THE ROLE OF REPUTATION IN PRICING

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Abstract

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We consider a bargaining game where firms produce goods and sell them to buyers. Quality of goods is unknown to buyers at the stage of purchasing. Firms make price offers, and buyers either accept or reject them. We analyze the effect of firms' reputation on buyers' willingness to pay and, as a consequence, on prices. In the first part of our thesis, the traditional game-theoretic approach is used. Here it is assumed that all information except quality of goods is common knowledge. In particular, agents know valuation of goods, costs of production and each others' preferences. In the second part, we use the evolutionary approach. Here some kind of information is assumed to be private, and agents try to reveal it by observing the market situation, which we describe as a game history, i.e. the history of trades occurred in recent times. We find equilibrium prices that appear the most frequently in long run, so called stochastically stable prices.

TABLE OF CONTENTS

LIST OF FIGURES

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GLOSSARY

Bargaining Game: A game that models a process through which players *on their own* try to reach an agreement. This process is typically time consuming and involves the players making offers and counteroffers to each other (Muthoo 1999).

Bounded rationality: If an agent is not *perfectly rational*, it is said that s/he is boundedly rational.

Common knowledge: Information is common knowledge if it is known to all players, each player knows that all of them know it, each of them knows that all of them know that all of them know it, and so forth ad infinitum (Rasmusen 1989).

Convention: By a convention, we mean a pattern of behavior that is customary, expected, and self-enforcing (Young 1996).

Discount factor: A factor that measures the value of a dollar received in the future relatively to the value of a dollar received today.

Estimation function: A function that describes how agents estimate some parameter, which is necessary for decision making, from empirical data.

Evolutionary game: A game where agents' decisions are based on learning the game history. Any evolutionary game includes *selection rules* (how agents choose strategies) and *mutation process* (agents choose non-optimal strategies by mistake); these components determine how the situation evolves.

Evolutionary stability: Evolutionary stability requires that any small deviation from the equilibrium be self-correcting. The system should, over time, evolve back to the equilibrium situation (Foster and Young 1990).

Game with incomplete information: A game in which some players do not know the payoffs of the others (Fudenberg and Tirole 1991).

Game with imperfect information: A game in which some information sets are not singletons, i.e. some players, when making decision, do not know *all* previous moves of all players (Fudenberg and Tirole 1991).

Mistake: Mistake is any strategy that does not maximize an agent's payoff subject to acquired information about the situation.

Mistake Probability Model: A model that describes the probability distribution over strategies to be played by mistake.

Perfect rationality: An agent is perfectly rational if the following assumptions hold:

(a) *Knowledge of the problem*. The agent has a clear picture of the choice problem s/he faces, s/he is fully aware of the set of alternatives;

(b) *Clear preferences*. The agent has a complete ordering over the entire set of alternatives;

(c) *Ability to optimize*. The agent has the skill necessary to whatever complicated calculations are needed to discover his optimal course of action.

(d) *Indifference to logically equivalent descriptions of alternatives and choice sets*. The choice is invariant to logically equivalent changes of descriptions of alternatives.

(Rubinstein 1998).

Stochastic stability: Stochastic stability requires that, in the long run, it is nearly certain that the system lies within every small neighborhood of the equilibrium as the noise tends slowly to zero (Foster and Young 1990). In contrast to the case of *evolutionary stability*, it resists not occasional small deviations but persistent pressure of mistakes (noise).

Worth of reputation: Total costs of establishing reputation within some period of time. If it is impossible to establish reputation within the specified period, it is said that the worth of reputation is infinite.

Chapter 1

INTRODUCTION

A firm which has to come to a decision about producing a new series of goods, or installing a new production line, or exploring a new market is concerned about costs and revenues of such a project. An essential part of revenues estimation is price forecasting. A precise determination of an expected price is extremely important, because a firm often makes very costly commitment, beginning a new project.

Both economic theory and empirical experience tell us that price dramatically depends on demand (unless it is heavily regulated). However, when a new good is offered, a firm has no empirical data that may be a basis of analysis of demand on the good. So, a firm analyzes prices and demand on similar goods, which have already been traded on a market, and makes inference about an expected price of its good.

Proper comparison of the firm's good and other (similar) goods must take into account difference in producers' reputation. Indeed, a price rather depends not on quality but on buyers' idea about quality. The objective of this research is to scrutinize the effect of reputation on pricing.

We consider a repeated *bargaining game* that has the following features:

(a) There are two classes of players, one of them is a class of long-run players. Such players are concerned not only in current payoff but also in all future payoffs throughout their (infinite) life.

(b) In each period, a long-run player offers a good at some price; a player from the other class may accept or reject the offer.

(c) Quality of goods is uncertain at the stage of bargaining.

(d) Long-run players may follow different patterns of behavior ("reputations") that are recognizable by players from the other class.

We analyze equilibrium prices and find that they substantially depend on "reputation".

In Chapter 2, we use traditional game-theoretic methodology for analysis of our game:

(a) Agents are assumed to be *perfectly rational*, i.e. they are always profit maximizers and they have unlimited computational and cognitive abilities.

(b) All information except quality of goods is *common knowledge*.

The result is that the better a firm's reputation, the greater the probability that it produces a high-quality good, therefore, the higher price may be accepted by buyers. However, buyers' expected gain from trade is zero, because firms can precisely determine buyers' valuation of a good.

Less conventional methodology is used in Chapter 3. Here we assume that agents are boundedly rational and information is *imperfect* and *incomplete*. As a result, players' decisions are based on study of the game history: They observe prices and quality of goods proposed by firms, which have similar reputation, in past.

The result of this model is quite unusual. Although it is shown that, as in Chapter 2, price depends on reputation, there is no another factor that directly influences a price. Demand, changes in factor prices, other external shocks can affect price only indirectly, through agents' preferences. The explanation is that a price is likely to become a standard: Agents get used to trade at some price and they are unwilling to change it without serious reasons.

An example is that the market for Initial Private Offering (IPOs) has dried up in the United States in 2001, because buyers have discovered that almost all IPOs are "lemons". Although the situation might change so that there may already be many "peaches" on the market for IPOs, buyers are not aware of this yet, so they continue to value IPOs low.

Another example: In both Russia and Ukraine, many new large firms and investment banks collapsed and stole all invested money in the first half of 1990s. Five years passed since then, however, people do not believe new firms and, especially, investment banks.

One more example is markets for currency (both Russian Rouble and Ukrainian Hrivna). Since the first half of 1990s, when many people lost much wealth due to (hyper) inflation, they are used to thinking that the currencies are very volatile, although they have been relatively stable for last years.

Of course, we do not assert that our model predicts that all prices in *absolute* terms tend to be constant and rarely change. In fact, we consider *relative* prices. So, prices of imports are very likely to be tied to exchange rate. In another problem, where landlords and tenants bargain over the share of crop, a price is expressed in terms of a fraction of crop.

Theoretic Background

In our thesis, two concepts are assembled. The first concept is the reputation effect. One of the first "reputation" researchers was Selten. He intuitively realises that "…Since reputations are like assets, a player is most likely to be willing to incur short-run costs to build up his reputation when he is patient and his planning horizon is long" (Fudenberg and Tirole 1991). However, he showed that for finitely repeated games players cannot benefit by maintaining reputation (Selten 1978). This result contradicted intuition, so it was called "Selten's paradox".

The solution to this paradox was proposed by Kreps and Wilson (1982) and Milgrom and Roberts (1982) (they simultaneously publish similar results). They assume that there are two types of players: "Tough" players who always play one specific strategy, and "sane" players who are payoff maximizers. If a "sane" agent pretends "tough" and plays the strategy peculiar to "tough" player in many successive periods, then an opponent tends to believe that she has faced "tough" player, so, if "sane" agent can benefit by pretending "tough", then she tries to establish such a reputation.

