Relative Performance of DEA and SFA in Response to Multicollinearity and Measurement Error Problems

by

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A thesis submitted in partial fulfillment of
the requirements for the degree of

Master of Arts in Economics

National University "Kyiv-Mohyla Academy"
Economics Education and Research Consortium
Master's Program in Economics

2006

Approved by ____
Mr. Serhiy Korablin (Head of the State Examination Committee)

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Program Authorized
to Offer Degree Master's Program in Economics, NaUKMA

Date ____

National University “Kyiv-Mohyla Academy”

Abstract

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MULTICOLLINEARITY AND
MEASUREMENT ERROR
PROBLEMS

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This study examines the relative performance of DEA VRS and SFA ML models in response to multicollinearity and measurement error in endogenous variables problem. We found no significant influence of multicorrelation even for realistically high correlation levels (ϱ=0.8) in the case of two inputs. Moreover, no clear direction of change of performance was observed with the introduction of the measurement error into endogenous variables, when the measurement error is moderate (not more than 20% of input).

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ABBREVIATIONS

DEA	Data Envelopment Analysis
DMU	Decision Making Unit
SFA	Stochastic Frontier Analysis
MAD	Mean Absolute Deviation $= \frac{\sum_{i=1}^{N} \lvert A_i - B_i \rvert}{N}$, where A and B are variables of interest
MLE	Maximum Likelihood Estimation
CRS	Constant Returns to Scale
VRS	Variable Returns to Scale

ACKNOWLEDGMENTS

The author is very grateful to Dr. Valentin Zelenyuk for his overall guidance, valuable comments, support and understanding. She also wishes to thank to Dr. Tom Coupé for his unprecedented patience. She is indebted to her beloved and intimate friends and relatives, this work would be impossible without their love and backing.

Chapter 1

INTRODUCTION

Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) are two widely used methodologies to conduct productivity analysis. DEA and SFA methodologies were first formulated in 60s/70s respectively. Since then they are widely used in various fields of economic research: at different levels of analysis – at productive unit of the firm, firm, industry, country levels; in different fields – labor economics, environmental economics, health economics, financial economics; to answer on a wide range of questions – to reveal relative efficiency of Decision Making Units (DMU) in the group, to estimate shadow prices of non-market goods/evils, to find out major determinants of growth rate slow-down in the world etc. Just search in Google Scholar gives us 17,500-29,600 results for papers that use DEA or SFA.

No wonder that a lot of investigation was made to examine the properties of the models in various settings. A number of prominent papers have proved analytically nice asymptotical properties of the methods. However, assessment of the properties of the models in small and medium samples is complicated. The problem is tackled by applying Monte Carlo experiments. There exist two kinds of studies of this kind. In the first one, the researches try to mimic artificial facts from a particular real-world industry. The aim of such studies is to asses the reliability of different methodologies. An example of such a work is the paper of Resti (2000). In the second one, researchers fix the problem of interest, such as problem of outliers, of high noise in dependent variable, try to investigate its influence in its "isolation" and "purity" An example of such a work is the paper of Banker *et.al.* (1993) . In the present work we follow the second approach. The problems under our investigation are multicollinearity and measurement error in the endogenous variables.

A lot of work was done to describe the performance of DEA and SFA in response to different problems. The scientists tackled such questions as presence of noise, problem of outliers, for SFA - misspecification of production function, misspecification of distribution of inefficiency term, heteroscedasticity in inefficiency scores, omission of relevant variables etc. No systematic investigation of the multicollinearity problem and measurement error in endogenous variables problem was performed yet. In the real-world, however, the named above problems are often to encounter. As Andrea Resti (2000) impartially stated: "… previous literature usually does not account for correlation among different products: [however] in real life, when a firm produced a large amount of one output it is likely to produce large quantities of the remaining ones also". The problem of multicollinearity is believed to have noticeable consequences. Let recall the words of Pedraja-Chaparra (1999) *et al.* :"The issue of correlation between inputs (or outputs) has received relatively little attention in the literature. However, … it is of <u>fundamental importance</u>…" (the underlining is mine). However, the scope of the multicollinearity problem and its consequences for a realistic range of correlation coefficients has not been examined yet. The problem of measurement error in endogenous variables was not considered in its "purity" and "isolation" either. Resti(2000) treated it coupled with a set of other problems, so that no specific conclusions on the measurement error problem in endogenous variables were made. On the other hand, the measurement error in endogenous variables could be treated as a problem of omitted variables. Such studies exist, for example, Banker *et al.* (1996) for DEA and Ruggiero (1999) for SFA. They find that both methods are sensitive to the problem of omitted variables. However, first, no direct comparison was made between two methods. Second, as a rule noise constitutes a small fraction of the observed variables, the above mentioned studies investigated the omission of variables of the same scope as the rest endogenous variables of the model. Since, conclusions from Monte Carlo experiments are very hard to generalize over its original model set-up, the question still exists: whether moderate (realistic) levels of noise constitute a significant problem for DEA and SFA and if yes, then, which model is more robust to the problem. To fill the gap on multicollinearity and measurement error in exogenous variables problem in the existing literature is the aim of our paper. We investigate performance of DEA VRS (with Variable Returns to Scale) and SFA ML (Maximum Likelihood) in small and medium samples: for 25, 50 and 100 DMU (Decision Making Units) pro sample.

The novelty of our work is not only in the subject of investigation, but also in method. To scrutinized influence of multicollinearity we fix a particular level of multicollinearity for each iteration of each experiment. So, for example, if multicollinearity of degree 0.5 is under study then it equals exactly to 0.5 in each iteration. In prior works if multicollinearity was inserted in the data generating process, it was done by implementing normal distribution of endogenous variables, relying on the fact that the sum of two normally distributed variables is also a normally distributed variable. In such a case, however, multicollinearity only at average equals to the desired value, which is inappropriate for the small samples.

Unlike many studies, we also apply a uniform distribution for endogenous variables and the measurement error in endogenous variables term. Thus, we avoid biased conclusions by separating "outlier problem" from our model set-up.

The paper proceeds as follows: first, we give a brief introduction into DEA and SFA methodologies. Then we make a literature review. In chapter 4 we illustrate, how large the discrepancy between results from DEA and SFA could be, using a real-world data set. Afterwards, in chapter 5 we provide a detailed description of the set-up of Monte Carlo experiments. Chapter 6 summarizes the results obtained from our experiments and chapter 7 concludes the paper.

Chapter 2

A BRIEF INTRODUCTION INTO DEA AND SFA METHODOLOGIES

The conceptual origins of productivity analysis could be traced to the classical economics. There was noticed, that firms could be inefficient not only in allocative, but in pure technical sense. In 1935 Hicks pointed: "people in monopolistic positions… are likely to exploit their advantage much more by not bothering to get very near the position of maximum profit, than by straining themselves to get very close to it". However, this remark was forgotten for a long time. With an impetuous development of neoclassical theory an assumption that firms, being profit-maximizers, always use their production possibilities in the most efficient way. In other words, it was widely believed that perfect technical efficiency is a reasonable assumption of real-world behavior of firms, inefficiencies could result only in allocation due to imperfections of the markets, such as monopoly, for example. The major doubts on such a point of view was cast by numerous reports of empirical studies in late 60s about decreases in average costs in American industries seemingly not attributable to any changes in organization or technology (Jameson, 1972). Thus, possibility of technical inefficiency was proved being possible. Intensive discussions in the literature produced several explanations of such observed behavior. Among them organizational entropy (X-inefficiency), bad motivation etc. Parallel to this conceptual disputes on existence and origins of technical inefficiency a mathematical definition appeared and was developing in the economic literature.

If firms could be inefficient, we would like to measure it. To move further, we should recall the core notions of the efficiency and productivity analysis are "production function" and "efficiency". Productive function is a relation between input and output vectors that shows the maximum product obtainable from the input endowment at the existing state of technological knowledge. Notion of efficiency was first introduced by Koopmans (1951). Guiding by the same logic, as in a Pareto-efficiency concept, the Decision Making Unit (DMU) is technically efficient if and only if it cannot produce more of some kind[s] of output without producing less of some other kind[s] of output, given vector of inputs. Hence, if there exists a possibility to increase output[s] given the input endowment or reduce input[s] given produced set of outputs then DMU is technically inefficient.

Based on Koopman's definition, a logical measure of technical inefficiency would be $\delta_i^{output} = \dfrac{potential \quad output_i}{actual \quad output_i}$ in output-oriented context or $\delta_i^{input} = \dfrac{actual \quad input_i}{potential \quad input_i}$ in input-oriented context. For output-oriented technical inefficiency measure, in case, we have several output, they should be aggregated in "one general output" by some aggregation function. Potential output is naturally to approximate by actual inputs of the DMU. *Mutatis mutandis*, this also holds for the input-oriented context. The further discussion would be restricted to the output-oriented context. Note that conclusion on the output-oriented DEA could be easily extended on the input-oriented DEA. The output oriented DEA was chosen was the convenience of the discussion.

Following this logic, Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) are two approaches to estimate the potential output.

Let give an exact mathematical definition of these two approaches.

Data Envelopment Analysis is a nonparametric technique to conduct productivity analysis. It has its roots in seminal works of Debreu (1951), Shephard (1953) and Farrel (1957). In the context of this paper (measuring technical inefficiencies of DMU's in the output-oriented context), DEA means implementation of non-parametric technique to approximate the aggregated potential output. DEA uses its basic assumptions[1] to estimate the potential output of the DMU from the output of the most efficient group of DMU's within the sample that are similar to the DMU of interest. In the context of the present paper, for DEA with variable returns to scale, DEA seeks for the most efficient group of DMU's within the sample from those, whose convex combination of input endowments, being equal to that of the DMU of interest, produces not less of each output than the DMU of interest. The correspondent mathematical formula is:

$$
\begin{aligned}
& TE^{output}{}_i \max_{\theta,\lambda} \phi, \\
& s.t. \quad X\lambda \le x_i , \\
& \phi y_i - Y\lambda \le 0, \\
& \lambda \ge 0 \\
& \sum_{i=1}^{n} \lambda_i = 1
\end{aligned}
\qquad \text{(model (1))}
$$

To the contrary of DEA, SFA is a parametric technique to conduct productivity analysis. It begins its existence from seminal papers of Aigner et al. (1977) and Meeusen et al. (1977).). In the context of this paper (measuring technical inefficiencies of DMU's in the output-oriented context), SFA uses maximum likelihood to estimate the production function, given an a-priori known functional forms of the production function and a-priori known distributions of the noise and technical inefficiency terms. In this work we use the model proposed by Aigner et al. (1977):

$Y = f(X) \cdot e^{-u+v}$, where $u_i \sim |N(0,\sigma_u|$ is an inefficiency term, $v_i \sim N(0,\sigma_v)$ is stochastic noise; $\hat{f}(x_i)$ is estimation of potential output for DMU *i*.

1

Note, that in SFA we cannot directly estimate individual technical efficiency scores, thus we use a proxy for it $E(e^{u_i} \mid \varepsilon_i)$, where ε_i is a composite error term ($\varepsilon_i = -u_i + v_i$).

The two main differences of DEA and SFA are that the former is non-parametric and ascribes the whole deviation from estimated potential output to inefficiency (does not take account for noise), whereas the latter is parametric and accounts for noise. Let have a closer look to these differences.

Being a parametric method, SFA relies heavily on the assumptions about the functional form of the production function, distributions of noise and inefficiency terms, whereas DEA, as a non-parametric method, does not. However, when the true functional form of the production function is known, SFA allows us to take account for it, which is of great plus.

DEA ascribes the whole deviation from the potential output to inefficiency. Let examine, what could this inefficiency measure be comprised of. The economic literature proposes the following explanations:

1. Measurement errors in measurement of related variables.
2. Pure random shocks in the production process. As poor weather conditions, natural disasters, accidental mistakes of workers etc;
3. Mistakes of the planners, engineers. Common explanation is information asymmetry.
4. Differences in quality of inputs (quality of machines, abilities of workers and entrepreneurs etc.)
5. Temporary losses in efficiency due to adaptation to the changed market conditions (Jameson, 1972)
6. X-efficiency (Leibenstein, 1965 etc.)
7. Differences in excluded factors of production/outputs

Apologists of SFA technique state that the inefficiency due to factors 1) and 2) is not of interest for the researchers, moreover, we can "выделить" it from the inefficiency measure: "The great virtue of stochastic production frontier models is that the impact on output of shocks due to variation in labor and machinery performance, vagaries of the weather, and just plain luck can at least in principle be separated from the contribution of variation in technical efficiency" (Kumbhakar, Lovell (2003)). Relying on the Central Limit Theorem and on the assumption that none of the factors, comprising 1) and 2), dominates, we receive that this part of inefficiency, called noise, could be captured by a normally distributed error term. So, we can "separate" it, given the a-priori known distribution of this noise term and the rest of inefficiency (inefficiency term).

Table 1. Comparison of DEA and SFA

DEA	**SFA**
Nonparametric technique	Parametric technique
Does not account for noise	Accounts for noise

Chapter 3

A LITERATURE REVIEW

Data Envelopment Analysis and Stochastic Frontier Analysis are widely used methods in the management science to measure technical efficiency of firms. But they could deliver different results. Some applications of different methodologies for the same data set, as those of Chirikos & Sear (2000), Ferrier & Lovell (1990) and others, found out that efficiency measurement depends on the employed methodology found out that efficiency measurement depends on the employed methodology. Thus, for example, Chirikos & Sear (2000) in their study of 186 American acute care hospitals received Pearson's correlation of cost efficiency scores received from different methods as low as $0.13 \leq \rho \leq 0.26$. Or similarly, Ferrier & Lovell (1990) investigating performance of 575 American financial institutions received Spearman's rank correlation as low as $0.014 \leq \rho \leq 0.017$. At the same time, there exist also studies that show a good compliance between technical scores/ ranks estimated by DEA and SFA. For example, Park & Lesourd (2000) received a very good compliance of DEA and SFA estimates for cost efficiency scores in their study of 64 power plants in South Korea. A recommendation was spelled in the literature to check robustness of the results obtained by one method by another one. A natural question arises, however, in which situations DEA outperforms SFA and *vice versa*, when these methods fail and when they produce reliable results?

A number of Monte Carlo comparisons were made to investigate the issue. The situations examined in these studies include presence of statistical noise, relative size of statistical noise to technical inefficiency, different functional forms of production function and true/false assumptions on it, different distributions of inefficiency term and true/false assumptions on it, correlation of inefficiency scores with explanatory variables, heterogeneity of inefficiency scores, problem of outliers, omission of relevant/ inclusion of irrelevant variables *etc*.

First of all, comparisons were made to investigate relative performance of DEA and SFA in presens of noise. It was expected DEA to be very sensitive to noise and perform worse than SFA even in small samples. The evidence was different, however. For example, in the experiments of Banker, Gadh & Gorr (1993) for DEA and COLS[2] DEA was found to be rather "robust" to noise. MAD (Mean Absolute Deviations) between efficiency scores from DEA and SFA and actual efficiency scores varied between 0.03 and 0.11. They even concluded that "DEA produce more accurate efficiency estimates … even with remarkably high errors present". SFA becomes more accurate, according to them, only if the noise level reaches ±17% to 45% of output values (depending on sample size, technology etc.) or given moderate noise if sample size exceeds 50. In presence of high levels of noise, however, neither of methods performed satisfactorily (MAD$\in[0.08;0.4]$).

Noise was also considered in experiments of Yu(1998) and Resti(2000). Yu examined high levels of noise, whereas Resti – "low", "medium" and "high" levels of noise. They both received rather good match of estimated and true cost efficiency scores and ranking - $MAD \leq 0.161, \quad 0.62 \leq \rho \leq 0.89$ and $0.004 \leq MAD \leq 0.63, \quad 0.63 \leq \rho \leq 1$ correspondently. The difference of the results illustrates importance of other factors that present in the experiments, such as number of observations, functional form of the production function etc.

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[2] COLS – Corrected Ordinary Least Squares – a variant of SFA

Evidence was also collected on the problem of outliers – points, in whose vicinity only few/none observations are available. For these points both DEA and SFA become les accurate. DEA VRS tends to treat them as efficient (Resti (2000), Yu (1998)). In case of SFA such points influence the slope of estimated production frontier and MAD around them is higher than around other points (Read & Thanassaillis (1996)). To be more specific, DEA and SFA are <u>both</u> inaccurate (fail) to estimate performance of outliers.

A range of experiments were made to examine the relative performance of DEA and SFA, when SFA uses an incorrect functional form of the production function (Banker, Charnes, Cooper & Maindiratte (1988), Gong & Sickles (1992), Banker, Chernes & Cooper (1996), Ruggiero (1999), Resti (2000)). An important conclusion was drawn from these experiments that the translog production function, a flexible production function that was suggested to use for approximation of the production frontier, when the true production function is unknown) delivers highly imprecise estimates if the sample is not large enough. The explanation given by Resti (2000) to this phenomenon is loss of precision due to multicollinearity between explanatory variables (cross-terms of higher orders).

Effects of inclusion of irrelevant variables and omission of relevant ones were treated by Banker, Charnes & Cooper (1996), Ruggiero (1999) for DEA and SFA separately. Both methods were sensitive to omission of a relevant variable and performed rather robust, when an irrelevant variable was included.

