}{n}\right)^{\mu-1} +$$

$$+\mu(\mu-1)\left(\frac{k}{n}\right)^{\mu-1}\tilde{k}_{t+1}$$

$$c^{-v} - vc^{-v}\tilde{c}_t = \delta E_t[(c^{-v} - vc^{-v}\tilde{c}_{t+1})(1-d+\mu\left(\frac{k}{n}\right)^{\mu-1} + \mu(\mu-1)\left(\frac{k}{n}\right)^{\mu-1}\tilde{k}_{t+1})]$$

$$c^{-v} - vc^{-v}\tilde{c}_t = E_t[\delta(c^{-v} - vc^{-v}\tilde{c}_{t+1})(1-d+\mu\left(\frac{k}{n}\right)^{\mu-1}) + \delta(c^{-v} -$$

$$-vc^{-v}\tilde{c}_{t+1})\mu(\mu-1)\left(\frac{k}{n}\right)^{\mu-1}\tilde{k}_{t+1}]$$

Using that $1 = \delta\left(1-d+\mu\left(\frac{k}{n}\right)^{\mu-1}\right)$ and neglecting cross-term like $\tilde{c}_{t+1}\tilde{k}_{t+1}$ or $\tilde{c}_{t+1}\tilde{n}_{t+1}$, we receive:

$$-v\tilde{c}_t = -vE_t(\tilde{c}_{t+1}) + \delta\mu(\mu-1)\left(\frac{k}{n}\right)^{\mu-1}E_t(\tilde{k}_{t+1})$$

Now, let's do the same, using the simpler way:

$$x_t = x\left(\frac{x_t}{x}\right) = x\exp\{\log\frac{x_t}{x}\} = x\exp\{\tilde{x}_t\}$$

Taking a first order Taylor approximation around the steady state we have:

$$x\exp\{\tilde{x}_t\} \approx xe^0 + xe^0(\tilde{x}_t - 0) \approx x(1+\tilde{x}_t)$$

We can write:

$$x_t y_t \approx x(1+\tilde{x}_t)y(1+\tilde{y}_t) \approx xy(1+\tilde{x}_t+\tilde{y}_t+\tilde{x}_t\tilde{y}_t) \approx xy(1+\tilde{x}_t+\tilde{y}_t)$$ where

$\tilde{x}_t\tilde{y}_t \approx 0$, since $\tilde{x}_t$ and $\tilde{y}_t$ are numbers close to zero.

$$f(x_t) \approx f(x) + f'(x)(x_t - x) \approx f(x) + f'(x)x(\frac{x_t}{x} - 1) \approx f(x) + f(x)\eta(\tilde{x}_t) \approx f(x)(1+\eta\tilde{x}_t)$$

, where $\eta = \dfrac{\partial f(x)}{\partial x}\dfrac{x}{f(x)}$

Now, let's log-linearize Euler equations using these approximations.

$$c_t^{-v} = \delta E_t\left[c_{t+1}^{-v}(1-d+r_{t+1})\right] = \delta E_t(c_{t+1}^{-v}) - \delta d E_t(c_{t+1}^{-v}) + \delta E_t(c_{t+1}^{-v} r_{t+1})$$

$$-vc^{-v}\tilde{c}_t = -vc^{-v}\delta E_t(\tilde{c}_{t+1}) + vc^{-v}\delta d E_t(c_{t+1}^{-v}) + \delta E_t c^{-v} r(-v\tilde{c}_{t+1} + \tilde{r}_{t+1})$$

$$-v\tilde{c}_t = -v\delta E_t(\tilde{c}_{t+1}) + v\delta d E_t(c_{t+1}^{-v}) + \delta E_t r(-v\tilde{c}_{t+1} + \tilde{r}_{t+1})$$