Another resolution to Selten's paradox is based on study of infinitely repeated prisoner's dilemma and is a particular case of Folk Theorem (Fudenberg and Maskin 1986), accordingly to which any individually rational outcome can be sustained for infinitely long time provided players are sufficiently patient. Aumann and Sorin (1989) used this approach for analysis of "repeated play of two-player stage games of *common interest*, which they define as stage games in which there is a payoff vector that strongly Pareto dominates other feasible payoffs" (Fudenberg and Tirole 1991).

The model presented in Chapter 2 is a modification of Kreps and Wilson's (1982) or Milgrom and Roberts's (1982). In contrast to them, our model has the following features:

(a) All players are "sane";

(b) Firms are heterogeneous (they are characterized with different costs of production);

(c) No mixed strategies are allowed;

(d) There are explicit costs of acquiring "good" reputation.

The second component of this thesis is the evolutionary approach, which appears in Chapter 3. Development of the theory of bargaining has inevitably led to weakening the concept of complete information and perfect rationality of players' behavior. An evolutionary approach in games was introduced by biologists of the 70s and adopted by economists and game theorists. It is based on learning the behavior of opponents. The essential concept of the evolutionary game theory is *evolutionary stability* of an equilibrium (due Maynard Smith 1974) that requires that equilibrium strategy remains the best reply against any opponent's play even if an opponent plays non-equilibrium strategy (by mistake), so that any small mistakes cannot change the evolutionary stable strategy. A fundamental work in this area was Fudenberg and Levine (1993), who explained attaining steady states of a system by learning the realized actions of opponents. In contrast to the concept of *evolutionary stability*, Foster and Young (1990) introduced the idea of the *stochastic stability*, the essence of which is that players make mistakes not occasionally but systematically. So, the evolutionary stable equilibrium is weaker concept than the stochastically stable equilibrium, which resists persistent pressure of mistakes. Later, this concept was applied to a bargaining game (Young 1993). Young's model (1993) generated several subsequent works, where:

(a) Other types of players involved in bargaining (besides myopic players that are in Young's model) were introduced. For example, Saez-Marti and Weibull (1999) considered "clever" agents; Kaniovski, Kryazhimskii, and Young (2000) considered "conformists" and "non-conformists";

(b) The same framework was used for wider class of games (Young 1998; Matros 2000). However, we ignore these works, because they are specific, they would contribute little advantage but much complexity to our research.

Our research is based on Young's evolutionary model of bargaining (Young 1993). Since our bargaining game is different from Young's, we adopt not Young's framework but approach to description of agents' behavior and his method of problem analysis. Besides, we use the concept of *stochastic stability* in our research.

However, we use less complicated *mistake probability model* than Young (1993) does. In fact, we use the simplest mistake probability model, adopted from Weibull (1995), where the probability of a mistake is proportional to the payoff obtained as a result of this mistake. In contrast, Young (1993) uses a model where the probability distribution of mistakes is arbitrary and has full support. An alternative model is the model where a probability of a mistake is proportional to the difference in payoffs in the current state and in the state played by mistake (state-dependent mistake probability, Weibull 1995). Another model describes the situation where players can control (in some extent) the frequency and/or harm of mistakes in expense of some utility loss (endogenous mistake probability, Van Damme and Weibull 1998).

Empirical Support

Empirical evidence for our research is very limited. Actually, the unique empirical study was done for the bargaining problem where landlords and tenants bargain over the share of crop (Young and Burke 2000). Authors found considerable support of the predicted results that the share of crop is a local standard which varies in different regions and is "roughly related to local economic fundamentals" (Young and Burke 2000), still, in each region, the share is stable over time.

Unfortunately, since the bargaining problem considered in our thesis differs from Young and Burke's (e.g. Young and Burke's model does not involve reputation at all), this empirical evidence is indirect and, thereby, is a weak support of our results.

Chapter 2

A MODEL WITH COMMON KNOWLEDGE

In this chapter, we consider a two-person repeated game, where a firm produces a good and sells it to a buyer. The buyer does not know real quality of the good at the stage of purchasing, so she inspects the firm's reputation. The reputation is a source of information about the probability that quality of a good is high. If the firm has "good" reputation, then the buyer is likely to believe that the quality of the good is high, thus she prefers to buy it. On the other hand, the firm that has "good" reputation may benefit by cheating, i.e. producing low-quality good and charging high price. However, cheating destroys the firm's reputation. So, a firm would rather avoid cheating, if costs of reputation improvement are high enough.

We assume that all information except quality of goods is common knowledge, i.e. agents know valuations of goods, costs of production, and each others' preferences. As a result, they are able to assess payoffs at all strategy combinations precisely. Besides, we will characterize a reputation by corresponding costs needed to attain it, so-called *worth of reputation*.

An intuition tells us that the better a firm's reputation is, the higher price of a good may be accepted by a buyer, because good standing is a kind of a guarantee that quality of a good is high. The objective of this chapter is to put mathematical background behind this intuitive inference and to show that:

(a) Each (equilibrium) price corresponds to a certain level of reputation;

(b) A firm needs the better reputation in order to charge the higher price.

The Model

Suppose that, in each period, a firm produces an indivisible good of quality chosen at the firm's discretion. We assume that the firm may produce either a good of high quality ("peach") or a good of low quality ("lemon"). Then, the firm makes price offer to a buyer. The buyer accepts the offer if the price is equal to or less than her valuation of the good. We assume that the buyer cannot detect the quality of the good at the stage of purchasing. So, the her valuation depends on her beliefs about the probability that the good is "peach".

Let v_L and v_H are a buyer's valuations of "lemon" and "peach" respectively, i.e. v_L is the maximum price that the buyer accepts if she believes that a good is "lemon", v_H is the maximum price that the buyer accepts if she believes that a good is "peach".
Let c_L and c_H are a firm's costs of production of "lemon" and "peach" respectively.

Assumptions:

(a) $v_H > v_L \geq 0$, $c_H > c_L \geq 0$, i.e. "peach" has higher value and costs of production than "lemon", all parameters are non-negative;

(b) $v_H \geq c_H$, $v_L \geq c_L$, i.e. gains from trade exist;

(c) If players disagree, then they obtain zero payoffs.

Since we assume that the buyer's valuations are common knowledge, a firm will charge prices exactly equal to the valuations.

Another important assumption is that agents have specific lexicographic preferences (Rubinstein 1998) that play a role at in the points of indifference:

(a) If a firm is indifferent between cheating or not, it lexicographically prefers not to cheat;

(b) If a buyer is indifferent between buying or not, it lexicographically prefers to purchase a good.

Thus, only pure strategies are allowed in our model that helps us to avoid significant complexity stemmed from mixed strategy analysis.

The Game:

Stage 1. Production. The firm produces a good (either “peach” or “lemon”).

Stage 2. Pricing. The firm makes price offer: (either p_H or p_L).

Stage 3. Purchasing. The buyer can either accept or reject the offer. She reject the offer only if the high price p_H is charged, whereas the firm’s reputation is not good enough so that a buyer does not believe that a proposed good is “peach”.