An interesting study was performed by Bojanic, Caudill & Ford (1998). They examined, how heteroscedastic noise (σ increases with the size of output) influenced the precision of DEA and SFA. In their studies SFA-based estimates consistently outperformed those of DEA. Both methods, however, overestimated the inefficiency parameter in the presence of high levels of noise and heteroscedasticity.

Design of Monte Carlo experiments is rather similar from paper to paper. Below we will briefly state different patterns of Monte Carlo studies.

Distribution of noise (if it is considered in the paper) is conventually taken as normal. In the study of Resti (2000) noise does not exceed 2% of the factor level in 95% of cases (σ_v=0.01; noise is introduced as $\tilde{y} = y(1+v), \quad v \sim N(0,\sigma_v)$). Yu (1998) chooses σ_v=0.15 (noise is no more than 34% of the output level in 95% of cases, where $\tilde{y} = ye^v, \quad v \sim N(0,\sigma_v)$ [i]); Banker *et al.* (1993) considers four different levels of noise: low (σ_v=0.0447 and σ_v=0.0632, when noise is not more than 9-13% of the output level with 95% level of confidence) and high (σ_v=0.14 and σ_v=0.2, when noise is not more than 25-50% of the output level with 95% level of confidence). The distributions of inefficiency scores could be half normal, truncated normal, exponential or their variants (for example, half-normal with 25% of true outputs on the frontier, exponential with 25% of output on the frontier, as in Banker *et al.* (1993)). In half-normal/truncated normal cases standard deviation of inefficiency scores σ_u was set to 0.36 by Yu (1998), so that relative importance of inefficiency to noise $\lambda = \frac{\sigma_u}{\sigma_v} =$ 2.4. Banker et al. (1993) for their half-normally distributed inefficiency scores took σ_u=0.2036 so that the mean inefficiency score was 1.15 and relative importance of inefficiency to noise varied from $\lambda = \frac{\sigma_u}{\sigma_v}$=1.018 to 4.55. An interesting experiment set-up was used by Bowlin *et al.* (1985) (he compared Ordinary Least Square model with input-oriented DEA), who specified the true production frontier as a linear function $y = Ax$, where A is a matrix of coefficients. Inefficiency in this model was introduced by reducing some coefficients of matrix A: $a_{ij}^{*} < a_{ij}$. Pedrajo-Chaparro et al (1999), who studied performance of DEA in different settings (as size of the sample, number of factors, degree of correlation between factors) used a simple Cobb-Douglas CRS function in their analysis. The studies of Banker with his colleagues (1988, 1993) implement a piecewise Cobb-Douglas function, i.e. with different parameters for 4 different intervals, chosen in such a way so that the production function remains continuous. In this way they checked how do DEA and Corrected Least Square model (COLS) react on violation of the following their basic assumptions: convexity for DEA and continuously differentiability for COLS. In work of Read & Thanassoulis (1996) the true production technology was specified as a 1-output Constant Elasticity of Substitution (CES) function. Gong & Sickles (1992) used a very flexible functional form for their experiments – CRESH technology (Constant Ratio of Elasticity of Substitution, Homothetic) with 1 output. Most commonly used by researchers production functions as Constant Elasticity of Substitution function (CES) and its limiting forms (Cobb-Douglas, linear, Leontief) are just particular cases of CRESH. The sample sizes under examination also vary from study to study. Resti (2000) chose 50 and 500-unit samples for her experiments. An interesting feature of her studies is that to make the results of Monte Carlo more relevant she chose as a backdrop a real-world industry – banking, and incorporated in her simulations returns to scale, correlation between different product lines. Banker *et al.* (1993) used samples of 25, 50, 100, 200 units in their simulations. Pedraja-Chaparra *et al.* (1999) considered samples of size 10, 20, 40, 80, 160. As criteria of comparison MAD, Pearson's correlation coefficient, Spearman's rank correlation coefficient, sometimes Kendall's rank correlation coefficient are commonly used throughout the papers.

Influence of multicollinearity on estimation results of DEA was pointed by Pedraja-Chaparra (1999) *et al.* The authors claimed that "The issue of correlation between inputs (or outputs) has received relatively little attention in the literature. However, … it is of <u>fundamental importance</u>…". The logic of their reasoning was the following: "If two inputs are positively correlated, then – other things being equal – this contribute less information to the DEA analysis than if they showed zero correlation". Indeed, if correlation between two exogenous variables equals to one, it is analogous to the situation, when we have one explanatory variable less. For SFA Resti (2000) pointed importance of multicollinearity in explanation of poor importance of the translog function. Overall, as Resti (2000) impartially mentioned, "… previous literature usually does not account for correlation among different products". Rare papers took in account multicollinearity by introducing it in explanatory variables. Thus, in her study of cost-efficiency DEA Resti (2000) mimicked the output vectors used for the simulations to the reality by introducing multicollinearity of 0.975-0.98 in them. Multicollinearity was inserted in data also by Banker (1996). But in both cases it was just one value of correlation coefficient for all the experiments. However, neither a systematic investigation of multicollinearity for SFA and DEA nor a comparison between relative performance of DEA and SFA in response to it was performed so far. What concerns measurement error in exogenous variables, the only known study for us, that of Resti(2000), used "noisy data" for her simulations to mimic reality (measurement error was normally distributed and only in 5% cases exceeded 2% of the factor level). Once more, it was one pattern of error (distribution, standard deviation) for all the experiments. So that to our best knowledge, neither a systematic investigation of measurement error in exogenous variables for SFA and DEA, nor a comparison between relative performance of DEA and SFA in response to them exists. To highlight these issues is the purpose of the present paper.

To illustrate, how significant could be discrepancy between results based on DEA and SFA Models we apply these methods to examine country's output efficiency scores.

C h a p t e r 4

ILLUSTRATION OF DISCREPANCIES BETWEEN DEA AND SFA

As the first step of our analysis and illustration of the significance of possible discrepancies in methodologies we compared performance of DEA and SFA on real-world data. The data set under examination includes real gross output, capital stock and number of working population for 57 countries in years 1965 and 1990. The data is taken from Penn World Tables (version 5.6). The countries under study are both developing, newly industrialized and developed (OECD). Real gross domestic product was constructed from Real gross domestic product per capita (RGDPCH) by multiplying it by population (POP). Number of workers was received by dividing Real GDP by Real GDP per worker (RGDPW). Capital stock was revealed from Capital stock per worker (KAPW) by using already computed Number of workers. In Penn methodology Real GDP and Capital stock are measured in 1985 international prices.

It is the same data set, as was used by Kumar & Russel (2002) for their study of convergence over countries over the time. The authors applied DEA methodology for their research. To the contrary, for our illustrative purpose we apply both DEA and SFA methodologies and compare them with a set of conventual statistics.

We used an output oriented <u>A</u>civity <u>A</u>nalysis <u>M</u>odel (AAM) both with variable returns to scale. AAM was specified in the following way:

$$TE^{output}{}_i = \max_{\theta,\lambda} \phi,$$

$$s.t. \quad X\lambda \le x_i ,$$

- $\phi y_i - Y\lambda \le 0,$ for the VRS model;

$$\lambda \ge 0$$

$$\sum_{i=1}^{n} \lambda_i = 1$$

where Y is the vector 1×57 of Real GDP, X is a matrix 2×57 of Capital Stock and Number of Workers. We use a measure $\frac{1}{\phi} = \frac{actual \quad output}{potential \quad output} \in [0;1]$ to compare with the SFA model.

For the SFA a Cobb-Douglas production was assumed with normally distributed two-sided error term and half-normally distributed inefficiency error term:

$\ln(Y_i) = a + \beta_1 \ln(K_i) + \beta_2 \ln(L_i) + (v_i - u_i)$, u_i is a half-normal distributed inefficiency term. The specified model was estimated by maximum likelihood techniques. We use measure $E(e^{u_i} \mid \varepsilon_i)$ to compare with the DEA VRS model.

Since DEA in both specifications and SFA we measure the same things, we expect their efficiency estimates to coincide.

Estimations of the models were received with DEAP vers. 2.1 and FRONTIER vers. 4.1.

According to the received technical efficiency scores the countries were rank from 1 to 57 in such a way, that countries with the same technical efficiency score received the same rank, a country with a higher technical efficiency score received a higher rank.

Table 2. Results of SFA-model

	coefficient	Standard-error	t-ratio
α	5.2	0.237	22.04
β_1	0.5	0.024	20.84
β_2	0.5	0.043	11.80
$\sigma^2=\sigma_V^2+\sigma_U^2$	0.45	0.099	4.53
$\gamma=\sigma_U^2/(\sigma_V^2+\sigma_U^2)$	0.9	0.028	31.56
log likelihood function			**-36.58**
LR test of the one-sided error			**48.94**

The summary of the results is presented in the table below. In this table you can see maximum absolute discrepancies ("-" direction is when estimates of the second models are greater than those of the first, "+" direction otherwise), and correlation of the estimates.

Table 3. Summary of the estimation results

Comparison b/w	**max "-" discrepancy**	**country**	**max "+" discrepancy**	**country**
SFA & VRS_DEA_65	-62.82%	India	21.06%	Hong Kong
SFA & VRS_DEA_65_rank	-20	Italy, Hong Kong	48	India
SFA & VRS_DEA_90	-59.11%	India	12.11%	New Zealand
SFA & VRS_DEA_90_rank	-23	New Zealand	48	India

As we see from the results, SFA produces both larger and smaller estimates for efficiency scores. This discrepancy could be substantial, for example, India was 100%efficient according to DEA VRS both in years 1965 and 1990, however, SFA estimated it to be very inefficient (efficency=37.2% in 1965 and 40.9% in 1990) . Summary statistics as Spearmen rank correlation coefficient (67.5% in 1965 and 65.9% in 1990), Pearson's rank correlation coefficient (75.5% in 1965 and 76.5% in 1990), Mean Absolute Deviation

(MAD) also show that correspondence between the results from two methods is poor in our case.

Therefore we have proven on a real data sample that conclusions could be substantially different, depending on whether a researcher uses DEA or SFA approach. And it is important to specify general recommendations, in which situations which model is better.

Chapter 5

THE DESIGN OF MONTE CARLO EXPERIMENTS

> "…any judgment on the quality of a … modal must be made in the light of the purposes for which the results are used. In such circumstances, what is ideally required is a measure of the expected costs of incorrect inferences. In practice, the magnitude of such costs is highly dependent on the precise context of the application [of the model]. For example, setting unachievable targets for a DMU might in one setting have few dysfunctional consequences, and might even be a speer to innovation and better performance, while in another setting it might lead to catastrophic crisis management and the collapse of morale".
> Pedraja-Chappara *et al.* (1999)

Critical feature of Monte Carlo experiments is dependence of results on the problem set-up and difficulty in generalizing conclusions. The first-best solution would be to derive analytically properties of the competing models and to confirm these conclusions via Monte Carlo method. A lot of progress was made in characterizing DEA and SFA, including the proof of consistency of DEA, finding confidence intervals etc. But all this is knowledge of asymptotic properties of the methods. Their small/medium sample characteristics still remain unknown and are very hard to obtain analytically. In the second best way is to fix the problem of interest try to investigate its influence in its "isolation" and "purity" by Monte Carlo simulation for the range values that are actual to the real world studies. The second best solution would be to design the experiment in such a way, that it would resemble the core characteristics of reality, but making the experiment set-up rather simple and transparent so that directions of dependence on different factors could be easily traced. We choose this second approach.

Our design of the Monte Carlo study includes two separate sets of experiments: for studying multicollinearity problem and those for studying measurement errors in exogenous variables problem.

We suggest the following mechanism to be under way in case of multicollinearity problem. For both DEA and SFA we see this problem as a technical one, which hampers the search of the optimal solution in LP (Linear Programming) and MLE algorithms, correspondently. Indeed, if correlation between two exogenous variables equals to one, it is analogous to the situation, when we have one explanatory variable less. But for each perfectly correlated variable we have a separate vector of coefficients, on which optimization occurs. The same holds for MLE. Thus, for both methods we expect estimates to be less robust and their range of values to be larger with increase in multicollinearity. The question is, how severe the reliability of our results deteriorates with multicollinearity and if it is so, then which method copes with the problem better. For DEA VRS, however, one more factor is in play. By high lower degrees of multicollinearity it is expected that more observations would be deemed to be efficient. In econometrics textbooks multicollinearity is often considered as inevitability with the first best advice to try to increase the sample size in hope to reduce the multicollinearity problem. Other solutions include dropping one-several variables, risking to end up with problem of bad specification. In the real life multicollinearity is very much a rule rather than exception. As Resti (2000) has pointed: "… previous literature usually does not account for correlation among different products: [however] in real life, when a firm produced a large amount of one output it is likely to produce large quantities of the remaining ones also".

Measurement error in exogenous variables, even if it is uncorrelated with noise or inefficiency terms biases our results in SFA. DEA overall, due to its specificity, is sensitive to noise/measurement errors. To feel it, we can conduct a simple mental exercise. If all observations are efficient in the world of CRS (1x1) technology and we have measurement errors in output (it is equivalent to noise in the efficiency score) then DEA would draw the estimated production frontier through the observation with the maximal error (see Figure 1). This observation with the maximal error will be chosen as a reference point and efficiency scores of other firms would be estimated in relation to it. A similar mechanism is in work if we have measurement error in inputs. Thus, measurement error problem in DEA_CRS tends to be exaggerated. This property is somewhat weaker, but also presents in DEA_VRS. For SFA and for DEA the problem of noise could be considered as a problem of omitted variables. Indeed, for a simple linear noise - linear production function example $y^{measured} = a + b(x + \varphi) - u + v$, where φ is the measurement error in x, is equivalent to $y^{true} = a + b(x + \varphi) - b\varphi - u + v$, hence our model suffers the omitted variable φ. Case of omitted variables was examined by Banker, Charnes & Cooper (1996), Ruggiero (1999) for DEA and SFA separately, who found both methods to be sensitive to omission of a relevant variable. In both studies, omission of an input of comparable size with other inputs was examined. Our aspect of study is very much different, however. Nobody expects the measurement error to be of a comparable size with inputs, otherwise, there would be no sense to measure these variables. In our study the size of measurement error is realistically assumed to vary from 0 to 20% of the size of input. Moreover, since any researcher conducts an eye-check of the data prior to analysis, the usage of infinite distribution for the modeling of measurement errors is not very much appropriate. We use a finite (uniform) distribution to investigate the problem. In econometric textbooks the two solutions are proposed for the measurement errors in endogenous variables problem. The first one is to assume it away by suggesting a small standard deviation of noise. The second one is to use proxy. However, "in practice it is not easy to find good proxies; we are often in the situation of complaining about the bad weather without being able to do much about it" (Gujarati (1995) p.470).

Thus, both problems are rather a rule than exception for the real-world data. The question of our study is how severe could these problems be and if it is, which model copes better with the problem.

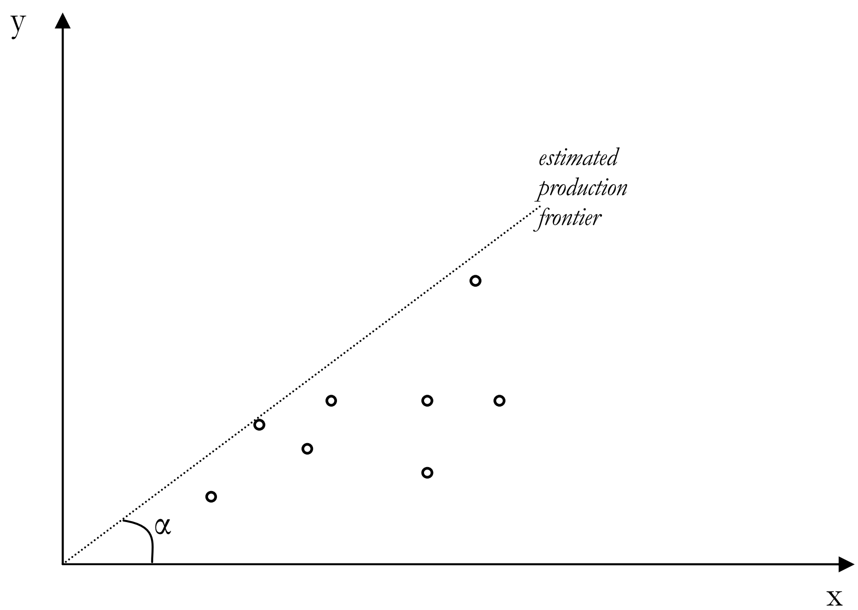

Figure 1. DEA CRS in presence of errors of measurement

In both sets of experiments we compare DEA_VRS and SFA_MLE. SFA_MLE assumes (correctly) a log-linear production function, half-normally distributed inefficiency term and normally distributed noise term:

$\ln y = \hat{\alpha} + \hat{\beta}_1 \ln x_1 + \hat{\beta}_2 \ln x_2 + (v \hat{-} u)$, where $u_i \sim |N(0, \sigma_u|$ is an inefficiency term, $v_i \sim N(0, \sigma_v)$ is noise term.