$$\tilde{r}_t = (\mu-1)\tilde{k}_t$$

$$r = \mu\left(\frac{k}{\overline{\overline{n}}}\right)^{\mu-1}$$

$$-v\tilde{c}_t = -v\delta E_t(\tilde{c}_{t+1}) + v\delta d E_t(c_{t+1}^{-v}) - \delta\mu\left(\frac{k}{\overline{\overline{n}}}\right)^{\mu-1} vE_t(\tilde{c}_{t+1}) + \delta\mu(\mu-1)\left(\frac{k}{\overline{\overline{n}}}\right)^{\mu-1} E_t(\tilde{k}_t)$$

$$-v\tilde{c}_t = -vE_t(\tilde{c}_{t+1})\left(\delta\left(1-d+\mu\left(\frac{k}{\overline{\overline{n}}}\right)^{\mu-1}\right)\right) + \delta\mu(\mu-1)\left(\frac{k}{\overline{\overline{n}}}\right)^{\mu-1} E_t(\tilde{k}_t)$$

Euler equation in the steady state has form: $\delta\left(1-d+\mu\left(\frac{k}{\overline{\overline{n}}}\right)^{\mu-1}\right) = 1$

$$-v\tilde{c}_t = -vE_t(\tilde{c}_{t+1}) + \delta\mu(\mu-1)\left(\frac{k}{\overline{\overline{n}}}\right)^{\mu-1} E_t(\tilde{k}_{t+1})$$

2) Log-linearization of the FOC:

$$\tilde{w}(S) + v_1(c_t)[\beta(\gamma_t)\rho_t w_t \bar{n} - \tau_t w_t \bar{n}] = 0$$

$$A\alpha_t = c_t^{-v}\tau_t w_t \bar{n} - c_t^{-v}\beta(\gamma_t)\rho w_t \bar{n}$$

$$A\alpha\tilde{\alpha}_t = \tau c^{-v} n w(-v\tilde{c}_t + \tilde{w}_t) - \beta(\gamma) c^{-v} n w \rho(-v\tilde{c}_t + \tilde{w}_t)$$

3) Log-linearization of the budget constraint:

$$c_t + k_{t+1} = (1 - d + r_t)k_t + w_t \bar{n} - w_t \bar{n}\alpha_t \tau_t - w_t \bar{n}\beta(\gamma_t)\rho + \alpha_t w_t \bar{n}\beta(\gamma_t)\rho$$

$$\tilde{w}_{t+1} = \mu \tilde{k}_{t+1}$$

$$\tilde{r}_t = (\mu - 1)\tilde{k}_t$$

$$c\tilde{c}_t + k\tilde{k}_{t+1} = (1 - d + r)k\tilde{k}_t + rk\tilde{r}_t + w\tilde{w}_t \bar{n} - w\bar{n}\alpha\tau(\tilde{w}_t + \tilde{\alpha}_t) - w\bar{n}\beta(\gamma)\rho\tilde{w}_t +$$
$$+ \alpha w\bar{n}\beta(\gamma)\rho(\tilde{\alpha}_t + \tilde{w}_t)$$

APPENDIX C. (Method of undetermined coefficients)

The decision functions have the following form:

$$\tilde{k}_{t+1} = \Phi_{KK}\tilde{k}_t$$

$$\tilde{c}_t = \Phi_{KC}\tilde{k}_t$$

$$\tilde{\alpha}_t = \Phi_{KA}\tilde{k}_t$$

The coefficients Φ_{KK}, Φ_{KC}, and Φ_{KA} are not known.