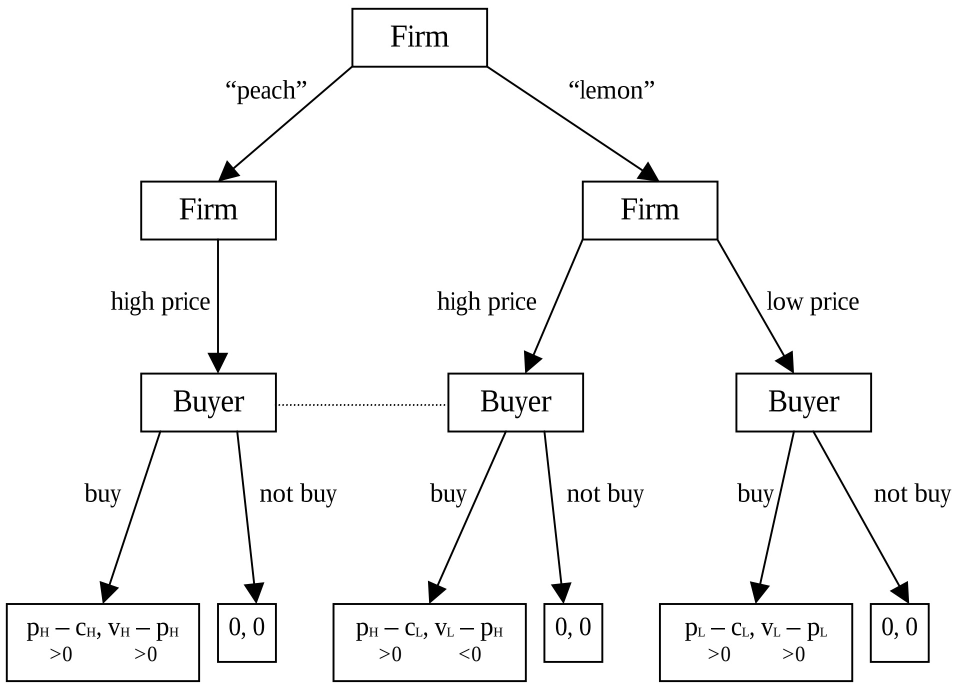

Figure 1. Game Structure

Suppose that there are two types of firms: *A* and *B*. Firms of both types may produce “peaches” and “lemons”, but they has different reputation:

- An *A-type* firm is in “good” standing so that buyers are likely to accept the high-price offers p_H, because they believe that such a firm produces high-quality goods.
- A *B-type* firm is in “bad” standing so that no buyers believe that it may produce goods of high quality, so buyers accept price offers less than or equal to p_L.

We assume that *B-type* firm may make some efforts to improve its reputation and become A-*type* firm in expense of some costs of improvement (*worth of reputation*). On the other hand, A-*type* firm may cheat and obtain a one-shot payoff equal to difference in costs of production ($c_H - c_L$) in expense of losing its reputation and becoming *B-type* firm.

Let W be *worth of reputation*, V_A and V_B be the present values of all future payoffs of A-*type* firm and *B-type* firm respectively, δ be discount factor.

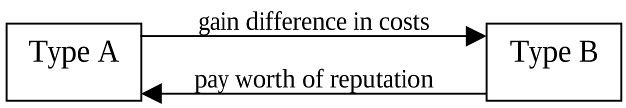

Figure 2. Transition from One Type to Another

A *B-type* firm has two alternatives:

(a) To stay *B-type* and produce low-quality goods only:

$$V_B = p_L - c_L + \delta V_B = \frac{p_L - c_L}{1-\delta}.$$

(b) To switch to A-*type* : $V_B = V_A - W$.

So, it tries to improve its reputation if the alternative *(b)* yields higher payoff than the alternative *(a)* does:

$$V_A - W \geq \frac{p_L - c_L}{1-\delta} \qquad \textbf{(2.1)}$$

An A-*type* firm has two alternatives as well:

(a) To produce “peaches” and to stay A-*type* :

$$V_A = p_H - c_H + \delta V_A = \frac{p_H - c_H}{1-\delta}.$$

(b) To cheat, i.e. to produce “lemon” and sell it as if it is “peach”:

$$V_A = p_H - c_L + \delta V_B .$$

So, it behaves “fairly” if the alternative *(a)* yields higher payoff than the alternative *(b)* does:

$$\frac{p_H - c_H}{1-\delta} \geq p_H - c_L + \delta V_A \qquad \textbf{(2.2)}$$

We will call this inequality the *continuation condition*, i.e. the condition under which an A-*type* firm maintains reputation.

Now let us scrutinize the continuation condition (2.2). We rewrite it substituting possible values of V_B:

(a) $V_B = \frac{p_L - c_L}{1-\delta}$. So, the continuation condition (2.2) is as follows:

$$\frac{p_H - c_H}{1-\delta} \geq p_H - c_L + \delta \frac{p_L - c_L}{1-\delta} \text{, or}$$

$$\frac{p_H - c_H}{1-\delta} \geq \frac{p_H - c_L - \delta p_H + \delta c_L + \delta p_L - \delta c_L}{1-\delta} \text{, or}$$

$$\delta(p_H - p_L) \geq c_H - c_L \qquad \textbf{(2.3)}$$

In this case, firms of both types have no incentive to change their types: A-type firm prefers not to cheat, provided (2.3) holds, *B-type* firm prefers to stay *B-type*, because benefits of being A-*type* do not outweigh the costs of improving reputation W.

(b) $V_B = V_A - W$. So, the continuation condition (2.2) is as follows:

$$V_A \geq p_H - c_L + \delta(V_A - W) \text{, or}$$

$$(1-\delta)V_A \geq p_H - c_L - \delta W.$$

Since $V_A = \frac{p_H - c_H}{1-\delta}$, we can rewrite the condition as $p_H - c_H \geq p_H - c_L - \delta W$, or

$$\delta W \geq c_H - c_L \qquad \textbf{(2.4)}$$

In this case, A-*type* firm produces high-quality goods and never cheat, *B-type* firm immediately pays the worth of reputation and becomes A-*type*.

Now suppose that the continuation condition (2.2) does not hold, so the value V_A of A-*type* firm is equal to the right side of the continuation condition (2.2): $V_A = p_H - c_L + \delta V_B$. Let us substitute this value to the inequality (2.1):

$$p_H - c_L + \delta V_B - W \geq \frac{p_L - c_L}{1-\delta}.$$

From this inequality, it follows that the value V_B is equal to the left side of the inequality above, since the corresponding payoff is higher. So,

$$V_B = p_H - c_L - W + \delta V_B = \frac{p_H - c_L - W}{1-\delta}.$$

Substituting V_B to the condition (2.1), we obtain:

$$\frac{p_H - c_L - W}{1-\delta} \geq \frac{p_L - c_L}{1-\delta} \text{ that implies}$$

$$p_H - p_L \geq W \qquad \textbf{(2.5)}$$

Summary:

(a) If $p_H - p_L \geq W$, i.e. it is beneficial to improve reputation for *B-type* firms (so $V_B = V_A - W$), then the continuation condition is (2.4): $\delta W \geq c_H - c_L$.

(b) If $p_H - p_L < W$, i.e. for *B-type* firms, it is better to stay *B-type* (so $V_B = \frac{p_L - c_L}{1-\delta}$), then the continuation condition is (2.3): $\delta(p_H - p_L) \geq c_H - c_L$.

Hence, the continuation condition can be presented in the following form:

$$\delta \min\{(p_H - p_L), W\} \geq c_H - c_L \qquad \textbf{(2.6)}$$

Game Equilibria

The assumption that almost all information is common knowledge implies that the buyer is aware of the firm's consideration of costs and benefits of each strategy. Thus, if the continuation condition holds, i.e. then the firm is not willing to cheat, then the buyer knows this fact and buys a good with probability one. Alternatively, if the continuation condition is violated, then the buyer concludes that a proposed good must be a "lemon", thereby, she does not buy it. The firm is also aware of the buyer's valuation of the good. So, if the continuation condition does not hold, it knows that there is no chance to sell the good at a price higher than the value of "lemon" p_L. In sum if there are one buyer and one seller, the latter *cannot* benefit by cheating.

Analyzing conditions (2.5) and (2.6), we obtain three possible *equilibria*:

1. $\delta(p_H - p_L) \geq \delta W \geq c_H - c_L$

   - An A-*type* firm stays A-*type* and produces high-quality goods, because the continuation condition (2.4) holds;

   - A *B-type* firm immediately switches to A-*type*, because it is beneficial to improve reputation for a *B-type* firm.