The main challenge in this approach in estimating of individual inefficiency scores is that we need to assess them having just estimates for distribution characteristics of v and *u* and individual estimations of a composed error term $\hat{\varepsilon}_i = (v_i \overset{\wedge}{-} u_i)$. Applying the approach of Jondrow *et al.*, we use the following proxy to access individual efficiency scores:

$$TE_i\_SFA\_1 = E\left(e^{u_{ii}} \mid \varepsilon_i\right) = \int_{u_i=0}^{\infty} e^{u_i} \cdot f\left(e^{u_{ii}} \mid \varepsilon_i\right) du_i =$$

$$= \frac{1-\Phi\left(-\sigma_* - \dfrac{\mu_*}{\sigma_*}\right)}{1-\Phi\left(-\dfrac{\mu_*}{\sigma_*}\right)} \cdot e^{\mu_* + \frac{1}{2}\sigma_*^2}, \text{ where } \sigma_* = \frac{\lambda \cdot \sigma}{\lambda^2+1},\ \mu_* = -\frac{\lambda^2}{\lambda^2+1} \cdot \varepsilon_i,\ \lambda = \frac{\sigma_u}{\sigma_v},$$

$$\sigma = \sqrt{\sigma_u^2 + \sigma_v^2}.$$

The true production function is Cobb-Douglas CRS with half-normally distributed inefficiency term and normally distributed noise term:

$\ln y = 0.5 + 0.5\ln x_1 + 0.5\ln x_2 + v - u$, where $u_i \sim \left|N(0, \sigma_u\right|$ is an inefficiency term, $v_i \sim N(0, \sigma_v)$ is noise term. We use this functional form for two reasons. Firstly, Cobb-Douglas production is proved to describe fairly well a plenty of real-world production processes. Secondly, it would allow us to make conclusions direct and transparent.

The parameters under variation include:

- The sample size N=25, 50, 100
- The standard deviation of the noise term σ_v=0; 0.02; 0.0447; 0.0976; 0.158; 0.2
- The bounds of the measurement error term (for the measurement error set of experiments): Abound=0; ±0.25; ±0.5; ±1
- The correlation coefficient between the explanatory variables (for the multicollinearity set of experiments): ϱ=0; 0.3; 0.5; 0.8.

The relative importance of inefficiency relative to noise λ is realistically set to 4.55. Thus, noise does not dominate inefficiency in none of experiments.

The explanatory variables in the measurement error set of experiments are independently uniformly distributed on the interval between 5 and 15: $x_1 \sim U(5;15)$, $x_2 \sim U(5;15)$.

For the multicollinearity set of experiments in order to fix correlation between explanatory variables the following data sets the correspondent data sets were generated and then pairs of variables were chosen from this set with correlation coefficients within a 0.01 range around the desired value (i.e. for ϱ=0.8, for example, pairs of variables with $corr(x_1, x_2) \in [0.795;0.805]$ were treated as acceptable) (see Table 2).

Table 4 Generation of explanatory variables for the multicollinearity set of experiments (note, thus generated x_2 is also distributed on the interval between 5 and 15).

$x_1 \sim U(5;15)$, $x_2 = a \cdot x_2 + U(A;B)$	
ϱ=-0.5	A=11; B=17; a=-2/5
ϱ=0	A=5; B=15; a=0
ϱ=0.3	A=3.7; B=11.1; a=1.3/5
ϱ=0.5	A=2/5; B=3; a=9
ϱ=0.8	A=3/5; B=2; a=6

Note, that correlation coefficients for (lnx_1, lnx_2) and (x_1;x_2) are very close to each other; so that, for example., for the generated data set 100x315 observations for each x_i and ϱ=0.8, exhibits correlation $corr(x_1, x_2) \in [0.7945;0.8045]$.

Our approach has several advantages over the common approach, where input variables are chosen from the normal distributions with the specified covariance matrix. Unlike the common approach, where correlation coefficients vary from study to study, on average tackling the desired value, in our study the correlation coefficient is fixed in each experiment. Moreover, uniform (finite) distribution both of the measurement error and inputs guarantees, that we don't have the problem of outliers. The outliers were proven to have a significant impact on the results (Resti (2000), Yu (1998), Read & Thanassaillis (1996)). Thus our conclusions will not be biased by the outlier-problem.

Parameters of distributions of noise and error terms are chosen in such a way to mimic possible real-world situations. The measurement error is included in the model as $\tilde{x}_i = x_i + \varphi_i$, $\varphi \sim U(-Abound, +Abound)$. It varies from 0 (the benchmark model) to ± 1, hence never exceeds 20% of the input level. The summary of the noise and inefficiency terms is presented in Table 3.

Table 5. Distribution parameters for noise and inefficiency terms

	$\lambda = 4.55$		$\lambda = 4.55$
$\sigma_v = 0$ no noise	$\lambda = \infty$ $\sigma_u = 0.2036$ $TE_{MEAN} = 1.186$ 95%: $TE < 1.490$ [ii]	$\sigma_v = 0.0976$ 95%: $< 21\%$	$\sigma_v = 0.4441$ $TE_{MEAN} = 1.482$ 95%: $TE < 2.388$
$\sigma_v = 0.02$ 95%: $< 4\%$	$\sigma_u = 0.0910$ $TE_{MEAN} = 1.077$ 95%: $TE < 1.195$	$\sigma_v = 0.1548$ 95%: $< 35\%$	$\sigma_u = 0.7043$ $TE_{MEAN} = 1.946$ 95%: $TE < 3.977$
$\sigma_v = 0.0447$ 95%: $< 9\%$	$\sigma_v = 0.2034$ $TE_{MEAN} = 1.1854$ 95%: $TE < 1.490$	$\sigma_v = 0.2$ 95%: $< 48\%$	$\sigma_v = 0.9100$ $TE_{MEAN} = 2.477$ 95%: $TE < 5.951$

Unfortunately, contrary to the claim of the epigraph, no proxy of costs of incorrect inference for the general decision making case is available for us. However, we treat here two basic situations, where productivity analysis is used. The first one is to determine the relative competitive position of the object of interest (ranking), the second one is to set the achievable output target for the DMU of interest (here absolute individual efficiency scores matter). For these purposes we use such criteria as Mean Absolute Deviation (MAD) between estimated and true technical efficiency scores, Pearson's correlation coefficient between estimated and true technical efficiency scores, Spearman's rank correlation coefficient between estimated and true rankings.

The experiment is conducted according to the following scenario to guarantee the robustness of results and appropriateness of conclusions. First we extendedly examine the behavior of DEA and SFA in samples of 25 and 50 units. Namely, we fix a pair of inputs with the given multicollinearity for a while and vary just significance of noise. Such an experiment should answer the following questions: how multicollinearity influences on the results and how this influence depends on the significance of the noise component. The appropriateness of results is checked then by making both multicollinearity and levels of noise variable, i.e. a different input pair and noise term is now chosen on each iteration of the experiment. Finally, the dependence of the results on multicollinearity is examined along the sample-number split. I.e., results of experiments for three basic noise levels are compared for number of sample sizes 25, 50 and 100. The set of measurement error experiments is conducted in the similar way. First, the measurement error term is fixed in the extended inquires for samples of 25 and 50 DMU's. Then we confirm our conclusions by examining the case, when both measurement error in inputs and outputs terms vary from iteration to iteration. And then we conclude with analyzing how sensitivity to the measurement error in endogenous variables is changed with the increase in the sample size.

Thus we run 216 experiments with 100 iterations pro experiment for experiments, when the sample size is 25 or 50, and 50 iterations pro experiment, when the sample size is 100. Hence, the overall number of "single" experiments is 24000.

Chapter 6

MAIN RESULTS

In our experiments DEA performs better than SFA according to MAD criterion. For example, in multicollinearity set of experiments, for σ_v=0, N=25 it ranges from 0.0305 to 0.1543, whereas SFA estimates show MAD form 0.1037 to 0.3093. In reality this would mean, that whereas you could mistake in determining efficiency of the DMU by 15% in the worst case in case of DEA, in case of SFA you mistake could be as large as 31%. Moreover, DEA is proven to be more robust for the increase in noise. Thus, in multicollinearity set of experiments, for σ_v=0, N=25 MAD in SFA is only two times higher than that of DEA, at high levels of noise for σ_v=0.2 MAD in SFA is about 2.5 times higher than that of DEA. In absolute values the discrepancy is even more pronounced. For σ_v=0.2 MAD for DEA ranges from) 0.25 to 1.37, whereas for SFA from 0.54 to 3.17. Both methods perform unsatisfactorily at high levels of noise. Our conclusions on a quite good robustness of DEA in response to noise and deterioration of performance of both DEA and SFA in response to increase in noise is in line with prior studies (Banker *et al.* (1993) etc.). However, in our experiments even at high levels of noise DEA outperforms SFA. The reason is that we use a rather high λ (relative significance of inefficiency in relation to noise) =4.55. As was pointed by Coelli *et al.* (1998), SFA tends to underestimate λ. At high levels of noise this effect becomes more pronounced. The pattern showed by MAD is widely confirmed by Pearson's and Spearman's correlation criteria. The value of correlation coefficients decreases with increase in noise. For example, in multicollinearity set of experiments, for σ_v=0, N=50, ϱ=0 for SFA Spearman=0.97, Pearson=0.99 at average, for σ_v=0.2 their average values are already 0.89 and 0.87 correspondently.

In both sets of experiments DEA and SFA show a substantial improvement in results with the increase of sample size. DEA improves its performance both in the level of MAD and spread of possible MAD. For example, for multicollinearity set of experiments for σ_v=0 if N=25 MAD ranges from 0.0471 to 0.1543 with its mean at 0.0939; for N=100 it is already 0.0313-0.0620 with its mean at 0.0447. Improvements of performance of SFA with increase of sample size are also substantial, but at low level of noise they are less pronounced. For example, in multicollinearity set of experiments for σ_v=0 the spread of MAD for SFA shrinks substantially with the increase of the sample size, with the widely same average level of MAD (at ϱ=0 MAD varies from 0.1037 to 0.2943 with average at 0.1845 for N=25, at N=100 it varies already from 0.1411 to 0.2103 with its average at 0.1862). For high levels of noise improvement in the level of MAD becomes more pronounced (for example, at σ_v=0.1548 ϱ=0 MAD is from 0.6951 to 0.3148 with its average at 1.5068 for N=25 and from 0.7307 to 1.2665 with its average at 0.9495 for N=100). Such pattern of results was expected at low levels of noise for DEA due to its consistency (Banker (1993)). Consistency of SFA as a maximum likelihood method also makes us to expect positive relation of the sample size to the quality of estimates. A good, rather unexpected piece of news is that improvement of the quality of results for DEA is present and is pronounced even in the presence of noise. The conclusions are confirmed by both Spearman and Pearson criteria. For example, in multicollinearity set of experiments σ_v=0.0976 ϱ=0.5 for DEA Spearman improves from 0.77 at N=25 to 0.83 at N=100, Pearson from 0.84 to 0.90 correspondently; SFA statistics follow the same pattern: Spearman increase its quality, varying from 0.85 to 0.90, Pearson – from 0.88 to 0.91 correspondently.

In our experiments, even having worse MAD, SFA consistently outperforms DEA by Spearman rank correlation and Pearson correlation coefficients. For example, in multicollinearity set of experiments for σ_v=0.0976

ϱ=0 N=25 for DEA average values of Spearman and Pearson are 0.73 and 0.77 correspondently, at the same time those of SFA are 0.86 and 0.88; however, MAD criteria shows a clear dominance of DEA for this case (MAD at average is 0.2133 for DEA and 0.4601 for SFA). The reason is that in DEA VRS production function is drawn through a set of observations chosen to be efficient according to the certain criteria, thus DEA consistently overestimates the number of 100% efficient DMUs. With the increase of the sample size this effect softens and asymptotically disappears.

Let examine the multicollinearity set of experiments more tediously. With the increase of multicollinearity MAD of DEA becomes on average smaller. For example, for N=25 σ_v=0 fixed inputs at ϱ=0 MAD DEA varies from 0.0351 to 0.1509 with average at 0.0965, at ϱ=0.8 it is somewhat lower and varies from 0.0323 to 0.1379 with average at 0.0720. In its scope, however, the effect is almost negligible. If by ϱ=0 we risk to make a mistake in 1.5% while estimating the inefficiency, at ϱ=0.8 we are "luckier" to reduce it to 1.4% being 0.1% exacter. Overall over experiments in different model settings this improvement never exceeds 1.5% in absolute value. The same direction of improvement is shown by Spearman and Pearson coefficients. For example for N=50 σ_v=0 fixed inputs at ϱ=0 Spearman and Pearson are at average 0.83 and 0.88 correspondently, at ϱ=0.8 their average values increase to 0.89 and 0.93 correspondently. This pattern is remarkably stable for all the experiments on multicollinearity. The reason, however, could be very simple. We noted in the preliminary analysis part (Chapter 5, Design of Monte Carlo Experiments) that 2 effects are expected to be in play in DEA: the first one, is decrease of robustness of results, the second one is decrease in the number of DMUs estimated to be efficient of DEA. The second effect is caught by our experiments. However, it is modest in its scope.

SFA shows no clear pattern of reaction in response to changes in multicollinearity in our experiments.

Overall, no significant influence of multicorrelation in the range from 0 to 0.8 in the case of two inputs was found. In view of believes of researches, described in the motivation and literature review sections it is an interesting and valuable result.

Let examine the measurement error set of experiments. Once more let recall that studies of omitted variable problem, when a variable of a comparable scope to the rest of the variables was omitted, for DEA and SFA showed that omission of a relevant variable has dramatic consequences. As was noted above (Chapter 1. Introduction) the measurement error in the endogenous variables could be considered as a special case of the omitted variable problem, where the "pseudo" omitted variable is the error term. However, commonly it is much less in scope then the endogenous variables. So, the question of interest is whether measurement errors of a realistic size (up to 20% of the endogenous variable) would cause any substantial problems to the estimated ranking and efficiency scores.

Having examined graphics we state that no clear direction of change of performance is observed with the introduction of the measurement error into endogenous variables, when the measurement error is moderate (not more than 20% of input). The expected deterioration in performance is not captured by the data even at low levels/no noise within the model.

It is also interesting to compare cases of no measurement error in endogenous variables, but presence of noise and cases of the presence of measurement error, but absence of noise. By doing so, we notice that uniformly distributed measurement error that lies in bound of -20% to 20% of the size of endogenous variables is roughly correspondent to the normally distributed noise that does not exceed 9% in 95 cases in 100. For example, for N=25 in case of no measurement errors in endogenous variables (Abound=0) and σ_v=0.0447 average MAD of DEA is 0.0891, Spearman and Pearson correlation coefficients are 0.6899 and 0.7684 correspondently. In case of no noise (σ_v=0) and Abound=1 average MAD for DEA is quite similar and equals to 0.0898, Spearman and Pearson coefficients somewhat higher and equal to 0.7857 and 0.8221 correspondently. The same pattern of correspondence is also observed for SFA. Lower correlation coefficients with the true efficiencies in case of noise along the same average value of MAD is a bit surprising, especially in case of SFA. Since exactly the normally distributed noise in endogenous variables is believed to be captured by the model. We see it as a confirmation of severance of underestimation of λ, as was pointed in Coelli (1998).

C h a p t e r 7

CONCLUSIONS

The aim of the present study was to examine the relative performance of DEA and SFA models in presence of realistic values of multicollinearity and the measurement error in endogenous variables. These problems were believed to have significant effects on the estimation results (Resti (2000), Pedraja-Chappara *et al.* (1999)). To examine the issue we carefully specified the underlying true data generation model to provide the "purity" of the experiment and conduct a number of experiments for different parameters of the sample size, significance of noise and also by fixing some variables for several iteration (by taking them out of loop) to check the robustness of the results. Our findings, however, are rather optimistic. We found no significant influence of multicorrelation in the range from 0 to 0.8 in the case of two inputs. Moreover, no clear direction of change of performance was observed with the introduction of the measurement error into endogenous variables, when the measurement error is moderate (not more than 20% of input). In view of believes of researches, described in the motivation and literature review sections, it is an interesting and valuable result.

We have performed a wide range of experiments to guarantee the robustness of our findings. The directions of research could be extended further, however. First of all, it would be interesting to examine consequences of multicollinearity as a function of number of endogenous variables. In our study we fixed the number of inputs to two. Secondly, it would be valuable to investigate also "extremely high" levels of multicollinearity and find the "critical" mass of multicollinearity for the models, when multicollinearity starts to pose serious problems. In case of measurement error set of experiments, it would be of value to examine the robustness of received results for different distributions of the measurement error term.

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APPENDIX A
TABLEA.1 COUNTRIES' EFFICIENCY. RESULTS FROM DEA VRS AND SFA.