Expectations:

$E_t[\tilde{k}_{t+1}] = \tilde{k}_{t+1}$ since we choose it in t

$$E_t[\tilde{c}_{t+1}] = E_t[\Phi_{KC}\tilde{k}_{t+1}] = \Phi_{KC}\tilde{k}_{t+1}$$

1. Euler equation:

$$-v\tilde{c}_t = -v\Phi_{KC}\tilde{k}_{t+1} + \delta\mu(\mu-1)\left(\frac{k}{\bar{n}}\right)^{\mu-1}\tilde{k}_{t+1}$$

$$-v\Phi_{KC}\tilde{k}_t = -v\Phi_{KC}\Phi_{KK}\tilde{k}_t + \delta\mu(\mu-1)\left(\frac{k}{\bar{n}}\right)^{\mu-1}\Phi_{KK}\tilde{k}_t$$

$$-v\Phi_{KC} = -v\Phi_{KC}\Phi_{KK} + \delta\mu(\mu-1)\left(\frac{k}{\bar{n}}\right)^{\mu-1}\Phi_{KK}$$

2. Budget constraint:

$$c\tilde{c}_t + k\tilde{k}_{t+1} = (1-d+r)k\tilde{k}_t + rk\tilde{r}_t + w\tilde{w}_t\bar{n} - w\bar{n}\alpha\tau(\tilde{w}_t + \tilde{\alpha}_t) - w\bar{n}\beta(\gamma)\rho\tilde{w}_t +$$
$$+ \alpha w\bar{n}\beta(\gamma)\rho(\tilde{\alpha}_t + \tilde{w}_t)$$

Using $\tilde{w}_t = \mu\tilde{k}_t$ and $\tilde{r}_t = (\mu-1)\tilde{k}_t$

$$c\tilde{c}_t + k\tilde{k}_{t+1} = (1-d+r)k\tilde{k}_t + rk(\mu-1)\tilde{k}_t + w\mu\tilde{k}_t\bar{n} - w\bar{n}\alpha\tau(\mu\tilde{k}_t + \tilde{\alpha}_t) - w\bar{n}\beta(\gamma)\rho\mu\tilde{k}_t +$$
$$+ \alpha w\bar{n}\beta(\gamma)\rho(\tilde{\alpha}_t + \mu\tilde{k}_t)$$

$$c\Phi_{KC} + k\Phi_{KK} = (1-d+r)k + rk(\mu-1) + w\mu\bar{n} - w\bar{n}\alpha\tau(\mu + \Phi_{KA}) - w\bar{n}\beta(\gamma)\rho\mu +$$
$$+ \alpha w\bar{n}\beta(\gamma)\rho(\Phi_{KA} + \mu)$$

3. FOC:

$$A\alpha\tilde{\alpha}_t = \tau c^{-v} nw(-v\tilde{c}_t + \tilde{w}_t) - \beta(\gamma)c^{-v} nw\rho(-v\tilde{c}_t + \tilde{w}_t)$$

$$A\alpha\tilde{\alpha}_t = \tau c^{-v} nw(-v\tilde{c}_t + \mu\tilde{k}) - \beta(\gamma)c^{-v} nw\rho(-v\tilde{c}_t + \mu\tilde{k})$$

$$A\alpha\Phi_{KA} = \tau c^{-v} nw(-v\Phi_{KC} + \mu) - \beta(\gamma)c^{-v} nw\rho(-v\Phi_{KC} + \mu)$$

Thus, we have linear system of 3 equations and 3 unknowns: Φ_{KK}, Φ_{KC}, Φ_{KA}. Let's solve this system.

$$-v\Phi_{KC} = -v\Phi_{KC}\Phi_{KK} + \delta\mu(\mu-1)\left(\frac{k}{\bar{n}}\right)^{\mu-1}\Phi_{KK}$$

$$c\Phi_{KC} + k\Phi_{KK} = (1-d+r)k + rk(\mu-1) + w\mu\bar{n} - w\bar{n}\alpha\tau(\mu+\Phi_{KA}) - w\bar{n}\beta(\gamma)\rho\mu +$$
$$+\alpha w\bar{n}\beta(\gamma)\rho(\Phi_{KA}+\mu)$$

$$A\alpha\Phi_{KA} = \tau c^{-v} nw(-v\Phi_{KC} + \mu) - \beta(\gamma)c^{-v} nw\rho(-v\Phi_{KC} + \mu)$$

The results are presented in the Table 5.4.