2. $\delta W \geq \delta(p_H - p_L) \geq c_H - c_L$

   - An A-*type* firm stays A-*type* and produces high-quality goods, because the continuation condition (2.3) holds;

   - A *B-type* firm stays *B-type* and produces low-quality goods, because, for *B-type* firms, it is better to stay *B-type.*

3. $\delta \min\{(p_H - p_L), W\} < c_H - c_L$

   - Despite having "good" reputation, an A-*type* firm produces low-quality goods and obtains the same payoff as a *B-type* firm, because the continuation condition does not hold, therefore, a buyer believe that only "lemons" are produced;

   - A *B-type* firm stays *B-type* and produces low-quality goods, because switching to A-*type* brings no benefit, but costs the worth of reputation W .

In the latter equilibrium, firms of both types have the same payoff irrespectively of reputation.

Now let us scrutinize the case where the continuation condition does not hold. We can distinguish two reasons why a firm is willing to cheat:

(a) *Non-Feasibility*: $\delta(p_H - p_L) < c_H - c_L$, or

$p_H - p_L - (1-\delta)(p_H - p_L) < c_H - c_L$, or

$(p_H - c_H) - (p_L - c_L) < (1-\delta)(p_H - p_L)$.

Dividing both sides of the inequality by $1-\delta$, we obtain:

$$\frac{p_H - c_H}{1-\delta} - \frac{p_L - c_L}{1-\delta} < p_H - p_L ,$$

in other words, life-time benefit of playing *type* A instead of *type* B is less than potential one-shot benefit of cheating. This condition depends only on costs and valuations, and it means that production of high-quality goods is not so profitable (comparing to production of low-quality goods) to be maintained in long run.

If the reverse inequality ($\delta(p_H - p_L) \geq c_H - c_L$) holds, we will say that the production of high-quality goods is *feasible.*

(b) *Irrationality:* $\delta W < c_H - c_L \leq \delta(p_H - p_L)$

This reason matters only if the production of high-quality goods is *feasible* and stems from the fact that the worth of reputation may be small enough to be easily paid, so a firm find it reasonable to cheat and then to improve reputation. Here, the worth of reputation W is an indicator. The buyer checks W and buy a "peach" if W is equal to or above some level $W^* = c_H - c_L$, and does not buy otherwise. The former case yields the Equilibrium 1 ("peaches" are produced, no cheating), the latter case yields the Equilibrium 3 (only "lemons" are produced).

In conclusion, as long as production of high-quality goods is feasible, it occurs if costs of reputation improvement are sufficiently large.

Many Firms

First, let us consider a case of two firms with different costs of production.

Let c_H^1, c_L^1 be costs of production of "peach" and "lemon" respectively of the firm 1, c_H^2, c_L^2 be costs of production of "peach" and "lemon" respectively of the firm 2. Suppose, for definiteness, that $c_H^1 - c_L^1 > c_H^2 - c_L^2$. Also we assume that production of "peaches" is feasible for both firms: $\delta(p_H - p_L) \geq c_H^i - c_L^i$, for i=1,2. (If the feasibility condition does not hold for either of the firms, then such a firm does not produce "peaches" whatever the worth of reputation is, so the situation is reduced to the case of one firm). Suppose that both firms are A-*type* with equal worth of reputation W , and a buyer may deal with each firm with equal probability.

With respect to the value W, there are three cases:

1. $\delta W \geq c_H^1 - c_L^1 > c_H^2 - c_L^2$, the continuation condition holds for both firms, only high-quality goods are produced.

2. $c_H^1 - c_L^1 > c_H^2 - c_L^2 > \delta W$, the continuation condition is violated for both firms, only low-quality goods are produced.

3. $c_H^1 - c_L^1 > \delta W \geq c_H^2 - c_L^2$, the continuation condition holds for the firm 2 and is violated the firm 1.

In the last case, a buyer does not know which firm she faces. So, the valuation of the proposed good corresponds to the expected utility of the lottery where the buyer receives "peach" with probability ½, and "lemon" with probability ½.

Let $u(p)$ be a buyer's Neumann-Morgenstern utility function such that $u(0) = 0$. Then, $u(p_H - p)$ and $u(p_L - p)$ are utilities of purchasing a high-quality good and a low-quality good respectively at price p. Then, the valuation $p*$ is such that buying a good of uncertain quality at price $p*$ is equivalent to rejecting the offer:

$$\frac{1}{2}u(p_H - p^*) + \frac{1}{2}u(p_L - p^*) = 0.$$

So, if the feasibility condition does not holds for the price $p*$ ($\delta(p^* - p_L) < c_H^2 - c_L^2$), then only "lemons" are produced for the given level of the worth of reputation W. Otherwise, both types of goods are produced, the firm 2 makes "peaches", whereas the firm 1 produces "lemons" and sells them as if they are "peaches".

Summary:

In the case of two firms, if a firm which offers a good does not have good reputation ($\delta W < c_H^2 - c_L^2$), then a buyer concludes that the proposed good is "lemon", so such a firm cannot charge a price higher than p_L. If a firm has splendid reputation ($\delta W \geq c_H^1 - c_L^1$), then a buyer believes to its fairness and values a proposed good as p_H, so such a firm can offer the highest possible price p_H. Finally, if a firm has average reputation

($c_H^1 - c_L^1 > \delta W \geq c_H^2 - c_L^2$), then it can charge average price p^*, where $p_L \leq p^* < p_H$, and $p_L = p^*$ if production of high-quality goods at price p^* is not feasible.

The continuation condition for a price p^* is as follows:

$$\delta \min\{(p^* - p_L), W\} \geq c_H - c_L \qquad \textbf{(2.7)}$$

Many firms:

If here are *n* firms with different costs of production, then we can number the firms such that $(c_H^{i-1} - c_L^{i-1}) \leq (c_H^i - c_L^i)$ for all $i = 1...n$, and any possible discounted worth of reputation will fall into one of the following intervals:

$$[0, c_H^1 - c_L^1), \ [c_H^1 - c_L^1, c_H^2 - c_L^2), \ldots, \ [c_H^{n-1} - c_L^{n-1}, c_H^n - c_L^n), \ [c_H^n - c_L^n, +\infty).$$

Suppose that the discounted worth of reputation W falls into the interval $[c_H^i - c_L^i, c_H^{i+1} - c_L^{i+1})$. Therefore, i firms will produce "peach", and at most $n - i$ firms find it reasonable to produce "lemons" and cheat ("at most", because some of them may find that it is not only unreasonable but also unfeasible to produce "peaches" – such firms produce "lemons" and do not cheat). So, the probability of purchasing a high-quality good is at least i/n, and, due to common knowledge, exact reservation price p_i^* can be calculated for all intervals $i = 1...n-1$, where $p_L \leq p_1^* \leq ... \leq p_{n-1}^* \leq p_H$.

In sum, if there are many firms with different costs of production, then equilibrium price is non-decreasing function of the worth of reputation.

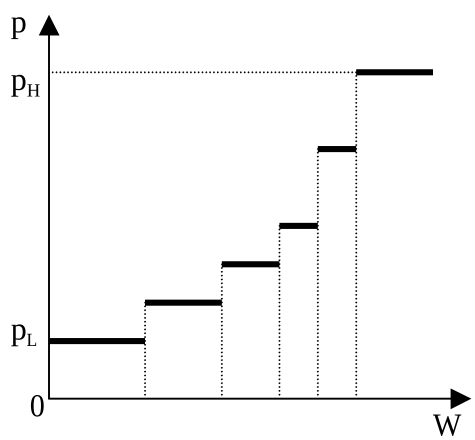

Figure 3. Price as a Function of Reputation

Conclusion

Two results:

1. For each level of reputation W, the maximal price that a buyer is willing to accept depends on the percentage of firms for which the given worth of reputation is large enough to deter cheating.
2. The higher the level of reputation, the fewer potential firms-swindlers are willing to pay such large costs of reputation improvement. This implies the greater probability that a buyer will purchase a high-quality good, therefore, a buyer is willing to accept high price.