	1965				1990			
	Efficiency (DEA)	DEA (Rank)	Efficiency (SFA)	SFA (Rank)	Efficiency (DEA)	DEA (Rank)	Efficiency (SFA)	SFA (Rank)
ARGENTINA	100.0%	1	79.2%	17	80.9%	24	80.9%	17
AUSTRALIA	76.4%	26	81.1%	13	82.4%	23	82.7%	13
AUSTRIA	85.3%	18	78.9%	18	73.2%	34	80.7%	18
BELGIUM	70.5%	32	81.4%	12	86.2%	18	83.0%	12
BOLIVIA	50.6%	45	39.5%	46	41.2%	50	43.2%	46
CANADA	79.7%	23	85.4%	6	93.5%	13	86.6%	6
CHILE	84.8%	19	71.1%	24	67.5%	39	73.4%	24
COLOMBIA	41.4%	53	43.6%	45	55.1%	45	47.2%	45
DENMARK	75.6%	27	77.3%	19	70.1%	37	79.2%	19
DOMINICAN REP.	75.1%	28	53.6%	36	51.7%	47	56.9%	36
ECUADOR	37.6%	54	36.6%	52	36.3%	53	40.3%	52
FINLAND	51.0%	44	63.4%	31	74.2%	31	66.2%	31
FRANCE	79.7%	24	83.0%	9	82.5%	22	84.5%	9
GERMANY, WEST	69.0%	33	74.7%	22	80.2%	26	76.8%	22
GREECE	54.8%	42	56.6%	35	59.9%	42	59.8%	35
GUATEMALA	85.7%	16	66.3%	29	76.5%	30	68.9%	29
HONDURAS	45.3%	49	37.0%	50	41.2%	51	40.8%	50
HONG KONG	**45.5%**	**48**	**66.7%**	**28**	100.0%	1	69.3%	28
ICELAND	100.0%	1	85.1%	7	100.0%	1	86.4%	7
INDIA	**100.0%**	**1**	**37.2%**	**49**	**100.0%**	**1**	**40.9%**	**49**
IRELAND	71.1%	30	75.4%	21	85.3%	19	77.5%	21
ISRAEL	60.2%	37	73.0%	23	84.5%	20	75.2%	23
ITALY	67.3%	**34**	79.8%	**14**	88.5%	17	81.5%	14
IVORY COAST	100.0%	1	66.1%	30	72.3%	35	68.7%	30
JAMAICA	57.0%	40	47.1%	41	52.1%	46	50.6%	41
JAPAN	76.7%	25	60.1%	32	61.5%	41	63.1%	32
KENYA	36.3%	55	26.8%	55	57.8%	44	30.4%	55
KOREA, REP.	55.4%	41	47.8%	39	67.9%	38	51.3%	39
LUXEMBOURG	100.0%	1	77.0%	20	100.0%	1	78.9%	20
MADAGASCAR	45.6%	47	25.2%	56	24.5%	57	28.8%	56
MALAWI	46.0%	46	36.9%	51	44.7%	49	40.6%	51
MAURITIUS	97.0%	12	79.7%	15	100.0%	1	81.4%	15
MEXICO	89.6%	14	79.5%	16	99.8%	12	81.2%	16
MAROCCO	93.5%	13	69.9%	26	100.0%	1	72.4%	26
NETHERLANDS	84.0%	21	86.8%	5	88.7%	16	88.0%	5

Proceeding…

NEW ZEALAND	87.4%	15	82.3%	10	**71.7%**	**36**	**83.8%**	**10**
NIGERIA	100.0%	1	45.5%	43	100.0%	1	49.0%	43
NORWAY	62.9%	36	58.8%	33	79.4%	27	61.8%	33
PANAMA	44.5%	50	38.0%	48	33.7%	55	41.7%	48
PARAGUAY	100.0%	1	94.2%	1	100.0%	1	94.7%	1
PERU	58.3%	38	46.9%	42	45.2%	48	50.4%	42
PHILIPPINES	51.6%	43	38.7%	47	74.0%	32	42.4%	47
PORTUGAL	71.1%	31	68.1%	27	80.6%	25	70.6%	27
SIERRA LEONE	100.0%	1	93.7%	2	100.0%	1	94.3%	2
SPAIN	97.8%	11	83.9%	8	82.9%	21	85.3%	8
SRI LANKA	32.6%	56	30.1%	54	36.6%	52	33.8%	54
SWEDEN	80.7%	22	81.7%	11	77.1%	29	83.3%	11
SWITZERLAND	85.4%	17	70.1%	25	89.1%	14	72.5%	25
SYRIA	42.2%	52	53.4%	37	65.6%	40	56.7%	37
TAIWAN	57.3%	39	51.7%	38	59.3%	43	55.1%	38
THAILAND	74.5%	29	45.0%	44	89.0%	15	48.5%	44
TURKEY	63.6%	35	47.2%	40	78.8%	28	50.7%	40
U.K.	100.0%	1	88.9%	4	100.0%	1	89.9%	4
U.S.A.	100.0%	1	92.5%	3	100.0%	1	93.2%	3
YUGOSLAVIA	84.4%	20	58.3%	34	73.8%	33	61.4%	34
ZAMBIA	42.9%	51	32.0%	53	34.1%	54	35.8%	53
ZIMBABWE	17.1%	57	18.1%	57	25.3%	56	21.3%	57

Table A.2 Correspondence of DEA VRS and SFA Estimates in the Countries' Example (Summary Statistics).

DEA 1965	SFA 1965	DEA 1990	SFA 1990
Mean	Mean	Mean	Mean
70.9%	62.2%	73.3%	64.8%
MAD		MAD	
0.13		0.11	
Pearson's correlation		Pearson's correlation	
75.5%		76.5%	
Spearman's correlation		Spearman's correlation	
67.5%		65.9%	

APPENDIX B

Table B.1

N=25

Only v and u vary from iteration to iteration; x (exogenous variables) are fixed

		min	mean	max	min	mean	max	min	mean	max
Abound/sigma_v			0			0.02			0.0447	
0										
	DEA									
	MAD	0.0337	0.0963	0.1737	0.0164	0.0417	0.0716	0.0352	0.0852	0.1408
	std_MAD	0.0285			0.0091			0.0193		
	Spearman	0.3638	0.7629	0.9523	0.2985	0.6519	0.8892	0.4546	0.7315	0.9615
	Pearson	0.0186	0.8095	0.9875	0.2029	0.7301	0.9682	0.3114	0.7927	0.9653
	SFA									
	MAD	0.1276	0.1929	0.3254	0.0497	0.0760	0.1022	0.1133	0.1851	0.2602
	std_MAD	0.0332			0.0119			0.0287		
	Spearman	0.6838	0.9317	0.9985	0.5231	0.8478	0.9569	0.4538	0.8401	0.9623
	Pearson	0.7859	0.9562	0.9955	0.6521	0.8875	0.9719	0.6930	0.8814	0.9738
0.3										
	DEA									
	MAD	0.0306	0.0834	0.1417	0.0210	0.0396	0.0658	0.0454	0.0897	0.1603
	std_MAD	0.0232			0.0087			0.0252		
	Spearman	0.4785	0.7895	0.9531	0.0862	0.6037	0.9308	0.2708	0.6666	0.9592
	Pearson	0.3004	0.8493	0.9847	0.2435	0.7319	0.9659	0.2077	0.7551	0.9746
	SFA									
	MAD	0.1244	0.1822	0.2483	0.0430	0.0768	0.1062	0.1274	0.1889	0.3250
	std_MAD	0.0268			0.0127			0.0324		
	Spearman	0.6785	0.9266	1.0000	0.4385	0.8435	0.9569	0.6538	0.8601	0.9646
	Pearson	0.7264	0.9521	0.9987	0.7308	0.8883	0.9741	0.7418	0.8911	0.9755
0.5										
	DEA									
	MAD	0.0308	0.0887	0.1766	0.0248	0.0404	0.0673	0.0411	0.0976	0.1510
	std_MAD	0.0253			0.0093			0.0234		
	Spearman	0.5315	0.7842	0.9346	0.4438	0.6582	0.8831	0.1692	0.6450	0.8592
	Pearson	0.4850	0.8356	0.9924	0.1964	0.7491	0.9697	0.2817	0.6856	0.9432
	SFA									
	MAD	0.1163	0.1858	0.2864	0.0425	0.0789	0.1073	0.1243	0.1841	0.2480
	std_MAD	0.0315			0.0125			0.0281		
	Spearman	0.7100	0.9267	0.9992	0.5300	0.8571	0.9492	0.6000	0.8555	0.9738
	Pearson	0.8318	0.9522	0.9987	0.6196	0.8947	0.9636	0.7241	0.8922	0.9629

Proceeding…

1									
DEA									
MAD	0.0449	0.0893	0.1417	0.0191	0.0401	0.0731	0.0573	0.0908	0.1668
std_MAD	0.0225			0.0096			0.0203		
Spearman	0.4662	0.7726	0.9362	0.1108	0.6365	0.9323	0.4462	0.7090	0.9115
Pearson	0.3151	0.8276	0.9794	0.3968	0.7488	0.9647	0.2645	0.7430	0.9394
SFA									
MAD	0.1093	0.1807	0.2680	0.0506	0.0775	0.1069	0.1162	0.1863	0.2860
std_MAD	0.0302			0.0129			0.0294		
Spearman	0.7592	0.9259	0.9985	0.5431	0.8517	0.9631	0.5200	0.8510	0.9646
Pearson	0.7726	0.9514	0.9982	0.6727	0.8947	0.9678	0.6499	0.8833	0.9721

Proceeding…

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
Abound/sigma_v		0.0976			0.1548			0.2	
0									
DEA									
MAD	0.0969	0.2081	0.3748	0.1960	0.4166	0.8367	0.3154	0.6318	1.2794
std_MAD	0.0595			0.1321			0.1956		
Spearman	0.2962	0.7171	0.9262	0.2662	0.7323	0.9400	0.4008	0.7190	0.9423
Pearson	0.1867	0.7967	0.9870	0.2417	0.7797	0.9796	0.0590	0.7858	0.9817
SFA									
MAD	0.2659	0.4828	0.6979	0.5223	0.9689	1.5757	0.6426	1.4049	2.4644
std_MAD	0.0927			0.2356			0.3898		
Spearman	0.4092	0.8505	0.9646	0.6508	0.8672	0.9808	0.6154	0.8479	0.9623
Pearson	0.7168	0.8860	0.9559	0.6552	0.8739	0.9436	0.6722	0.8546	0.9472
0.3									
DEA									
MAD	0.1011	0.2100	0.3558	0.1890	0.4150	0.7527	0.2419	0.6655	1.3510
std_MAD	0.0533			0.1331			0.2289		
Spearman	0.3938	0.7409	0.9392	0.1838	0.7384	0.9515	0.3638	0.7366	0.9208
Pearson	0.1678	0.8200	0.9884	0.3478	0.8313	0.9830	0.2721	0.8154	0.9897
SFA									
MAD	0.3120	0.4969	0.7307	0.4850	0.9481	1.6351	0.7728	1.4957	2.6250
std_MAD	0.0914			0.2303			0.4095		
Spearman	0.6092	0.8551	0.9654	0.4869	0.8490	0.9638	0.6485	0.8542	0.9638
Pearson	0.6631	0.8858	0.9584	0.6745	0.8649	0.9529	0.6819	0.8485	0.9543

Proceeding…

0.5									
DEA									
MAD	0.1195	0.2147	0.3707	0.1876	0.4097	0.9828	0.2929	0.6039	1.4640
std_MAD	0.0578			0.1360			0.1963		
Spearman	0.1362	0.7218	0.9100	0.3185	0.6923	0.9138	0.3823	0.7704	0.9646
Pearson	0.1422	0.7970	0.9616	0.0386	0.7911	0.9759	0.1378	0.8528	0.9951
SFA									
MAD	0.2352	0.4863	0.7387	0.3703	0.9182	1.4583	0.7552	1.4679	3.5347
std_MAD	0.0995			0.2178			0.4535		
Spearman	0.6208	0.8485	0.9623	0.5492	0.8427	0.9462	0.5262	0.8504	0.9554
Pearson	0.6183	0.8756	0.9557	0.6581	0.8696	0.9499	0.6470	0.8561	0.9585
1									
DEA									
MAD	0.1052	0.2117	0.3786	0.1974	0.4062	0.7209	0.2365	0.5938	1.2738
std_MAD	0.0534			0.1146			0.1745		
Spearman	0.3377	0.7101	0.9377	0.1862	0.7347	0.9385	0.4977	0.7727	0.9631
Pearson	0.1065	0.7797	0.9580	0.3375	0.8176	0.9864	0.2283	0.8497	0.9944
SFA									
MAD	0.2776	0.4800	0.7111	0.4842	0.9449	1.6367	0.8512	1.4582	2.4230
std_MAD	0.0872			0.2179			0.3517		
Spearman	0.6369	0.8529	0.9608	0.6185	0.8479	0.9723	0.6408	0.8579	0.9577
Pearson	0.7298	0.8829	0.9569	0.6889	0.8670	0.9539	0.6206	0.8552	0.9584

Table B.2

N=50

Only v and u vary from iteration to iteration; x (exogenous variables) are fixed

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
Abound/sigma_v		0			0.02			0.0447	
0									
DEA									
MAD	0.0366	0.0668	0.1076	0.0185	0.0311	0.0532	0.0415	0.0650	0.0977
std_MAD	0.0146			0.0054			0.0105		
Spearman	0.6971	0.8548	0.9741	0.4836	0.7102	0.8762	0.5929	0.7873	0.9301
Pearson	0.5316	0.8992	0.9884	0.4427	0.7900	0.9360	0.6091	0.8473	0.9547
SFA									
MAD	0.1416	0.1889	0.2496	0.0540	0.0770	0.1021	0.1423	0.1835	0.2351
std_MAD	0.0206			0.0089			0.0214		
Spearman	0.8482	0.9694	0.9988	0.7335	0.8863	0.9535	0.7901	0.8929	0.9510
Pearson	0.8689	0.9782	0.9976	0.8115	0.9149	0.9650	0.8271	0.9163	0.9666

Proceeding…

0.25										
	DEA									
	MAD	0.0309	0.0669	0.1100	0.0187	0.0315	0.0430	0.0408	0.0666	0.0991
	std_MAD	0.0157			0.0053			0.0110		
	Spearman	0.5563	0.8132	0.9520	0.4995	0.7368	0.9014	0.5371	0.7530	0.9045
	Pearson	0.5502	0.8674	0.9828	0.5350	0.8114	0.9421	0.5906	0.8384	0.9639
	SFA									
	MAD	0.1323	0.1853	0.2418	0.0521	0.0780	0.1012	0.1370	0.1857	0.2393
	std_MAD	0.0226			0.0079			0.0213		
	Spearman	0.8348	0.9665	0.9995	0.7475	0.8885	0.9563	0.7157	0.8821	0.9472
	Pearson	0.8247	0.9763	0.9980	0.8187	0.9177	0.9725	0.7839	0.9114	0.9647
0.5										
	DEA									
	MAD	0.0456	0.0719	0.1129	0.0203	0.0316	0.0465	0.0454	0.0709	0.1053
	std_MAD	0.0141			0.0062			0.0118		
	Spearman	0.6109	0.7949	0.9435	0.3593	0.7113	0.9005	0.5073	0.7415	0.9110
	Pearson	0.5465	0.8435	0.9679	0.3130	0.7863	0.9472	0.4261	0.8122	0.9567
	SFA									
	MAD	0.1485	0.1845	0.2582	0.0588	0.0761	0.1015	0.1355	0.1847	0.2357
	std_MAD	0.0181			0.0082			0.0177		
	Spearman	0.8712	0.9684	0.9988	0.7200	0.8799	0.9450	0.7660	0.8868	0.9499
	Pearson	0.9177	0.9781	0.9984	0.8031	0.9110	0.9616	0.7980	0.9157	0.9573
1										
	DEA									
	MAD	0.0286	0.0623	0.0985	0.0185	0.0306	0.0448	0.0414	0.0707	0.1043
	std_MAD	0.0157			0.0057			0.0131		
	Spearman	0.6094	0.8600	0.9772	0.5230	0.7383	0.8783	0.5557	0.7534	0.9149
	Pearson	0.5674	0.9016	0.9843	0.5866	0.8044	0.9442	0.5845	0.8279	0.9611
	SFA									
	MAD	0.1320	0.1843	0.2483	0.0464	0.0760	0.0940	0.1421	0.1869	0.2487
	std_MAD	0.0228			0.0084			0.0207		
	Spearman	0.8275	0.9638	0.9985	0.7380	0.8868	0.9482	0.7650	0.8855	0.9568
	Pearson	0.8464	0.9750	0.9979	0.8294	0.9131	0.9589	0.8117	0.9146	0.9545