Chapter 3

AN EVOLUTIONARY MODEL

The model presented in the previous chapter is based on strong assumptions. Indeed, it is very strong assumption that buyers know all firms' costs of production and can precisely calculate the probability of buying a high-quality good. And so is the assumption that firms know buyers' valuations of each lottery.

In this chapter, we relax these assumptions. Instead, we propose a mechanism which allows agents to acquire necessary information. So, a buyer who faced a firm with some level of reputation observes results of a number of previous trades in which firms with similar reputation are involved. She believes that the probability of buying a "peach" is equal to the relative frequency of "peaches" bought in the observed past trades. Similarly, a seller's decision about charging a price is based on the information obtained by means of observation of results of past bargains. She believes that the probability that some price will be accepted by a buyer is equal to the relative frequency of acceptance of the same price offered by firms with similar reputation in past.

An important point is that agents do not observe all past bargains. They manage to obtain information about only small part of all the history. Therefore, acquired information may not reflect true state of nature.

This approach is called the *evolutionary approach*, due to the following features:

1. *Inertia*. Because agents' decisions are based on historical data, the system has substantial inertia and cannot be changed instantly.
2. *Selection*. Agents maximize their payoff. So, they select strategies which yield high payoffs and avoid the others. However, they maximize payoff subject to acquired information, thus their "optimal" strategies could be non-optimal if they had complete information.

3. *Mutation*. Sometimes agents experiment or make mistakes, i.e. they choose non-optimal strategies. Nevertheless, new strategies played by mistake may yield even higher payoff than those chosen by the selection rule, due to incomplete information.

So, in this chapter, we will scrutinize our model put into the evolutionary framework. The objective of this chapter is to answer the following questions:

(a) What are stable prices? How are they related to reputation?

(b) What is a long-run (stochastically stable) equilibrium prices? What factors influence the equilibrium?

(c) How do results of this model differ from those of the model in Chapter 2?

Young's Evolutionary Model

We adopt the evolutionary mechanism from Young's evolutionary model of bargaining (Young 1993). Since our framework is substantially different from that in Young's paper (1993), we will apply not Young's theorems and results but the methodology. So, we will analyze our model in the manner proposed by Young.

The essence of Young's evolutionary model of bargaining is as follows.
"Young (1993) considered two finite populations, A and B, and a finite set of feasible decisions $D(\delta)$, where $D(\delta)=\{\delta,2\delta,...,1-\delta\}$ … The parameter δ is called the *precision* of the set of feasible decisions" (Saez-Marti and Weibull 1999). In each period, two individuals drawn by random, one from each population, meet and bargain for shares of a unit "pie".

Features of all (homogenous) individuals in population A:

- von Neumann–Morgenstern utility function $u:[0,1]\to R$ is differentiable, strictly increasing, and concave, $u(0)=0$;
- $u(x)$ is utility of obtaining the share x.

Features of all (homogenous) individuals in population B:

- von Neumann–Morgenstern utility function $v:[0,1] \rightarrow R$ is differentiable, strictly increasing, and concave, $v(0) = 0$;
- $u(y)$ is utility of obtaining the share y.

In order to divide a pie "individuals play the Nash demand game: The individual from population A demands some share $x \in D(\delta)$, the individual from population B demands some share $y \in D(\delta)$, and they obtain their demanded shares if $x + y \leq 1$, otherwise they obtain nothing" (Saez-Marti and Weibull 1999).

Since a priori agents know nothing about the other population, their decisions are based on (1) utility function, and (2) expected behavior of the opponent which they try to learn. Information available about the opposite population is a history of plays (of length m) that we denote as

$$h_t = ((x_t, y_t), (x_{t-1}, y_{t-1}), ..., (x_{t-m+1}, y_{t-m+1}))$$

Now suppose that some players $\alpha \in A$ and $\beta \in B$ meet and bargain in period $t+1$. The individual α tries to predict β's demand: She draws a sample (of size k_α) of agent β's previous moves from the history of moves h_t. Then she "makes a demand x_{t+1} that maximizes her expected payoff against the sampled distribution of demands from population *B*" (Saez-Marti and Weibull 1999):

$$x_{t+1} = \arg\max_{x \in D(\delta)} u(x) F_y(1-x),$$

where $F_y(1-x)$ is an empirical cumulative distribution function of agent β's bids, i.e. the probability that the player β will demand no more than $(1-x)$.

The same does the individual β: She draws a sample (of size k_β) of agent a's previous moves from the history of moves h_t and looks for the optimal bid y_{t+1} such that

$$y_{t+1} = \arg\max_{y \in D(\delta)} v(y) F_x(1-y),$$

where $F_x(1-y)$ is an empirical cumulative distribution function of agent a's demands. When new bargain (in period $t+1$) occurs, the process (called *evolutionary bargaining process*) moves to the next state and, irrespectively of bargain result, the history becomes

$$h_{t+1} = ((x_{t+1}, y_{t+1}), (x_t, y_t), ..., (x_{t-m+2}, y_{t-m+2})) .$$

Because the history is of limited depth m, the pair of demands made at $t-m+1$ is considered to be obsolete and irrelevant to current situation. Therefore, it disappears.

Convention (Young 1993).

A state h is a *convention* if it consists of some fixed division $(x,1-x)$ repeated m times in succession, where $x \in D(\delta)$. We shall denote this convention by x.

Indeed, an empirical distribution function constructed by the agent β who has drawn the sample from history of convention $h = ((x,1-x),(x,1-x),...,(x,1-x))$ is

$$F_x(1-y) = \begin{cases} 0, \text{ if } (1-y) < x \\ 1, \text{ if } (1-y) \geq x \end{cases},$$

and utility is maximized only at $y = 1-x$. The same holds for the agent α. So, unique outcome is $(x,1-x)$, and the state h repeats. Any convention is evidently absorbing state.

Theorem 1 (Young 1993).

If at least one agent in each class samples at most half of the surviving records, then from any initial state the evolutionary bargaining process converges almost surely to a convention.

The intuition behind this theorem is that if there is positive (possibly, very small) probability of coming to a convention, then on infinite horizon the probability of not coming to a convention vanishes. Thus, the process converges to a convention with probability 1. The requirement that players sample at most half of the history is sufficient to show that such probability exists.

Mistakes and Experimentation.

The next step of Young was the assumption that agents are not necessarily perfectly rational, i.e. utility maximizers. They may make mistakes or experiment. So, they sometimes do not choose their best replies.

Suppose that any share in $D(\delta)$ may be demanded with positive probability, i.e. the probability distribution of mistakes has full support. Evidently, at any convention x, there might occur a sequence of mistakes that leads the process out of the convention.

Let ε be a common level of mistake probability such that if $\varepsilon = 0$, then no mistakes ever occur. Since the probability distribution of mistakes has full support, the process is irreducible and has stationary distribution μ^{ε}, where, in particular, μ_x^{ε} is an expected relative frequency with which the convention x is observed.

Stochastically Stable Convention (Young 1993).

A convention x is stochastically *stable* if $\lim_{\varepsilon \to 0} \mu_x^{\varepsilon}$ exists and is positive. It is *strongly stable* if $\lim_{\varepsilon \to 0} \mu_x^{\varepsilon} = 1$.

"Over the long run, stable conventions will be observed much more frequently than unstable conventions when the probability ε of the perturbations is small. A strongly stable convention will be observed almost all of the time when ε is small" (Young 1993).

Mistake (Young 1993).

Let $h_t = ((x_t, y_t), (x_{t-1}, y_{t-1}), ..., (x_{t-m+1}, y_{t-m+1}))$ be some state,

and let $h_{t+1} = ((x, y), (x_t, y_t), ..., (x_{t-m+2}, y_{t-m+2}))$ be a successor of h_t. x is a *mistake* in the transition $h_t \to h_{t+1}$ if, for every individual α, x is not a best reply by α to any sample of size k_α drawn from h_t. Similarly, y is a mistake if, for every individual β, y is not a best reply by β to any sample of size k_β drawn from h_t.

Resistance (Young 1993).

If h_{t+1} is a successor of h_t, then the *resistance* $r(h_t, h_{t+1})$ of the one-period transition $h_t \to h_{t+1}$ is the minimum number of mistakes involved in the transition. Clearly $r(h_t, h_{t+1})$ =0, 1, or 2. For every two states h and h', the resistance $r(h, h')$ is the least total number of mistakes in any sequence of one-period transitions that leads from h to h'.

Now define a graph G as follows. "There is one vertex for each convention x, and a directed edge from every vertex to every other. The "weight" or resistance of the directed edge $x \to x'$ is the resistance $r(x, x')$ of moving from the convention x to the convention x'" (Young 1993).

x-tree (Young 1993).

An x-*tree* is a collection of edges in G such that, from every vertex $x' \neq x$ there is a unique path to x, and there are no cycles. Let Γ_x be the set of all x-trees.

Stochastic Potential (Young 1993).

The stochastic potential of the convention x is the least resistance among all x-trees. Evidently, a process moves out of a convention x with zero probability without mistakes. Stochastic potential is the least number of mistakes such that there appears some out-of-convention best reply and, therefore, there appears positive probability of leaving the convention x.

Theorem 2 (Young 1993).

The sequence of stationary distributions μ^{ε} converges to a stationary distribution μ^0 as $\varepsilon \to 0$. Moreover, state h is stochastically stable if and only if $h = x$ is a convention and has minimum stochastic potential among all conventions.

Summary.

We will analyze our model, following the sequence of steps proposed by Young:

(a) Framework description;

(b) Agents' decision rules;

(c) Convention: What it is, how it is established;

(d) Mistakes and experiments;

(e) Stochastic stability and long-run equilibria.

The Evolutionary Model of Pricing with Reputation Effect

The Model.

Suppose that there are a finite population of homogeneous buyers B and many finite populations of firms, each of which is characterized by a specific level of reputation. Let us consider some population of firms A. Each firm from this population has the same reputation level W. In each period $t = 1,2,...$, a firm drawn at random from A produces a good (either "peach" or "lemon") and offers it to a buyer drawn at random from B. We assume that a price charged for a good is chosen from the discrete set of feasible prices $P(\Delta) = \{\Delta, 2\Delta, 3\Delta, ...\}$, where Δ is the precision of the set of feasible prices. If a buyer accepts the offer, then she pays the assigned price and receives a lottery of obtaining "peach" or "lemon", a seller receives a difference between the price and costs of production. Otherwise, if the buyer rejects the offer, then both players receive zero payoffs.

If a trade at some period t occurs, a buyer obtains the realization of the lottery, i.e. either "peach" or "lemon". So, information about each period t is (p_t, σ_t, π_t), where:

(a) p_t is the price offered at the period t, $p_t \in P(\Delta)$;

(b) σ_t is the Boolean variable which indicates whether a good has been bought or not at the period t:

$$\sigma_t = \begin{cases} 1, \text{if a good is bought,} \\ 0, \text{otherwise.} \end{cases}$$

(c) π_t is the Boolean variable which indicates whether a good has been "peach" or "lemon" at the period t:

$$\pi_t = \begin{cases} 1, \text{if a good is "peach", or if trade has not occurred,} \\ 0, \text{if a good is "lemon".} \end{cases}$$

Suppose that a length of the game history is m. So, the game history after the period t is as follows:

$$h_t = \left((p_t, \sigma_t, \pi_t), (p_{t-1}, \sigma_{t-1}, \pi_{t-1}), ..., (p_{t-m+1}, \sigma_{t-m+1}, \pi_{t-m+1})\right).$$

A Buyer's Selection Rule.

A firm moves first: It produces a good, quality of which is unknown to a buyer, then it offers some price $p*$.

A buyer observes the offered price. Then she estimates the probability of purchasing a "peach", using her *estimation function* (we will discuss it later).

The buyer selection rule is as follows:

$$\hat{\pi}(p^*) \cdot u(p_H - p^*) + \left(1 - \hat{\pi}(p^*)\right) \cdot u(p_L - p^*) \geq 0, \qquad \textbf{(3.1)}$$

where $\hat{\pi}(\cdot)$ is the estimation function,

$u(\cdot)$ is Neuman-Morgenstern utility function such that $u(0) = 0$ and $u(\cdot)$ is strictly concave,

$p*$ is the offered price,

p_H and p_L are a buyer's reservation prices of "peach" and "lemon" respectively.

If (3.1) holds, then the offer is accepted ($\sigma = 1$), otherwise the offer is rejected ($\sigma = 0$).

A Buyer's Estimation Function.

A buyer's estimation function serves for assessing the probability of buying a high-quality good. It is based on the historical data. We assume that a buyer cannot observe all the game history. She draws a sample S_B of size k_B from the history, so, she observes only k_B trades of the total number of trades m. Analyzing acquired information, she supposes that:

(a) If some firm charged price p ($p \leq p*$) and a "peach" was produced, the firm would produce "peach" at any price higher than p as well (accordingly to the continuation condition (2.7)), in particular, at the price $p*$.

(b) If some firm charged price p ($p \geq p*$) and a "lemon" was produced, the firm would produce "lemon" at any price lower than p as well, in particular, at the price $p*$.

Let $H(p^*)$ be the number of high-quality goods produced by firms at all prices less than or equal to $p*$ observed in the buyer's sample:

$$H(p^*) = \sum_{t \in S'_B} \pi_t \text{ , where } S'_B = \{(p_t, \sigma_t, \pi_t) \mid (p_t, \sigma_t, \pi_t) \in S_B \text{ and } p_t \le p^*\}.$$

Let $L(p^*)$ be the number of low-quality goods produced by firms at all prices greater than or equal to p^* observed in the buyer's sample:

$$L(p^*) = \sum_{t \in S''_B} (1 - \pi_t) \text{ , where } S''_B = \{(p_t, \sigma_t, \pi_t) \mid (p_t, \sigma_t, \pi_t) \in S_B \text{ and } p_t \ge p^*\}.$$

A buyer's estimation function:

$$\hat{\pi}(p^*) = \frac{H(p^*)}{H(p^*) + L(p^*)}.$$

A Firm's Selection Rule.

A firm selects a price in order to maximize its expected payoff. Since it knows that the higher the price, the less the probability that such an offer will be accepted, its selection rule is as follows:

$$\max_{p \in P'} \left[\hat{\sigma}(p) \cdot v(p - c) + (1 - \hat{\sigma}(p)) \cdot v(0)\right],$$

where $\hat{\sigma}(\cdot)$ is the estimation function,

$v(\cdot)$ is Neuman-Morgenstern utility function such that $v(0) = 0$ and $v(\cdot)$ is strictly concave,

c is costs of production, specific for each firm,

p is price,

P' is the set of observed prices, $P' \subset P(\Delta)$.

We assume that the selection rule cannot result a new price that is not observed in past, the firm simply selects the best price among given ones. So, if no offers have been accepted in the observed trades, then the firm chooses the lowest price among the observed prices, supposing that the lowest price has the highest *objective* probability to be accepted.

A Firm's Estimation Function.

A firm's estimation function serves for assessing the probability that some price offer will be accepted. Similarly to a buyer's estimation function, it is based on the historical data, we assume that a firm cannot observe all the game history. She draws a sample S_A of size k_A from the history, so, she observes only k_A trades of the total number of trades m. Analyzing acquired information, she supposes that:

(c) If some firm charged price p ($p \geq p*$) and a buyer accepted this offer, she would accept any price offer lower than p as well, in particular, the price $p*$.