Proceeding…

		min	mean	max	min	mean	max	min	mean	max
Abound/sigma_v			0.0976			0.1548			0.2	
0										
	DEA									
	MAD	0.1177	0.1695	0.2894	0.1905	0.3689	0.6882	0.3168	0.5757	1.2006
	std_MAD	0.0305			0.0931			0.1478		
	Spearman	0.4394	0.7665	0.9161	0.4983	0.7530	0.9207	0.6359	0.7860	0.9498
	Pearson	0.4354	0.8452	0.9609	0.3792	0.8355	0.9826	0.3536	0.8763	0.9883
	SFA									
	MAD	0.3024	0.4673	0.6871	0.5498	0.9403	1.3164	0.3893	1.4657	2.5581
	std_MAD	0.0634			0.1603			0.3023		
	Spearman	0.7322	0.8877	0.9503	0.7619	0.8838	0.9598	0.6980	0.8819	0.9503
	Pearson	0.8077	0.9032	0.9544	0.7350	0.8754	0.9338	0.7574	0.8595	0.9726
0.25										
	DEA									
	MAD	0.1089	0.1748	0.2457	0.2176	0.3588	0.5668	0.3274	0.5479	0.8661
	std_MAD	0.0303			0.0739			0.1155		
	Spearman	0.5648	0.7679	0.9201	0.5721	0.7694	0.9517	0.5420	0.8122	0.9550
	Pearson	0.4860	0.8382	0.9787	0.4735	0.8518	0.9685	0.6316	0.9025	0.9849
	SFA									
	MAD	0.3370	0.4826	0.6273	0.5961	0.9494	1.3899	1.0180	1.5273	2.1443
	std_MAD	0.0608			0.1572			0.2542		
	Spearman	0.7857	0.8895	0.9477	0.7119	0.8828	0.9442	0.7900	0.8855	0.9576
	Pearson	0.8056	0.9065	0.9525	0.7777	0.8803	0.9448	0.7400	0.8555	0.9214
0.5										
	DEA									
	MAD	0.1125	0.1846	0.3242	0.2015	0.3598	0.6634	0.3155	0.5212	0.9476
	std_MAD	0.0391			0.0863			0.1262		
	Spearman	0.4014	0.7329	0.8946	0.5155	0.7736	0.9375	0.6903	0.8477	0.9381
	Pearson	0.4187	0.8246	0.9649	0.3750	0.8449	0.9870	0.5107	0.9138	0.9860
	SFA									
	MAD	0.3375	0.4819	0.6460	0.2948	0.9531	1.3085	0.9669	1.4895	2.5598
	std_MAD	0.0624			0.1627			0.2742		
	Spearman	0.7921	0.8881	0.9640	0.7475	0.8874	0.9500	0.6976	0.8869	0.9447
	Pearson	0.8349	0.9065	0.9560	0.7371	0.8800	0.9382	0.7059	0.8659	0.9340

Proceeding…

1									
DEA									
MAD	0.0874	0.1655	0.2663	0.2156	0.3643	0.6383	0.2976	0.5760	1.0876
std_MAD	0.0294			0.0834			0.1561		
Spearman	0.6206	0.8056	0.9253	0.5887	0.7690	0.9093	0.6173	0.8022	0.9237
Pearson	0.6047	0.8665	0.9672	0.3236	0.8547	0.9895	0.2807	0.8707	0.9867
SFA									
MAD	0.3515	0.4923	0.6657	0.6340	0.9609	1.4812	0.9697	1.4622	2.3244
std_MAD	0.0718			0.1506			0.2754		
Spearman	0.7773	0.8936	0.9483	0.7256	0.8876	0.9534	0.7670	0.8827	0.9419
Pearson	0.8318	0.9090	0.9454	0.7147	0.8788	0.9467	0.7125	0.8533	0.9225

Table B.3

N=25

All variables vary from iteration to iteration

	min	mean	max	min	mean	max	min	mean	max
Abound/sigma_v		0			0.02			0.0447	
0									
DEA									
MAD	0.0456	0.0919	0.1735	0.0186	0.0414	0.0651	0.0441	0.0891	0.1682
std_MAD	0.0270			0.0097			0.0222		
Spearman	0.4815	0.7703	0.9554	0.2854	0.6465	0.9000	0.4146	0.6899	0.8985
Pearson	0.3720	0.8061	0.9850	0.2243	0.7419	0.9434	0.3374	0.7684	0.9784
SFA									
MAD	0.1046	0.1855	0.2593	0.0430	0.0769	0.1043	0.1079	0.1850	0.2555
std_MAD	0.0335			0.0125			0.0325		
Spearman	0.6646	0.9273	0.9992	0.5200	0.8476	0.9638	0.5831	0.8590	0.9738
Pearson	0.7711	0.9534	0.9981	0.7370	0.8980	0.9737	0.7744	0.8977	0.9779
0.25									
DEA									
MAD	0.0416	0.0915	0.1880	0.0218	0.0412	0.0650	0.0509	0.0890	0.1674
std_MAD	0.0263			0.0095			0.0211		
Spearman	0.4315	0.7608	0.9431	0.2623	0.6782	0.9123	0.2992	0.6776	0.9000
Pearson	0.3203	0.8160	0.9901	0.3702	0.7423	0.9443	0.3028	0.7438	0.9639
SFA									
MAD	0.1118	0.1862	0.2817	0.0544	0.0789	0.1067	0.0867	0.1818	0.2596
std_MAD	0.0324			0.0119			0.0306		
Spearman	0.7308	0.9316	0.9985	0.5262	0.8563	0.9715	0.5123	0.8416	0.9577
Pearson	0.7397	0.9566	0.9979	0.7054	0.8999	0.9777	0.7246	0.8893	0.9738

Proceeding…

0.5									
DEA									
MAD	0.0427	0.0928	0.1722	0.0187	0.0412	0.0795	0.0348	0.0891	0.1411
std_MAD	0.0279			0.0114			0.0221		
Spearman	0.3746	0.7782	0.9700	0.2400	0.6549	0.8685	0.1846	0.6909	0.9046
Pearson	0.3377	0.8278	0.9873	0.3496	0.7466	0.9630	0.2501	0.7549	0.9629
SFA									
MAD	0.1183	0.1866	0.2991	0.0425	0.0776	0.1073	0.1156	0.1858	0.2659
std_MAD	0.0361			0.0132			0.0310		
Spearman	0.7531	0.9247	0.9969	0.6515	0.8492	0.9685	0.5208	0.8491	0.9615
Pearson	0.8294	0.9530	0.9983	0.6696	0.8926	0.9670	0.7501	0.8878	0.9653
1									
DEA									
MAD	0.0324	0.0898	0.1588	0.0228	0.0409	0.0658	0.0451	0.0873	0.1424
std_MAD	0.0263			0.0092			0.0203		
Spearman	0.5138	0.7857	0.9708	0.3238	0.6656	0.9392	0.3454	0.7085	0.9046
Pearson	0.4175	0.8221	0.9868	0.3150	0.7330	0.9632	0.2111	0.7930	0.9574
SFA									
MAD	0.1165	0.1878	0.2770	0.0483	0.0782	0.1094	0.1042	0.1854	0.2547
std_MAD	0.0318			0.0131			0.0316		
Spearman	0.7477	0.9249	0.9985	0.5546	0.8599	0.9615	0.5046	0.8490	0.9631
Pearson	0.7981	0.9459	0.9967	0.6347	0.8939	0.9737	0.7206	0.8969	0.9696

Proceeding…

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
Abound/sigma_v		0.0976			0.1548			0.2	
0									
DEA									
MAD	0.0745	0.2187	0.4073	0.2016	0.4264	0.7697	0.2768	0.6302	1.1381
std_MAD	0.0563			0.1118			0.2042		
Spearman	0.3215	0.7034	0.9262	0.3354	0.6961	0.9262	0.1915	0.7459	0.9400
Pearson	0.2191	0.7711	0.9748	0.0863	0.7655	0.9850	0.2959	0.8264	0.9858
SFA									
MAD	0.2653	0.4832	0.7765	0.5867	0.9557	1.4468	0.7903	1.5372	2.6993
std_MAD	0.0895			0.1982			0.3936		
Spearman	0.6885	0.8505	0.9531	0.5669	0.8595	0.9600	0.5408	0.8564	0.9708
Pearson	0.7129	0.8724	0.9704	0.6993	0.8670	0.9558	0.5949	0.8563	0.9527

Proceeding…

0.25										
	DEA									
	MAD	0.1125	0.2139	0.4100	0.1973	0.4062	0.7340	0.2642	0.6284	1.3860
	std_MAD	0.0582			0.1179			0.2101		
	Spearman	0.3985	0.7239	0.9638	0.3462	0.7223	0.9623	0.4792	0.7543	0.9423
	Pearson	0.2346	0.7834	0.9849	0.1798	0.7766	0.9852	0.1644	0.8417	0.9902
	SFA									
	MAD	0.2737	0.4792	0.7500	0.5330	0.9484	1.4314	0.5776	1.4911	2.7697
	std_MAD	0.0998			0.2124			0.4164		
	Spearman	0.5885	0.8439	0.9554	0.6738	0.8622	0.9562	0.6354	0.8469	0.9638
	Pearson	0.6301	0.8678	0.9577	0.5900	0.8629	0.9500	0.6558	0.8512	0.9604
0.5										
	DEA									
	MAD	0.0979	0.2078	0.4793	0.1982	0.4146	0.7240	0.2437	0.6416	1.2493
	std_MAD	0.0654			0.1043			0.1898		
	Spearman	0.2446	0.7075	0.9162	0.4015	0.7246	0.9531	0.1685	0.7208	0.9569
	Pearson	0.1400	0.7744	0.9743	0.2147	0.8083	0.9951	0.0446	0.8256	0.9829
	SFA									
	MAD	0.2670	0.4679	0.7391	0.1752	0.9459	1.5186	0.7342	1.4707	2.4531
	std_MAD	0.0967			0.2174			0.3641		
	Spearman	0.5515	0.8559	0.9746	0.5915	0.8527	0.9685	0.4600	0.8431	0.9754
	Pearson	0.7015	0.8903	0.9668	0.7477	0.8792	0.9552	0.6064	0.8503	0.9469
1										
	DEA									
	MAD	0.1191	0.2035	0.3303	0.2225	0.4073	0.7293	0.2679	0.6811	1.4576
	std_MAD	0.0517			0.1087			0.2161		
	Spearman	0.4669	0.7408	0.9608	0.3454	0.7233	0.9392	0.2731	0.7237	0.9469
	Pearson	0.3257	0.8038	0.9894	-0.0251	0.7934	0.9794	0.1415	0.8200	0.9858
	SFA									
	MAD	0.2993	0.4731	0.7329	0.5715	0.9340	1.6740	0.8122	1.5640	3.3550
	std_MAD	0.0827			0.1994			0.4418		
	Spearman	0.6277	0.8473	0.9654	0.6408	0.8471	0.9662	0.5962	0.8525	0.9615
	Pearson	0.7330	0.8804	0.9569	0.6734	0.8624	0.9495	0.6758	0.8501	0.9461

Table B.4

N=50

All variables vary from iteration to iteration

Abound/sigma_v		min	mean	max	min	mean	max	min	mean	max
			0			0.02			0.0447	
0										
	DEA									
	MAD	0.0293	0.0624	0.1081	0.0197	0.0315	0.0457	0.0462	0.0720	0.1256
	std_MAD	0.0136			0.0057			0.0137		
	Spearman	0.6249	0.8390	0.9749	0.3978	0.7047	0.8717	0.4572	0.7385	0.9297
	Pearson	0.6783	0.8989	0.9859	0.4112	0.7885	0.9471	0.3303	0.8107	0.9657
	SFA									
	MAD	0.1395	0.1833	0.2582	0.0548	0.0751	0.0943	0.0702	0.1873	0.2383
	std_MAD	0.0206			0.0082			0.0263		
	Spearman	0.8578	0.9706	0.9997	0.7726	0.8877	0.9532	0.7990	0.8936	0.9649
	Pearson	0.9103	0.9793	0.9983	0.8529	0.9183	0.9681	0.8507	0.9207	0.9702
0.25										
	DEA									
	MAD	0.0330	0.0615	0.1021	0.0196	0.0317	0.0479	0.0419	0.0712	0.1128
	std_MAD	0.0159			0.0059			0.0135		
	Spearman	0.6131	0.8438	0.9754	0.4817	0.7081	0.8723	0.4848	0.7491	0.9217
	Pearson	0.5384	0.8822	0.9824	0.4807	0.7861	0.9318	0.4942	0.8170	0.9527
	SFA									
	MAD	0.1320	0.1850	0.2395	0.0531	0.0775	0.0981	0.1423	0.1851	0.2434
	std_MAD	0.0193			0.0096			0.0235		
	Spearman	0.7937	0.9656	0.9997	0.7288	0.8819	0.9522	0.7662	0.8791	0.9619
	Pearson	0.8527	0.9756	0.9978	0.8317	0.9144	0.9671	0.8008	0.9112	0.9635
0.5										
	DEA									
	MAD	0.0336	0.0690	0.1257	0.0189	0.0309	0.0482	0.0396	0.0690	0.1018
	std_MAD	0.0180			0.0058			0.0126		
	Spearman	0.6429	0.8277	0.9627	0.3591	0.7250	0.9345	0.5953	0.7505	0.9255
	Pearson	0.2166	0.8296	0.9930	0.5277	0.8008	0.9550	0.5699	0.8471	0.9485
	SFA									
	MAD	0.1414	0.1887	0.2386	0.0518	0.0758	0.0980	0.1302	0.1866	0.2406
	std_MAD	0.0235			0.0086			0.0230		
	Spearman	0.8652	0.9685	0.9986	0.7339	0.8778	0.9540	0.7108	0.8796	0.9691
	Pearson	0.9229	0.9782	0.9979	0.5181	0.9090	0.9673	0.8026	0.9156	0.9644

Proceeding…

1										
	DEA									
	MAD	0.0253	0.0700	0.1261	0.0199	0.0320	0.0489	0.0454	0.0697	0.1076
	std_MAD	0.0216			0.0064			0.0131		
	Spearman	0.6391	0.8381	0.9798	0.4986	0.7260	0.9031	0.5693	0.7542	0.8995
	Pearson	0.3644	0.8358	0.9898	0.5744	0.7954	0.9618	0.5541	0.8276	0.9521
	SFA									
	MAD	0.1425	0.1853	0.2437	0.0540	0.0773	0.0938	0.1191	0.1863	0.2426
	std_MAD	0.0236			0.0088			0.0211		
	Spearman	0.8679	0.9627	0.9997	0.6672	0.8805	0.9401	0.7219	0.8858	0.9511
	Pearson	0.8867	0.9740	0.9982	0.7285	0.9093	0.9595	0.7994	0.9132	0.9523

Proceeding…

		min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
Abound/sigma_v			0.0976			0.1548			0.2	
0										
	DEA									
	MAD	0.1174	0.1778	0.3366	0.2221	0.3574	0.5867	0.2833	0.5982	1.2269
	std_MAD	0.0378			0.0762			0.1589		
	Spearman	0.4776	0.7610	0.9122	0.4695	0.7643	0.9484	0.5134	0.7625	0.9368
	Pearson	0.4873	0.8297	0.9586	0.2747	0.8464	0.9825	0.2577	0.8631	0.9918
	SFA									
	MAD	0.3573	0.4854	0.7566	0.3631	0.9325	1.3859	0.9018	1.4907	2.3316
	std_MAD	0.0651			0.1503			0.2803		
	Spearman	0.7620	0.8888	0.9536	0.7694	0.8871	0.9481	0.7647	0.8861	0.9550
	Pearson	0.8119	0.9039	0.9531	0.7808	0.8785	0.9376	0.7200	0.8567	0.9512
0.25										
	DEA									
	MAD	0.1247	0.1765	0.2544	0.1927	0.3508	0.5560	0.3486	0.5720	0.9986
	std_MAD	0.0323			0.0721			0.1411		
	Spearman	0.5100	0.7693	0.9128	0.5946	0.7698	0.8967	0.5673	0.7626	0.9286
	Pearson	0.4962	0.8460	0.9623	0.5249	0.8836	0.9743	0.4044	0.8510	0.9826
	SFA									
	MAD	0.3620	0.4898	0.6797	0.5959	0.9555	1.2923	0.8596	1.4556	2.4473
	std_MAD	0.0599			0.1542			0.3086		
	Spearman	0.7952	0.8909	0.9544	0.7123	0.8794	0.9695	0.7609	0.8833	0.9510
	Pearson	0.8379	0.9034	0.9554	0.7929	0.8854	0.9334	0.7691	0.8627	0.9350

Proceeding…

0.5										
	DEA									
	MAD	0.1116	0.1758	0.2861	0.2262	0.3496	0.5159	0.3054	0.5887	1.0706
	std_MAD	0.0364			0.0683			0.1497		
	Spearman	0.5740	0.7713	0.8945	0.5697	0.7743	0.9314	0.6020	0.7719	0.9549
	Pearson	0.3629	0.8459	0.9618	0.5341	0.8720	0.9765	0.4109	0.8700	0.9949
	SFA									
	MAD	0.3518	0.4777	0.6474	0.2522	0.9355	1.2656	0.7948	1.4695	2.2419
	std_MAD	0.0690			0.1462			0.2907		
	Spearman	0.7842	0.8881	0.9523	0.6918	0.8753	0.9573	0.7277	0.8846	0.9557
	Pearson	0.8048	0.9050	0.9694	0.7926	0.8790	0.9586	0.7320	0.8544	0.9149
1										
	DEA									
	MAD	0.1117	0.1700	0.2618	0.2157	0.3588	0.6076	0.3306	0.5805	1.0145
	std_MAD	0.0264			0.0742			0.1345		
	Spearman	0.5848	0.7854	0.9307	0.4810	0.7804	0.9254	0.4778	0.7801	0.9382
	Pearson	0.4257	0.8604	0.9738	0.5801	0.8681	0.9786	0.4423	0.8688	0.9843
	SFA									
	MAD	0.3316	0.4886	0.6875	0.6726	0.9797	1.5278	0.8911	1.4811	2.4466
	std_MAD	0.0633			0.1644			0.2929		
	Spearman	0.7263	0.8889	0.9612	0.7623	0.8899	0.9623	0.6672	0.8783	0.9463
	Pearson	0.8316	0.9082	0.9524	0.8004	0.8854	0.9457	0.6037	0.8557	0.9232

Table B.5

sigma_v=0   lambda=inf   sigma_u=0.2036
All variables vary from iteration to iteration

		min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
Abound/N			25			50			100	
0										
	DEA									
	MAD	0.0456	0.0919	0.1735	0.0293	0.0624	0.1081	0.0243	0.0466	0.0702
	std_MAD	0.027			0.0136			0.0109		
	Spearman	0.4815	0.7703	0.9554	0.6249	0.839	0.9749	0.736	0.8933	0.9653
	Pearson	0.372	0.8061	0.985	0.6783	0.8989	0.9859	0.7186	0.9327	0.9894
	SFA									
	MAD	0.1046	0.1855	0.2593	0.1395	0.1833	0.2582	0.1543	0.1895	0.2335
	std_MAD	0.0335			0.0206			0.0204		
	Spearman	0.6646	0.9273	0.9992	0.8578	0.9706	0.9997	0.9087	0.9835	0.9997
	Pearson	0.7711	0.9534	0.9981	0.9103	0.9793	0.9983	0.9509	0.9863	0.9977

Proceeding…

0.25										
	DEA									
	MAD	0.0416	0.0915	0.188	0.033	0.0615	0.1021	0.019	0.0452	0.0686
	std_MAD	0.0263			0.0159			0.0094		
	Spearman	0.4315	0.7608	0.9431	0.6131	0.8438	0.9754	0.7656	0.8925	0.9691
	Pearson	0.3203	0.816	0.9901	0.5384	0.8822	0.9824	0.7997	0.9323	0.9904
	SFA									
	MAD	0.1118	0.1862	0.2817	0.132	0.185	0.2395	0.1544	0.187	0.2281
	std_MAD	0.0324			0.0193			0.0168		
	Spearman	0.7308	0.9316	0.9985	0.7937	0.9656	0.9997	0.9017	0.9817	0.9996
	Pearson	0.7397	0.9566	0.9979	0.8527	0.9756	0.9978	0.9455	0.9854	0.9966
0.5										
	DEA									
	MAD	0.0427	0.0928	0.1722	0.0336	0.069	0.1257	0.0284	0.0438	0.0725
	std_MAD	0.0279			0.018			0.0089		
	Spearman	0.3746	0.7782	0.97	0.6429	0.8277	0.9627	0.7694	0.9041	0.967
	Pearson	0.3377	0.8278	0.9873	0.2166	0.8296	0.993	0.8433	0.9437	0.987
	SFA									
	MAD	0.1183	0.1866	0.2991	0.1414	0.1887	0.2386	0.16	0.1874	0.2205
	std_MAD	0.0361			0.0235			0.0138		
	Spearman	0.7531	0.9247	0.9969	0.8652	0.9685	0.9986	0.9317	0.9839	0.9988
	Pearson	0.8294	0.953	0.9983	0.9229	0.9782	0.9979	0.948	0.9874	0.9974
1										
	DEA									
	MAD	0.0324	0.0898	0.1588	0.0253	0.07	0.1261	0.0222	0.0431	0.0618
	std_MAD	0.0263			0.0216			0.0102		
	Spearman	0.5138	0.7857	0.9708	0.6391	0.8381	0.9798	0.8071	0.8952	0.9754
	Pearson	0.4175	0.8221	0.9868	0.3644	0.8358	0.9898	0.7898	0.9308	0.9916
	SFA									
	MAD	0.1165	0.1878	0.277	0.1425	0.1853	0.2437	0.1411	0.1855	0.2165
	std_MAD	0.0318			0.0236			0.0149		
	Spearman	0.7477	0.9249	0.9985	0.8679	0.9627	0.9997	0.912	0.9809	0.9988
	Pearson	0.7981	0.9459	0.9967	0.8867	0.974	0.9982	0.9478	0.9851	0.997

Table B.6

sigma_v=0.0976
All variables vary from iteration to iteration

Abound/N		min	mean (25)	max	min	mean (50)	max	min	mean (100)	max
0										
	DEA									
	MAD	0.0745	0.2187	0.4073	0.1174	0.1778	0.3366	0.1285	0.1669	0.2425
	std_MAD	0.0563			0.0378			0.0255		
	Spearman	0.3215	0.7034	0.9262	0.4776	0.7610	0.9122	0.6288	0.8161	0.9115
	Pearson	0.2191	0.7711	0.9748	0.4873	0.8297	0.9586	0.6339	0.8854	0.9681
	SFA									
	MAD	0.2653	0.4832	0.7765	0.3573	0.4854	0.7566	0.3935	0.5018	0.6413
	std_MAD	0.0895			0.0651			0.0622		
	Spearman	0.6885	0.8505	0.9531	0.7620	0.8888	0.9536	0.8329	0.9082	0.9486
	Pearson	0.7129	0.8724	0.9704	0.8119	0.9039	0.9531	0.8742	0.9126	0.9411
0.25										
	DEA									
	MAD	0.1125	0.2139	0.4100	0.1247	0.1765	0.2544	0.1240	0.1681	0.2416
	std_MAD	0.0582			0.0323			0.0222		
	Spearman	0.3985	0.7239	0.9638	0.5100	0.7693	0.9128	0.6988	0.8038	0.8979
	Pearson	0.2346	0.7834	0.9849	0.4962	0.8460	0.9623	0.6893	0.8752	0.9604
	SFA									
	MAD	0.2737	0.4792	0.7500	0.3620	0.4898	0.6797	0.3890	0.4805	0.5580
	std_MAD	0.0998			0.0599			0.0433		
	Spearman	0.5885	0.8439	0.9554	0.7952	0.8909	0.9544	0.8208	0.9004	0.9432
	Pearson	0.6301	0.8678	0.9577	0.8379	0.9034	0.9554	0.8603	0.9069	0.9386
0.5										
	DEA									
	MAD	0.0979	0.2078	0.4793	0.1116	0.1758	0.2861	0.1263	0.1603	0.2152
	std_MAD	0.0654			0.0364			0.0200		
	Spearman	0.2446	0.7075	0.9162	0.5740	0.7713	0.8945	0.6970	0.8327	0.9134
	Pearson	0.1400	0.7744	0.9743	0.3629	0.8459	0.9618	0.7599	0.8996	0.9624
	SFA									
	MAD	0.2670	0.4679	0.7391	0.3518	0.4777	0.6474	0.3510	0.4855	0.5914
	std_MAD	0.0967			0.0690			0.0459		
	Spearman	0.5515	0.8559	0.9746	0.7842	0.8881	0.9523	0.8597	0.9077	0.9478
	Pearson	0.7015	0.8903	0.9668	0.8048	0.9050	0.9694	0.8634	0.9163	0.9514

Proceeding…

1									
DEA									
MAD	0.1191	0.2035	0.3303	0.1117	0.1700	0.2618	0.1130	0.1580	0.2167
std_MAD	0.0517			0.0264			0.0227		
Spearman	0.4669	0.7408	0.9608	0.5848	0.7854	0.9307	0.7109	0.8279	0.9256
Pearson	0.3257	0.8038	0.9894	0.4257	0.8604	0.9738	0.8378	0.8966	0.9500
SFA									
MAD	0.2993	0.4731	0.7329	0.3316	0.4886	0.6875	0.4007	0.4755	0.5719
std_MAD	0.0827			0.0633			0.0393		
Spearman	0.6277	0.8473	0.9654	0.7263	0.8889	0.9612	0.8164	0.9053	0.9484
Pearson	0.7330	0.8804	0.9569	0.8316	0.9082	0.9524	0.8673	0.9114	0.9463

Table B.7

sigma_v=0.1548
All variables vary from iteration to iteration

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
Abound/N		25			50			100	
0									
DEA									
MAD	0.2016	0.4264	0.7697	0.2221	0.3574	0.5867	0.2523	0.3558	0.5117
std_MAD	0.1118			0.0762			0.0698		
Spearman	0.3354	0.6961	0.9262	0.4695	0.7643	0.9484	0.6397	0.8205	0.9087
Pearson	0.0863	0.7655	0.9850	0.2747	0.8464	0.9825	0.3759	0.8842	0.9775
SFA									
MAD	0.5867	0.9557	1.4468	0.3631	0.9325	1.3859	0.7361	0.9417	1.1487
std_MAD	0.1982			0.1503			0.1030		
Spearman	0.5669	0.8595	0.9600	0.7694	0.8871	0.9481	0.8102	0.9040	0.9495
Pearson	0.6993	0.8670	0.9558	0.7808	0.8785	0.9376	0.7910	0.8840	0.9295
0.25									
DEA									
MAD	0.1973	0.4062	0.7340	0.1927	0.3508	0.5560	0.2405	0.3400	0.4814
std_MAD	0.1179			0.0721			0.0568		
Spearman	0.3462	0.7223	0.9623	0.5946	0.7698	0.8967	0.6312	0.8217	0.8965
Pearson	0.1798	0.7766	0.9852	0.5249	0.8836	0.9743	0.6637	0.8936	0.9655
SFA									
MAD	0.5330	0.9484	1.4314	0.5959	0.9555	1.2923	0.7356	0.9398	1.1563
std_MAD	0.2124			0.1542			0.1066		
Spearman	0.6738	0.8622	0.9562	0.7123	0.8794	0.9695	0.8276	0.9014	0.9409
Pearson	0.5900	0.8629	0.9500	0.7929	0.8854	0.9334	0.8334	0.8807	0.9152

Proceeding…

0.5									
DEA									
MAD	0.1982	0.4146	0.7240	0.2262	0.3496	0.5159	0.2437	0.3522	0.5258
std_MAD	0.1043			0.0683			0.0655		
Spearman	0.4015	0.7246	0.9531	0.5697	0.7743	0.9314	0.6932	0.8207	0.8847
Pearson	0.2147	0.8083	0.9951	0.5341	0.8720	0.9765	0.7023	0.8957	0.9717
SFA									
MAD	0.1752	0.9459	1.5186	0.2522	0.9355	1.2656	0.7388	0.9463	1.1132
std_MAD	0.2174			0.1462			0.1017		
Spearman	0.5915	0.8527	0.9685	0.6918	0.8753	0.9573	0.8310	0.9044	0.9470
Pearson	0.7477	0.8792	0.9552	0.7926	0.8790	0.9586	0.8331	0.8832	0.9301
1									
DEA									
MAD	0.2225	0.4073	0.7293	0.2157	0.3588	0.6076	0.2494	0.3434	0.5242
std_MAD	0.1087			0.0742			0.0622		
Spearman	0.3454	0.7233	0.9392	0.4810	0.7804	0.9254	0.7162	0.8211	0.9152
Pearson	-0.0251	0.7934	0.9794	0.5801	0.8681	0.9786	0.6392	0.9022	0.9734
SFA									
MAD	0.5715	0.9340	1.6740	0.6726	0.9797	1.5278	0.6879	0.9560	1.2884
std_MAD	0.1994			0.1644			0.1206		
Spearman	0.6408	0.8471	0.9662	0.7623	0.8899	0.9623	0.8690	0.9051	0.9501
Pearson	0.6734	0.8624	0.9495	0.8004	0.8854	0.9457	0.8346	0.8898	0.9393

APPENDIX C

Table C.1

N=25

Only v and u vary from iteration to iteration; x (exogenous variables) are fixed

	min	mean	max	min	mean	max	min	mean	max
rho/sigma_v		0			0.02			0.0447	
0									
DEA									
MAD	0.0351	0.0965	0.1509	0.0209	0.0437	0.0689	0.0485	0.0936	0.1776
std_MAD	0.0245			0.0089			0.0254		
Spearman	0.4277	0.7622	0.9400	0.2815	0.6516	0.8992	0.2908	0.6947	0.8969
Pearson	0.2230	0.8167	0.9883	0.1746	0.7374	0.9406	0.3478	0.7673	0.9651
SFA									
MAD	0.1066	0.1866	0.2558	0.0448	0.0776	0.1062	0.1114	0.1908	0.2888
std_MAD	0.0323			0.0118			0.0311		
Spearman	0.6154	0.9295	0.9962	0.6323	0.8545	0.9685	0.4777	0.8642	0.9600
Pearson	0.8234	0.9539	0.9979	0.7501	0.8936	0.9758	0.6923	0.8949	0.9673
0.3									
DEA									
MAD	0.0341	0.0918	0.1760	0.0240	0.0405	0.0684	0.0409	0.0945	0.1660
std_MAD	0.0245			0.0088			0.0223		
Spearman	0.5485	0.7594	0.9215	0.3392	0.6636	0.8923	0.3769	0.6669	0.9408
Pearson	0.4374	0.8342	0.9862	0.2704	0.7414	0.9494	0.1714	0.7415	0.9668
SFA									
MAD	0.1085	0.1832	0.2654	0.0458	0.0757	0.1058	0.1276	0.1884	0.2949
std_MAD	0.0329			0.0129			0.0300		
Spearman	0.6100	0.9337	0.9977	0.6892	0.8614	0.9646	0.5469	0.8532	0.9608
Pearson	0.8003	0.9594	0.9988	0.6990	0.8971	0.9739	0.6976	0.8918	0.9701
0.5									
DEA									
MAD	0.0291	0.0829	0.1393	0.0187	0.0355	0.0670	0.0474	0.0797	0.1441
std_MAD	0.0239			0.0092			0.0202		
Spearman	0.4046	0.8024	0.9654	0.3415	0.6912	0.8900	0.3915	0.7289	0.9400
Pearson	0.4409	0.8324	0.9938	0.2109	0.7747	0.9502	0.2413	0.7970	0.9667
SFA									
MAD	0.1141	0.1890	0.2702	0.0470	0.0756	0.1330	0.1138	0.1798	0.2522
std_MAD	0.0336			0.0123			0.0306		
Spearman	0.7154	0.9260	0.9969	0.6169	0.8577	0.9723	0.5600	0.8418	0.9715
Pearson	0.7816	0.9473	0.9974	0.7336	0.9004	0.9705	0.6858	0.8831	0.9750

Proceeding…

0.8									
DEA									
MAD	0.0323	0.0720	0.1379	0.0178	0.0344	0.0580	0.0311	0.0762	0.1387
std_MAD	0.0209			0.0084			0.0222		
Spearman	0.5315	0.8355	0.9777	0.3669	0.7058	0.8846	0.4908	0.7585	0.9392
Pearson	0.5101	0.8808	0.9900	0.3909	0.7779	0.9704	0.3753	0.8290	0.9855
SFA									
MAD	0.1309	0.1848	0.2638	0.0519	0.0761	0.1157	0.1183	0.1862	0.2606
std_MAD	0.0292			0.0129			0.0307		
Spearman	0.7115	0.9230	1.0000	0.6354	0.8546	0.9546	0.4915	0.8432	0.9669
Pearson	0.8154	0.9508	0.9988	0.7113	0.8935	0.9758	0.6370	0.8856	0.9645

Proceeding…

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
rho/sigma_v		0.0976			0.1548			0.2	
0									
DEA									
MAD	0.1129	0.2224	0.3973	0.1749	0.4086	0.7824	0.2847	0.6400	1.2802
std_MAD	0.0610			0.1238			0.2157		
Spearman	0.3531	0.7226	0.9431	0.3585	0.7305	0.9331	0.4246	0.7152	0.9631
Pearson	0.2845	0.7724	0.9642	0.3060	0.8285	0.9872	0.1821	0.7771	0.9925
SFA									
MAD	0.2515	0.4889	0.8439	0.3703	0.9471	1.6367	0.4594	1.4120	2.6153
std_MAD	0.1037			0.2173			0.4039		
Spearman	0.3446	0.8414	0.9569	0.6146	0.8528	0.9754	0.6846	0.8527	0.9646
Pearson	0.5321	0.8719	0.9733	0.6946	0.8735	0.9592	0.6958	0.8614	0.9487
0.3									
DEA									
MAD	0.1071	0.2194	0.4057	0.1576	0.4069	0.8409	0.3069	0.6399	1.3828
std_MAD	0.0577			0.1193			0.2073		
Spearman	0.2854	0.7258	0.9046	0.4369	0.7576	0.9408	0.4123	0.7402	0.9538
Pearson	0.2496	0.7943	0.9510	0.2745	0.8306	0.9944	-0.0250	0.8260	0.9884
SFA									
MAD	0.2765	0.4912	0.8102	0.5349	0.9909	1.8379	0.9186	1.5100	3.2092
std_MAD	0.0986			0.2326			0.3779		
Spearman	0.5962	0.8497	0.9738	0.5838	0.8509	0.9769	0.6000	0.8593	0.9562
Pearson	0.6859	0.8794	0.9584	0.7053	0.8609	0.9477	0.6725	0.8551	0.9451

Proceeding…

0.5									
DEA									
MAD	0.0794	0.1846	0.3461	0.1923	0.3852	0.7889	0.2598	0.5742	1.0565
std_MAD	0.0522			0.1164			0.1800		
Spearman	0.4100	0.7681	0.9800	0.3515	0.7533	0.9462	0.2538	0.7478	0.9577
Pearson	0.4311	0.8450	0.9752	0.2801	0.8185	0.9904	0.1916	0.8542	0.9961
SFA									
MAD	0.3108	0.4716	0.6819	0.6052	0.9582	2.0274	0.6802	1.4015	2.4616
std_MAD	0.0817			0.2285			0.3724		
Spearman	0.5862	0.8613	0.9869	0.5808	0.8592	0.9646	0.6477	0.8409	0.9492
Pearson	0.6866	0.8899	0.9656	0.7368	0.8622	0.9584	0.7025	0.8530	0.9399
0.8									
DEA									
MAD	0.0840	0.1799	0.3296	0.1796	0.3614	0.7806	0.2789	0.5990	1.3708
std_MAD	0.0433			0.1133			0.2041		
Spearman	0.4200	0.7695	0.9438	0.4046	0.7788	0.9377	0.2200	0.7549	0.9508
Pearson	0.2313	0.8408	0.9813	0.1193	0.8534	0.9863	-0.0147	0.8139	0.9877
SFA									
MAD	0.1837	0.4835	0.7673	0.5463	0.9483	1.7058	0.3072	1.4402	2.7504
std_MAD	0.0979			0.2195			0.4569		
Spearman	0.6038	0.8471	0.9523	0.5615	0.8619	0.9569	0.5100	0.8561	0.9631
Pearson	0.7085	0.8807	0.9618	0.6271	0.8709	0.9518	0.7040	0.8553	0.9639