(d) If some firm charged price p ($p \leq p*$) and a buyer rejected this offer, she would reject any price higher than p, in particular, the price $p*$.

Let $A(p*)$ be the number of accepted offers of all prices higher than or equal to $p*$ observed in the firm's sample:

$$A(p*) = \sum_{t \in S'_A} \sigma_t \text{, where } S'_A = \{(p_t, \sigma_t, \pi_t) \mid (p_t, \sigma_t, \pi_t) \in S_A \text{ and } p_t \geq p*\}.$$

Let $R(p*)$ be the number of rejected offers of all prices less than or equal to $p*$ observed in the firm's sample:

$$R(p*) = \sum_{t \in S''_A} (1 - \sigma_t) \text{, where } S''_A = \{(p_t, \sigma_t, \pi_t) \mid (p_t, \sigma_t, \pi_t) \in S_A \text{ and } p_t \leq p*\}.$$

A firm's estimation function:

$$\hat{\sigma}(p*) = \frac{A(p*)}{A(p*) + R(p*)}.$$

The Evolutionary Process.

The selection rules described above define Markov chain on a discrete (numerable) set of states (a set of all possible game histories). The initial state is an arbitrary history h_m:

$$h_m = ((p_m, \sigma_m, \pi_m), (p_{m-1}, \sigma_{m-1}, \pi_{m-1}), ..., (p_1, \sigma_1, \pi_1))$$

In each period, the process moves to another state, a successor. A state h_{t+1} is a successor to a state h_t if:

$$h_{t+1} = ((p, \sigma, \pi), (p_t, \sigma_t, \pi_t), (p_{t-1}, \sigma_{t-1}, \pi_{t-1}), ..., (p_{t-m+2}, \sigma_{t-m+2}, \pi_{t-m+2})) \text{ given that}$$

$$h_t = ((p_t, \sigma_t, \pi_t), (p_{t-1}, \sigma_{t-1}, \pi_{t-1}), ..., (p_{t-m+2}, \sigma_{t-m+2}, \pi_{t-m+2}), (p_{t-m+1}, \sigma_{t-m+1}, \pi_{t-m+1})),$$

and (p, π) is a best reply of some firm against some sample drawn from the history h_t, σ is a best reply of a buyer against some sample drawn from the history h_t.

Equilibrium Analysis

Let s_t be a set of states h with a specific sequence of prices $(p_t, p_{t-1},..., p_{t-m+1})$, but arbitrary sequences of events: Acceptance/rejection sequence $(\sigma_t, \sigma_{t-1},..., \sigma_{t-m+1})$ is arbitrary, "peach"/"lemon" sequence $(\pi_t, \pi_{t-1},..., \pi_{t-m+1})$ is arbitrary as well:

$$s_t = \{h_t : h = ((p^*, \sigma_t, \pi_t), (p^*, \sigma_{t-1}, \pi_{t-1}),..., (p^*, \sigma_{t-m+1}, \pi_{t-m+1}))$$

Convention.

A set of states s is a *convention* if the same price p^* has been offered throughout m past trades: $s = \{h : h = ((p^*, \sigma, \pi), (p^*, \sigma_{-1}, \pi_{-1}),..., (p^*, \sigma_{-m+1}, \pi_{-m+1}))$. We denote such a convention as $s(p^*)$.

Apparently, any convention is an absorbing set of states, because of the assumption that a set of a firm's price strategies is a set of observed prices. Only one price p^* is observed in a convention, therefore, a firm can choose only p^*.

Proposition 1 (Proof in Appendix).[1]

If at least one firm samples at most half of historical observations of past bargains, then from any initial state the evolutionary process of pricing converges almost surely to a convention.

Thus, sooner or later, agents will come to some price standard (convention) and never break it down. Such a price standard is completely stable and cannot be altered by any means. There is no evolution in this process. In contrast, evolution necessarily involves a mutation mechanism that allows agents to adapt prices accordingly to external changes (in demand, technology, etc.). We introduce such a mechanism as a possibility that agents do not always maximize, instead, they experiment or make mistakes from time to time.

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[1] This proposition is an analogy of Young's Theorem 1 (1993). The proof is similar to that proposed by Young.

Mistakes.

Suppose that agents sometimes make mistakes. We assume that a probability distribution of mistakes has full support and is state-independent, i.e. any strategy may be played by mistake irrespectively of the current state (history).

Let $I^+ = \{1,2,...\}$ be the set of positive integers, and i be an element of this set ($i \in I^+$). Let $p_i = i \cdot \Delta$, thereby, p_i is a feasible price $p_i \in P(\Delta) = \{\Delta, 2\Delta, 3\Delta, ...\}$, and the set of feasible prices may be represented as $P(\Delta) = \{p_1, p_2, p_{3,} ...\}$

Let us denote εq_i^α as the probability that the firm α will offer price p_i by mistake, where ε is a general level of perturbations, i.e. the probability of mistake occurrence; q_i^α is a conditional probability that the firm α will offer price p_i given that mistake has occurred. We assume that:

(a) $\varepsilon > 0$. If $\varepsilon = 0$, then there is no mutation in the process.

(b) $\varepsilon q_i^\alpha > 0$, for all $\alpha \in A$, $i \in I^+$.

(c) $\sum_{i \in I^+} q_i^\alpha = 1$, for all $\alpha \in A$.

(d) q_i^α is proportional to an objective expected payoff which the firm α obtains by offering price p_i.

Expected (Asymptotic) Utility

Let $\sigma(p^*)$ be the true (asymptotic) probability that the price p^* will be accepted:

$$\sigma(p^*) = \lim_{k \to \infty} \hat{\sigma}_k(p^*),$$

where $\hat{\sigma}_k(p^*)$ is a firm's estimation function with the size of sample k.

The true *expected (asymptotic) utility* $V_\alpha(p^*)$ of the firm α (with costs of production c_α) when it charges the price p^* is as follows:

$$V_\alpha(p^*) = \sigma(p^*) \cdot v(p^* - c_\alpha)$$

where $v(p^* - c_\alpha)$ is the firm's utility of selling a good at the price p^*.

A Firm with Greatest Costs of Production

Let α^* be a firm which costs of production are the greatest among those of all firms:

$$\alpha^* \in A \text{ such that } c_{\alpha^*} \geq c_{\alpha} \text{ for all } \alpha \in A$$

Let p^* be the price that maximizes the expected utility of this firm, given a current convention is $s(p)$:

$$p^* = \arg\max_{p \in P(\Delta)} \sigma(p) \cdot v(p - c_{\alpha^*})$$

Let $z^*(p)$ be the least number of mistakes of playing p^* (instead of p) so that there is a positive probability that the firm α^* will *select* the new price p^*.

A Firm with Lowest Costs of Production

Let α^{**} be a firm which costs of production are the least among those of all firms:

$$\alpha^{**} \in A \text{ such that } c_{\alpha^{**}} \leq c_{\alpha} \text{ for all } \alpha \in A$$

Let p^{**} be the price that maximizes the expected utility of this firm, given a current convention is $s(p)$:

$$p^{**} = \arg\max_{p \in P(\Delta)} \sigma(p) \cdot v(p - c_{\alpha^{**}})$$

Let $z^{**}(p)$ be the least number of mistakes of playing p^{**} (instead of p) so that there is a positive probability that the firm α^{**} will *select* the new price p^{**}.

Proposition 2 (Proof in Appendix).

There are at least one and at most two stochastically stable prices that converge to the unique stochastically stable price when the precision of the set of feasible prices converges to zero ($\Delta \to 0$). The set of stochastically stable prices is completely determined by only two firms (α^*, α^{**}) which have the greatest and the least costs of production among all firms.