Table C.2

N=50

Only v and u vary from iteration to iteration; x (exogenous variables) are fixed

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
rho/sigma_v		0			0.02			0.0447	
0									
DEA									
MAD	0.0276	0.0665	0.1362	0.0204	0.0310	0.0490	0.0465	0.0712	0.1083
std_MAD	0.0184			0.0053			0.0127		
Spearman	0.5835	0.8271	0.9654	0.4672	0.7166	0.9348	0.5280	0.7504	0.8898
Pearson	0.4982	0.8765	0.9890	0.4310	0.7941	0.9469	0.5438	0.8179	0.9399
SFA									
MAD	0.1306	0.1850	0.2408	0.0528	0.0752	0.1015	0.0931	0.1869	0.2379
std_MAD	0.0234			0.0085			0.0234		
Spearman	0.8294	0.9664	0.9980	0.6853	0.8817	0.9577	0.7030	0.8860	0.9523
Pearson	0.8664	0.9774	0.9971	0.7787	0.9124	0.9628	0.8416	0.9157	0.9669

Proceeding…

0.3										
	DEA									
	MAD	0.0249	0.0586	0.0988	0.0148	0.0310	0.0446	0.0433	0.0666	0.1010
	std_MAD	0.0135			0.0051			0.0101		
	Spearman	0.6525	0.8621	0.9779	0.5097	0.7152	0.9093	0.4286	0.7735	0.9102
	Pearson	0.6420	0.9101	0.9803	0.5374	0.7987	0.9624	0.5601	0.8360	0.9489
	SFA									
	MAD	0.1224	0.1840	0.2390	0.0577	0.0770	0.0949	0.1251	0.1848	0.2398
	std_MAD	0.0235			0.0079			0.0211		
	Spearman	0.7553	0.9633	0.9993	0.7615	0.8888	0.9603	0.7953	0.8904	0.9439
	Pearson	0.8382	0.9732	0.9978	0.8301	0.9190	0.9644	0.8379	0.9143	0.9599
0.5										
	DEA									
	MAD	0.0300	0.0566	0.0911	0.0200	0.0289	0.0396	0.0458	0.0655	0.1013
	std_MAD	0.0135			0.0052			0.0106		
	Spearman	0.6793	0.8582	0.9655	0.5523	0.7444	0.9078	0.6033	0.7699	0.8892
	Pearson	0.6492	0.8980	0.9896	0.5119	0.8137	0.9528	0.4925	0.8505	0.9422
	SFA									
	MAD	0.0431	0.1854	0.2298	0.0578	0.0759	0.0963	0.1236	0.1868	0.2467
	std_MAD	0.0259			0.0089			0.0200		
	Spearman	0.8754	0.9688	0.9993	0.7381	0.8829	0.9444	0.7770	0.8888	0.9473
	Pearson	0.9057	0.9770	0.9985	0.7897	0.9123	0.9696	0.8420	0.9192	0.9668
0.8										
	DEA									
	MAD	0.0269	0.0523	0.0910	0.0180	0.0278	0.0391	0.0361	0.0594	0.0853
	std_MAD	0.0144			0.0048			0.0105		
	Spearman	0.6903	0.8856	0.9842	0.5681	0.7666	0.9289	0.5764	0.8003	0.9474
	Pearson	0.6259	0.9279	0.9912	0.6120	0.8364	0.9523	0.5841	0.8660	0.9667
	SFA									
	MAD	0.1375	0.1876	0.2479	0.0573	0.0769	0.1019	0.1213	0.1839	0.2445
	std_MAD	0.0190			0.0079			0.0215		
	Spearman	0.8471	0.9643	0.9971	0.7678	0.8803	0.9536	0.7562	0.8902	0.9503
	Pearson	0.8981	0.9762	0.9976	0.8317	0.9145	0.9644	0.7811	0.9190	0.9729

Proceeding…

		min	mean	max	min	mean	max	min	mean	max
rho/sigma_v			0.0976			0.1548			0.2	
0										
	DEA									
	MAD	0.1094	0.1751	0.3171	0.2349	0.3574	0.6049	0.3326	0.6004	1.1125
	std_MAD	0.0337			0.0696			0.1479		
	Spearman	0.5644	0.7664	0.9333	0.5042	0.7522	0.9191	0.5167	0.7569	0.9091
	Pearson	0.4820	0.8487	0.9706	0.5362	0.8631	0.9771	0.3982	0.8491	0.9965
	SFA									
	MAD	0.3516	0.4930	0.6658	0.2863	0.9324	1.2657	0.7257	1.4821	2.5317
	std_MAD	0.0732			0.1578			0.3018		
	Spearman	0.7292	0.8894	0.9565	0.5962	0.8799	0.9396	0.7608	0.8832	0.9486
	Pearson	0.8039	0.9052	0.9537	0.7073	0.8859	0.9652	0.7197	0.8572	0.9349
0.3										
	DEA									
	MAD	0.1141	0.1622	0.2652	0.2089	0.3418	0.6387	0.3074	0.5619	1.1308
	std_MAD	0.0317			0.0789			0.1568		
	Spearman	0.5978	0.7965	0.9108	0.4828	0.8150	0.9395	0.5658	0.8015	0.9249
	Pearson	0.6644	0.8800	0.9738	0.4559	0.8900	0.9865	0.2893	0.8737	0.9832
	SFA									
	MAD	0.3562	0.4796	0.6828	0.6727	0.9673	1.5275	0.4086	1.4335	2.4725
	std_MAD	0.0679			0.1618			0.2847		
	Spearman	0.7242	0.8823	0.9582	0.7598	0.8894	0.9600	0.7825	0.8900	0.9648
	Pearson	0.8192	0.9039	0.9615	0.7532	0.8797	0.9401	0.7884	0.8668	0.9596
0.5										
	DEA									
	MAD	0.1080	0.1614	0.2872	0.1806	0.3374	0.7192	0.3337	0.5707	1.1533
	std_MAD	0.0335			0.0767			0.1453		
	Spearman	0.5953	0.7950	0.9547	0.5577	0.8044	0.9260	0.5438	0.7967	0.9324
	Pearson	0.5783	0.8760	0.9684	0.3805	0.8824	0.9775	0.4515	0.8741	0.9837
	SFA									
	MAD	0.3229	0.4890	0.6382	0.5355	0.9574	1.5684	0.3734	1.4538	2.3661
	std_MAD	0.0698			0.1546			0.2675		
	Spearman	0.7099	0.8793	0.9591	0.7545	0.8898	0.9647	0.7842	0.8890	0.9524
	Pearson	0.8218	0.9035	0.9528	0.7549	0.8854	0.9397	0.7489	0.8627	0.9580

Proceeding…

0.8									
DEA									
MAD	0.0967	0.1555	0.2260	0.1973	0.3375	0.6641	0.2967	0.5813	1.3873
std_MAD	0.0267			0.0798			0.1911		
Spearman	0.5752	0.8139	0.9732	0.5195	0.8205	0.9411	0.6634	0.8168	0.9307
Pearson	0.5588	0.8927	0.9811	0.5893	0.8882	0.9904	0.5643	0.8938	0.9933
SFA									
MAD	0.3242	0.4769	0.6161	0.6666	0.9515	1.5361	0.6458	1.4714	2.3120
std_MAD	0.0626			0.1688			0.2747		
Spearman	0.7915	0.8902	0.9549	0.7763	0.8907	0.9525	0.7564	0.8827	0.9483
Pearson	0.8007	0.9061	0.9610	0.7770	0.8822	0.9352	0.7488	0.8574	0.9361

Table C.3

N=25

All variables vary from iteration to iteration

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
rho/sigma_v		0			0.02			0.0447	
0									
DEA									
MAD	0.0471	0.0939	0.1543	0.0269	0.0447	0.0695	0.0498	0.0923	0.1773
std_MAD	0.0244			*0.0098*			0.0243		
Spearman	0.3023	0.7723	0.9577	0.2154	0.6330	0.8892	0.2946	0.7067	0.9277
Pearson	0.4461	0.8216	0.9853	-0.0789	0.7152	0.9565	0.1986	0.7659	0.9553
SFA									
MAD	0.1037	0.1845	0.2943	0.0448	0.0779	0.1101	0.1086	0.1860	0.2772
std_MAD	0.0300			*0.0141*			0.0302		
Spearman	0.7854	0.9329	0.9969	0.5492	0.8444	0.9623	0.6877	0.8624	0.9738
Pearson	0.8184	0.9525	0.9982	0.6966	0.8843	0.9673	0.7651	0.8948	0.9648
0.3									
DEA									
MAD	0.0369	0.0904	0.1511	0.0206	0.0420	0.0682	0.0493	0.0902	0.1750
std_MAD	0.0239			*0.0088*			0.0224		
Spearman	0.4938	0.7659	0.9669	0.2892	0.6126	0.9223	0.4131	0.6774	0.8823
Pearson	0.3696	0.8142	0.9920	0.2010	0.7174	0.9649	0.4223	0.7697	0.9611
SFA									
MAD	0.1046	0.1829	0.2785	0.0502	0.0766	0.1021	0.1142	0.1842	0.2563
std_MAD	0.0322			*0.0103*			0.0289		
Spearman	0.7308	0.9334	0.9985	0.5885	0.8655	0.9600	0.6015	0.8543	0.9685
Pearson	0.7947	0.9554	0.9981	0.6374	0.8974	0.9706	0.7229	0.8935	0.9720

Proceeding…

0.5									
DEA									
MAD	0.0305	0.0821	0.1597	0.0201	0.0382	0.0620	0.0369	0.0763	0.1384
std_MAD	0.0259			*0.0087*			0.0216		
Spearman	0.5077	0.8186	0.9692	0.2523	0.6639	0.9192	0.4354	0.7589	0.9592
Pearson	0.3887	0.8644	0.9893	0.3477	0.7527	0.9643	0.3650	0.8236	0.9815
SFA									
MAD	0.1230	0.1907	0.3073	0.0477	0.0766	0.1150	0.1233	0.1828	0.2541
std_MAD	0.0364			*0.0118*			0.0280		
Spearman	0.6777	0.9340	0.9985	0.6354	0.8527	0.9754	0.5077	0.8567	0.9623
Pearson	0.8211	0.9574	0.9989	0.7285	0.8931	0.9725	0.7610	0.8915	0.9666
0.8									
DEA									
MAD	0.0325	0.0748	0.1155	0.0156	0.0363	0.0598	0.0385	0.0784	0.1518
std_MAD	0.0196			*0.0087*			0.0211		
Spearman	0.5454	0.8255	0.9715	0.3546	0.6975	0.9515	0.3900	0.7401	0.9377
Pearson	0.3864	0.8738	0.9948	0.2622	0.7679	0.9711	0.2405	0.8058	0.9827
SFA									
MAD	0.1194	0.1818	0.3093	0.0414	0.0769	0.1110	0.1262	0.1912	0.2681
std_MAD	0.0301			*0.0113*			0.0293		
Spearman	0.5731	0.9236	0.9985	0.5838	0.8489	0.9700	0.6185	0.8613	0.9654
Pearson	0.7800	0.9511	0.9986	0.7028	0.8916	0.9800	0.7466	0.8967	0.9721

Proceeding…

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
rho/sigma_v		0.0976			0.1548			0.2	
0									
DEA									
MAD	0.1130	0.2133	0.5839	0.2189	0.4386	0.8375	0.2659	0.6510	1.5371
std_MAD	0.0609			*0.1361*			*0.2442*		
Spearman	0.3408	0.7290	0.9077	0.3738	0.7125	0.9485	0.1592	0.7285	0.9115
Pearson	0.2265	0.7716	0.9662	0.3755	0.8060	0.9827	0.1186	0.8160	0.9889
SFA									
MAD	0.2709	0.4601	0.9088	0.4593	0.9672	1.9346	0.6951	1.5068	3.1648
std_MAD	0.0988			*0.2444*			*0.4169*		
Spearman	0.4854	0.8562	0.9615	0.6638	0.8511	0.9508	0.7085	0.8626	0.9700
Pearson	0.7062	0.8779	0.9659	0.7125	0.8677	0.9615	0.6764	0.8586	0.9490

Proceeding…

0.3										
	DEA									
	MAD	0.1144	0.2214	0.3955	0.1828	0.4020	0.9498	0.2704	0.5995	1.1294
	std_MAD	0.0614			*0.1294*			*0.1757*		
	Spearman	0.3231	0.7261	0.9331	0.4269	0.7304	0.9623	0.4838	0.7636	0.9354
	Pearson	0.3361	0.8058	0.9919	0.0672	0.8244	0.9870	0.2229	0.8497	0.9938
	SFA									
	MAD	0.1595	0.4974	0.7900	0.4902	0.9519	1.6064	0.7455	1.4402	2.5025
	std_MAD	0.1083			*0.2505*			*0.3513*		
	Spearman	0.6369	0.8517	0.9646	0.5338	0.8453	0.9415	0.5546	0.8414	0.9600
	Pearson	0.6755	0.8880	0.9621	0.7390	0.8676	0.9527	0.6892	0.8535	0.9529
0.5										
	DEA									
	MAD	0.0910	0.1913	0.3786	0.1840	0.3775	0.7889	0.2501	0.5938	1.1958
	std_MAD	0.0547			*0.1199*			*0.1895*		
	Spearman	0.3031	0.7694	0.9454	0.4700	0.7596	0.9469	0.5262	0.7570	0.9623
	Pearson	0.2796	0.8407	0.9828	0.3301	0.8246	0.9845	0.0666	0.8488	0.9832
	SFA									
	MAD	0.3028	0.4912	0.6901	0.4603	0.9518	1.7404	0.5417	1.5021	2.8039
	std_MAD	0.0857			*0.2495*			*0.4052*		
	Spearman	0.5146	0.8490	0.9638	0.6246	0.8542	0.9538	0.6754	0.8602	0.9485
	Pearson	0.6012	0.8840	0.9576	0.7035	0.8662	0.9542	0.6777	0.8619	0.9943
0.8										
	DEA									
	MAD	0.0987	0.1916	0.3767	0.1735	0.3659	0.6279	0.2610	0.5753	1.3691
	std_MAD	0.0596			*0.1020*			*0.1951*		
	Spearman	0.4715	0.7608	0.9523	0.3946	0.7758	0.9385	0.5008	0.7928	0.9377
	Pearson	0.3604	0.8440	0.9841	0.3666	0.8518	0.9813	0.1764	0.8681	0.9903
	SFA									
	MAD	0.1368	0.4964	0.7490	0.5860	0.9870	1.7892	0.7281	1.5243	2.9160
	std_MAD	0.0990			*0.2068*			*0.4335*		
	Spearman	0.6223	0.8571	0.9692	0.6062	0.8422	0.9662	0.5385	0.8554	0.9500
	Pearson	0.7658	0.8945	0.9702	0.5232	0.8620	0.9415	0.6990	0.8596	0.9601

Table C.4

N=50

All variables vary from iteration to iteration

	min	mean	max	min	mean	max	min	mean	max
rho/sigma_v		0			0.02			0.0447	
0									
DEA									
MAD	0.0311	0.0657	0.1014	0.0209	0.0306	0.0452	0.0447	0.0683	0.1123
std_MAD	0.0153			0.0053			0.0124		
Spearman	0.6453	0.8281	0.9716	0.4224	0.7228	0.9120	0.4463	0.7542	0.9215
Pearson	0.6130	0.8741	0.9860	0.6059	0.8147	0.9370	0.4123	0.8273	0.9521
SFA									
MAD	0.1323	0.1848	0.2373	0.0567	0.0757	0.0984	0.1275	0.1856	0.2759
std_MAD	0.0203			0.0082			0.0218		
Spearman	0.8389	0.9676	0.9994	0.7027	0.8850	0.9560	0.7520	0.8915	0.9521
Pearson	0.9148	0.9787	0.9980	0.8395	0.9199	0.9656	0.8361	0.9186	0.9693
0.3									
DEA									
MAD	0.0286	0.0568	0.0948	0.0200	0.0299	0.0420	0.0428	0.0663	0.1057
std_MAD	0.0130			0.0046			0.0114		
Spearman	0.6812	0.8685	0.9748	0.5334	0.7362	0.9049	0.5577	0.7775	0.9098
Pearson	0.6888	0.9178	0.9908	0.5546	0.8145	0.9586	0.6039	0.8454	0.9516
SFA									
MAD	0.1400	0.1861	0.2422	0.0548	0.0771	0.1018	0.1355	0.1868	0.2364
std_MAD	0.0226			0.0092			0.0193		
Spearman	0.7581	0.9651	0.9983	0.7365	0.8877	0.9525	0.7470	0.8901	0.9505
Pearson	0.9135	0.9776	0.9981	0.8344	0.9171	0.9662	0.7698	0.9179	0.9614
0.5									
DEA									
MAD	0.0277	0.0550	0.1063	0.0166	0.0286	0.0400	0.0437	0.0628	0.0927
std_MAD	0.0151			0.0044			0.0093		
Spearman	0.6771	0.8745	0.9871	0.5125	0.7508	0.9047	0.4937	0.7789	0.9118
Pearson	0.4947	0.9152	0.9928	0.5594	0.8294	0.9501	0.4354	0.8473	0.9525
SFA									
MAD	0.1422	0.1864	0.2575	0.0547	0.0768	0.1061	0.1407	0.1824	0.2357
std_MAD	0.0244			0.0080			0.0195		
Spearman	0.8767	0.9679	0.9998	0.6434	0.8854	0.9554	0.7639	0.8904	0.9528
Pearson	0.9033	0.9766	0.9983	0.8213	0.9192	0.9747	0.7951	0.9129	0.9585

Proceeding…

0.8									
DEA									
MAD	0.0212	0.0517	0.1075	0.0186	0.0279	0.0473	0.0375	0.0611	0.0861
std_MAD	0.0164			0.0052			0.0109		
Spearman	0.6990	0.8886	0.9822	0.6076	0.7654	0.9022	0.5564	0.7948	0.9408
Pearson	0.6996	0.9310	0.9961	0.6324	0.8354	0.9478	0.4202	0.8517	0.9540
SFA									
MAD	0.1246	0.1850	0.2492	0.0599	0.0768	0.0981	0.0456	0.1818	0.2341
std_MAD	0.0215			0.0093			0.0250		
Spearman	0.8843	0.9706	0.9998	0.6879	0.8794	0.9384	0.7132	0.8856	0.9595
Pearson	0.9288	0.9788	0.9980	0.8279	0.9158	0.9561	0.8288	0.9102	0.9578

Proceeding…

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
rho/sigma_v		0.0976			0.1548			0.2	
0									
DEA									
MAD	0.1051	0.1791	0.2855	0.2361	0.3356	0.5847	0.3108	0.5548	0.9350
std_MAD	0.0354			0.0588			0.1222		
Spearman	0.4461	0.7487	0.9368	0.5621	0.7799	0.9381	0.5756	0.7666	0.9563
Pearson	0.3514	0.8382	0.9725	0.5663	0.8681	0.9832	0.3744	0.8575	0.9908
SFA									
MAD	0.3306	0.4823	0.6456	0.6935	0.9411	1.3588	0.7719	1.4535	2.2070
std_MAD	0.0616			0.1484			0.2603		
Spearman	0.7572	0.8816	0.9487	0.7743	0.8890	0.9545	0.7083	0.8861	0.9507
Pearson	0.8029	0.9023	0.9564	0.7971	0.8839	0.9477	0.7501	0.8617	0.9333
0.3									
DEA									
MAD	0.1052	0.1667	0.2524	0.2007	0.3282	0.5530	0.3429	0.5690	1.0673
std_MAD	0.0311			0.0661			0.1442		
Spearman	0.5865	0.8046	0.9358	0.6281	0.8081	0.9314	0.4819	0.7983	0.9380
Pearson	0.4838	0.8655	0.9703	0.5644	0.8882	0.9729	0.6390	0.8889	0.9892
SFA									
MAD	0.2022	0.4851	0.6496	0.6136	0.9450	1.4903	0.7233	1.4884	2.1573
std_MAD	0.0708			0.1566			0.2851		
Spearman	0.7495	0.8912	0.9631	0.7310	0.8870	0.9527	0.7976	0.8837	0.9496
Pearson	0.8190	0.9023	0.9527	0.7617	0.8902	0.9513	0.7384	0.8537	0.9304

Proceeding…

0.5									
DEA									
MAD	0.0899	0.1622	0.2385	0.1933	0.3402	0.7756	0.3277	0.5737	1.5593
std_MAD	0.0324			0.0833			0.1725		
Spearman	0.4921	0.7923	0.9449	0.6130	0.7948	0.9455	0.6005	0.8003	0.9422
Pearson	0.4619	0.8586	0.9819	0.4033	0.8662	0.9837	0.5681	0.8940	0.9807
SFA									
MAD	0.3322	0.4683	0.5982	0.5953	0.9446	1.4726	0.9393	1.4855	2.3756
std_MAD	0.0613			0.1715			0.2724		
Spearman	0.7526	0.8864	0.9544	0.7626	0.8829	0.9586	0.7818	0.8832	0.9566
Pearson	0.7859	0.9045	0.9623	0.7815	0.8774	0.9390	0.7390	0.8583	0.9478
0.8									
DEA									
MAD	0.0993	0.1591	0.2779	0.1833	0.3205	0.5382	0.3045	0.5731	1.1577
std_MAD	0.0349			0.0694			0.1551		
Spearman	0.5674	0.8276	0.9255	0.6356	0.8283	0.9349	0.6255	0.8025	0.9346
Pearson	0.6429	0.8872	0.9731	0.6008	0.9069	0.9894	0.4481	0.8835	0.9886
SFA									
MAD	0.3158	0.4817	0.7040	0.2646	0.9379	1.3659	0.7895	1.4991	2.2838
std_MAD	0.0720			0.1798			0.3019		
Spearman	0.7815	0.8926	0.9551	0.6667	0.8867	0.9511	0.7433	0.8898	0.9678
Pearson	0.8382	0.9048	0.9614	0.7383	0.8847	0.9354	0.7181	0.8554	0.9253

Table C.5

sigma_v=0 lambda=inf sigma_u=0.2036
All variables vary from iteration to iteration

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
rho/N		25			50			100	
0									
DEA									
MAD	0.0471	0.0939	0.1543	0.0311	0.0657	0.1014	0.0313	0.0447	0.0620
std_MAD	0.0244			0.0153			0.0082		
Spearman	0.3023	0.7723	0.9577	0.6453	0.8281	0.9716	0.7833	0.8866	0.9510
Pearson	0.4461	0.8216	0.9853	0.6130	0.8741	0.9860	0.8339	0.9282	0.9801
SFA									
MAD	0.1037	0.1845	0.2943	0.1323	0.1848	0.2373	0.1411	0.1862	0.2103
std_MAD	0.0300			0.0203			0.0137		
Spearman	0.7854	0.9329	0.9969	0.8389	0.9676	0.9994	0.9117	0.9806	0.9993
Pearson	0.8184	0.9525	0.9982	0.9148	0.9787	0.9980	0.9531	0.9850	0.9976

Proceeding…

0.3										
	DEA									
	MAD	0.0369	0.0904	0.1511	0.0286	0.0568	0.0948	0.0223	0.0384	0.0562
	std_MAD	0.0239			0.0130			0.0078		
	Spearman	0.4938	0.7659	0.9669	0.6812	0.8685	0.9748	0.8479	0.9183	0.9704
	Pearson	0.3696	0.8142	0.9920	0.6888	0.9178	0.9908	0.8291	0.9493	0.9862
	SFA									
	MAD	0.1046	0.1829	0.2785	0.1400	0.1861	0.2422	0.1572	0.1872	0.2165
	std_MAD	0.0322			0.0226			0.0145		
	Spearman	0.7308	0.9334	0.9985	0.7581	0.9651	0.9983	0.9221	0.9798	0.9988
	Pearson	0.7947	0.9554	0.9981	0.9135	0.9776	0.9981	0.9566	0.9844	0.9968
0.5										
	DEA									
	MAD	0.0305	0.0821	0.1597	0.0277	0.0550	0.1063	0.0210	0.0382	0.0664
	std_MAD	0.0259			0.0151			0.0087		
	Spearman	0.5077	0.8186	0.9692	0.6771	0.8745	0.9871	0.8329	0.9194	0.9913
	Pearson	0.3887	0.8644	0.9893	0.4947	0.9152	0.9928	0.8008	0.9500	0.9961
	SFA									
	MAD	0.1230	0.1907	0.3073	0.1422	0.1864	0.2575	0.1522	0.1863	0.2225
	std_MAD	0.0364			0.0244			0.0185		
	Spearman	0.6777	0.9340	0.9985	0.8767	0.9679	0.9998	0.8942	0.9807	0.9999
	Pearson	0.8211	0.9574	0.9989	0.9033	0.9766	0.9983	0.9177	0.9845	0.9980
0.8										
	DEA									
	MAD	0.0325	0.0748	0.1155	0.0212	0.0517	0.1075	0.0195	0.0356	0.0524
	std_MAD	0.0196			0.0164			0.0084		
	Spearman	0.5454	0.8255	0.9715	0.6990	0.8886	0.9822	0.8496	0.9291	0.9816
	Pearson	0.3864	0.8738	0.9948	0.6996	0.9310	0.9961	0.8665	0.9536	0.9931
	SFA									
	MAD	0.1194	0.1818	0.3093	0.1246	0.1850	0.2492	0.1415	0.1876	0.2185
	std_MAD	0.0301			0.0215			0.0147		
	Spearman	0.5731	0.9236	0.9985	0.8843	0.9706	0.9998	0.9298	0.9792	0.9985
	Pearson	0.7800	0.9511	0.9986	0.9288	0.9788	0.9980	0.9487	0.9833	0.9969

Table C.6

sigma_v=0.0976
All variables vary from iteration to iteration

rho/N		min	mean	max	min	mean	max	min	mean	max
			25			50			100	
0										
	DEA									
	MAD	0.1130	0.2133	0.5839	0.1051	0.1791	0.2855	0.1329	0.1655	0.2081
	std_MAD	0.0609			0.0354			0.0211		
	Spearman	0.3408	0.7290	0.9077	0.4461	0.7487	0.9368	0.7134	0.8133	0.9019
	Pearson	0.2265	0.7716	0.9662	0.3514	0.8382	0.9725	0.7336	0.8819	0.9514
	SFA									
	MAD	0.2709	0.4601	0.9088	0.3306	0.4823	0.6456	0.3510	0.4795	0.5884
	std_MAD	0.0988			0.0616			0.0410		
	Spearman	0.4854	0.8562	0.9615	0.7572	0.8816	0.9487	0.8704	0.9106	0.9499
	Pearson	0.7062	0.8779	0.9659	0.8029	0.9023	0.9564	0.8826	0.9171	0.9400
0.3										
	DEA									
	MAD	0.1144	0.2214	0.3955	0.1052	0.1667	0.2524	0.1084	0.1601	0.2264
	std_MAD	0.0614			0.0311			0.0265		
	Spearman	0.3231	0.7261	0.9331	0.5865	0.8046	0.9358	0.7196	0.8324	0.9154
	Pearson	0.3361	0.8058	0.9919	0.4838	0.8655	0.9703	0.7603	0.8844	0.9615
	SFA									
	MAD	0.1595	0.4974	0.7900	0.2022	0.4851	0.6496	0.4007	0.4797	0.6056
	std_MAD	0.1083			0.0708			0.0436		
	Spearman	0.6369	0.8517	0.9646	0.7495	0.8912	0.9631	0.8381	0.9060	0.9421
	Pearson	0.6755	0.8880	0.9621	0.8190	0.9023	0.9527	0.8603	0.9140	0.9529
0.5										
	DEA									
	MAD	0.0910	0.1913	0.3786	0.0899	0.1622	0.2385	0.1132	0.1636	0.2240
	std_MAD	0.0547			0.0324			0.0268		
	Spearman	0.3031	0.7694	0.9454	0.4921	0.7923	0.9449	0.6945	0.8321	0.9074
	Pearson	0.2796	0.8407	0.9828	0.4619	0.8586	0.9819	0.7849	0.9038	0.9706
	SFA									
	MAD	0.3028	0.4912	0.6901	0.3322	0.4683	0.5982	0.3935	0.4978	0.6413
	std_MAD	0.0857			0.0613			0.0629		
	Spearman	0.5146	0.8490	0.9638	0.7526	0.8864	0.9544	0.8174	0.9041	0.9410
	Pearson	0.6012	0.8840	0.9576	0.7859	0.9045	0.9623	0.8808	0.9134	0.9415

Proceeding…

0.8									
DEA									
MAD	0.0987	0.1916	0.3767	0.0993	0.1591	0.2779	0.1186	0.1609	0.2272
std_MAD	0.0596			0.0349			0.0239		
Spearman	0.4715	0.7608	0.9523	0.5674	0.8276	0.9255	0.7335	0.8444	0.9233
Pearson	0.3604	0.8440	0.9841	0.6429	0.8872	0.9731	0.7464	0.8979	0.9570
SFA									
MAD	0.1368	0.4964	0.7490	0.3158	0.4817	0.7040	0.3890	0.4810	0.5586
std_MAD	0.0990			0.0720			0.0416		
Spearman	0.6223	0.8571	0.9692	0.7815	0.8926	0.9551	0.8430	0.8976	0.9378
Pearson	0.7658	0.8945	0.9702	0.8382	0.9048	0.9614	0.8476	0.9074	0.9432

Table C.7

sigma_v=0.1548
All variables vary from iteration to iteration

	min	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
rho/N		25			50			100	
0									
DEA									
MAD	0.2659	0.6510	1.5371	0.2361	0.3356	0.5847	0.2220	0.3646	0.6413
std_MAD	*0.2442*			0.0588			0.0682		
Spearman	0.1592	0.7285	0.9115	0.5621	0.7799	0.9381	0.6326	0.8139	0.9332
Pearson	0.1186	0.8160	0.9889	0.5663	0.8681	0.9832	0.5804	0.8842	0.9743
SFA									
MAD	0.6951	1.5068	3.1648	0.6935	0.9411	1.3588	0.7307	0.9495	1.2665
std_MAD	*0.4169*			0.1484			0.1186		
Spearman	0.7085	0.8626	0.9700	0.7743	0.8890	0.9545	0.8412	0.9051	0.9456
Pearson	0.6764	0.8586	0.9490	0.7971	0.8839	0.9477	0.8116	0.8911	0.9326
0.3									
DEA									
MAD	0.2704	0.5995	1.1294	0.2007	0.3282	0.5530	0.2147	0.3517	0.7380
std_MAD	*0.1757*			0.0661			0.0880		
Spearman	0.4838	0.7636	0.9354	0.6281	0.8081	0.9314	0.7092	0.8474	0.9129
Pearson	0.2229	0.8497	0.9938	0.5644	0.8882	0.9729	0.8364	0.9170	0.9595
SFA									
MAD	0.7455	1.4402	2.5025	0.6136	0.9450	1.4903	0.7700	0.9435	1.2394
std_MAD	*0.3513*			0.1566			0.1012		
Spearman	0.5546	0.8414	0.9600	0.7310	0.8870	0.9527	0.8571	0.9051	0.9466
Pearson	0.6892	0.8535	0.9529	0.7617	0.8902	0.9513	0.7951	0.8835	0.9321

Proceeding…

0.5										
	DEA									
	MAD	0.2501	0.5938	1.1958	0.1933	0.3402	0.7756	0.2248	0.3615	0.5879
	std_MAD	*0.1895*			0.0833			0.0818		
	Spearman	0.5262	0.7570	0.9623	0.6130	0.7948	0.9455	0.6698	0.8364	0.9227
	Pearson	0.0666	0.8488	0.9832	0.4033	0.8662	0.9837	0.5130	0.9003	0.9683
	SFA									
	MAD	0.5417	1.5021	2.8039	0.5953	0.9446	1.4726	0.7534	0.9792	1.3617
	std_MAD	*0.4052*			0.1715			0.1526		
	Spearman	0.6754	0.8602	0.9485	0.7626	0.8829	0.9586	0.8611	0.9052	0.9456
	Pearson	0.6777	0.8619	0.9943	0.7815	0.8774	0.9390	0.8227	0.8820	0.9177
0.8										
	DEA									
	MAD	0.2610	0.5753	1.3691	0.1833	0.3205	0.5382	0.2488	0.3667	0.5180
	std_MAD	*0.1951*			0.0694			0.0645		
	Spearman	0.5008	0.7928	0.9377	0.6356	0.8283	0.9349	0.6790	0.8470	0.9239
	Pearson	0.1764	0.8681	0.9903	0.6008	0.9069	0.9894	0.6946	0.9061	0.9836
	SFA									
	MAD	0.7281	1.5243	2.9160	0.2646	0.9379	1.3659	0.7468	0.9520	1.1262
	std_MAD	*0.4335*			0.1798			0.0981		
	Spearman	0.5385	0.8554	0.9500	0.6667	0.8867	0.9511	0.8155	0.9033	0.9351
	Pearson	0.6990	0.8596	0.9601	0.7383	0.8847	0.9354	0.7922	0.8831	0.9240

[i] If noise is introduced in the following way (as by Yu(1998), Banker et al. (1993), in my work etc.): $\tilde{y} = ye^{v}, \quad v \sim N(0, \sigma_v)$, the 95% confidence level for noise is calculated in the following way: $\alpha = \left(e^{1.96 \cdot \sigma_v} - 1\right) \cdot 100\%$. We receive that noise is not larger than α% of the output level in 95% of cases.

[ii] Confidence intervals for technical efficiency scores are computed by the same formula as in 1) : $\alpha = \left(e^{1.96 \cdot \sigma_{uv}} - 1\right) \cdot 100\%$