More precisely, we claim that:

(a) A stochastic potential of a convention $s(p)$ is

$$z(p) = \min\{z^*(p), z^{**}(p)\}.$$

A stochastically stable convention $s(\hat{p})$ has the greatest stochastic potential among all conventions, so any stochastically stable price $\hat{p}$ is such that $z(\hat{p}) = \max_{p \in P(\Delta)} z(p)$.

(b) There are at least one and at most two stochastically stable prices that converge to a unique stochastically stable price $\tilde{p}$ such that $z^*(\tilde{p}) = z^{**}(\tilde{p})$, as $\Delta \to 0$.

Intuition behind this theorem is the following: If some price is played by mistake several times, the number of mistakes required to a firm to choose the new price is less if a gain in expected utility is higher. So, if a price of a current convention is low, then a firm with the largest costs of production receives the highest gain in utility, thus requires the least number of mistakes. On the other hand, if a price of a current convention is high, then a firm with the lowest costs of production receives the highest gain in utility, thus requires the least number of mistakes. As a result, for any given convention and any price played by mistake, the highest gain in utility is received by one of these two “marginal” firms, and a stochastically stable price is somewhere in the middle so that “weights” of the “marginal” firms are nearly equal.

Thus, if some convention is changed (due to agents’ mistakes), a new convention is very likely to be closer to the stochastically stable one, and if the level of perturbations ε is negligible, then the stochastically stable price will be observed most time in long run.

Bigger and Smaller Firms.

We say that that a firm β is *bigger* than the firm α if a firm β can produce high-quality goods at lower costs of production (economy of scale).

Corollary of Proposition 2.

Let us assume:

(a) Costs of production of high-quality goods c^H are such that, for any two firms α and β $(\alpha, \beta \in A)$ $c^H_\alpha > c^H_\beta$,

(b) Costs of production of high-quality goods are the same for all firms: $c^L_\alpha = c^L_\beta = c^L$, for any two firms α and β $(\alpha, \beta \in A)$.

So, *bigger* firms have smaller difference in costs of production ($c_\beta^H - c_\beta^L$) than *smaller* firms have.

We claim that the worth of reputation W (peculiar to a class of firms A) almost completely determines a stochastically stable price, provided the discount factor δ is fixed.

Proof.

Let α be the *smallest* firm among all firms such that the *continuation condition* $\delta W \geq c_\alpha^H - c^L$ holds. So, for any firm smaller than α the *continuation condition* does not hold, and it produces "lemons" (with the lowest costs of production c^L). On the other hand, any firm β *bigger* than α has lower costs of production ($c_\beta^H < c_\alpha^H$), so the *continuation condition* holds for β as well, and it produces "peaches".

Thus, a firm with difference in costs of production which is the first greater than δW is the firm with the lowest costs of production c^L; a firm with difference in costs of production which is the first less than δW is the firm α with the greatest costs of production c_α^H. These two firms determine a stochastically stable price (by Proposition 2). This concludes the proof.

Reputation Consistency.

Equilibrium prices are said to be *reputation consistent* if the better reputation is, the higher price corresponds to it.

When we compare stochastically stable prices of different levels of reputation, we find that they are reputation consistent. The reason of this is that better reputation corresponds to smaller difference in costs of production $c^H - c^L$. Therefore, accordingly to Corollary of Proposition 2, a stochastically stable price is higher.

Chapter 4

CONCLUSION

The first result of our research is that higher prices correspond to better reputation. There are three consequences of that:

1. A firm assesses an expected price of a new product by observing prices of goods produced by other firms. However, because a price depends on reputation, this has to be taken into account for appropriate price comparison.

2. A firm must charge a *reputation consistent* price of its good, i.e. prices of goods produced by firms with better reputation than that of the analyzed firm must be higher, and, symmetrically, prices of goods produced by firms with worse reputation must be lower.

3. When a firm has a possibility to improve its reputation, it compares costs of such an improvement and benefits of having "good" reputation, then it chooses the level of reputation that maximizes its outcome. Normally, firms with lower costs of production have higher reputation, as the model results. However, there may be mutual effect: When a market is not satiated, a firm with "good" reputation may grow and economize on scale, thus lowering costs of production.

The second result is that *relative* prices are locally stable (standardized). This means that, although prices are "roughly related to economic fundamentals" (Young and Burke 2000), they resemble agreement among firms and buyers, so they are not volatile. They usually do not react on small short-run changes (trembles) of environment (fluctuations of demand, factor prices, etc.). However, in long run, prices tend to some long-run equilibrium, a stochastically stable price, which is peculiar to some locale and some specific class of firms.

We found that the stochastically stable price is related to reputation, and the better reputation, the higher the stochastically stable price.

An heuristic example that illustrates results of our model is as follows. There were almost no transactions on the market for Initial Public Offering (IPOs) in the United States in 2001. In terms of the model, by the end of 2000, buyers had discovered that almost all IPOs were “lemons”. Therefore, buyers’ willingness to pay for IPOs dropped. Nevertheless, prices of IPOs are local standards, they have not declined. This results that buyers do not buy such expensive IPOs which are likely to be “lemons”. Perhaps, the situation have already changed so that there are many ”peaches” on the market, still, buyers are not aware of this yet. Only after some risky trials, they will discover that the situation have been improved.

Drawbacks and Suggestions.

1. The main deficiency of this research is insufficient empirical support. So, empirical study of wide variety of markets is one of the most important suggestions for future research.
2. Entering new market, a firm often has a possibility to establish its reputation (for example, by means of advertising), moreover, it often has to do this. The problem of determination of the level of reputation that maximizes a firm’s expected utility is raised in our work, however, it is not stated explicitly and scrutinized. Actually, the question of reputation analysis is very deep and difficult. It includes the following subordinate questions:
   (a) What are constituents of reputation?
   (b) How can reputation be established/improved?
   (c) What are costs of reputation establishment/improvement? How can they be estimated?
   (d) How does reputation relate to time?

   …and many other questions. This field is explored a little, thus gives many opportunities for future research.

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MATHEMATICAL APPENDIX

Proof of Proposition 1

To prove the proposition, we will show that there exists a positive probability q that the process converges to a convention from an arbitrary initial state within at most N periods of time. This implies that the probability on not reaching a convention within N periods is $(1-q)$, and the probability on not reaching a convention within rN periods is $(1-q)^r$ that goes to zero as $r \to \infty$. So, on infinite horizon, the process almost surely converges to a convention.

Let $h_m = \big((p_m, \sigma_m, \pi_m), (p_{m-1}, \sigma_{m-1}, \pi_{m-1}), ..., (p_1, \sigma_1, \pi_1)\big)$ be an arbitrary initial state. A firm $\alpha \in A$ draws a sample of size k_α. Let $\alpha *$ be the firm with the minimal sample size among all firms: $\alpha * = \arg\min_{\alpha \in A} k_\alpha$, and let $k = k_{\alpha *} = \min_{\alpha \in A} k_\alpha \le m/2$.

(1) Since firms may participate in a trade with equal probability, there is a positive probability that the firm $\alpha *$ will trade in successive k periods. Moreover, because $k \le m/2$, there is a positive probability that the firm $\alpha *$ will draw the same sample throughout these k periods (it cannot do it if $k > m/2$, because at the period $[m/2]+1$ more than a half of the history will be new, and the initial sample of size $k > m/2$ will not exist any more). As a result, there is a positive probability that the same best reply $p *$ will appear in the latter k entries of the history:

$$h_{m+k} = \big((p*, \sigma_{m+k}, \pi_{m+k}), ..., (p*, \sigma_{m+1}, \pi_{m+1}), (p_m, \sigma_m, \pi_m)..., (p_{k+1}, \sigma_{k+1}, \pi_{k+1})\big)$$

(2) After k periods, the firm $\alpha *$ may trade in successive $m-k$ periods with a positive probability. Moreover, it may draw the same sample with repeated price $p *$:

$$\{(p*, \sigma_{m+k}, \pi_{m+k}), ..., (p*, \sigma_{m+1}, \pi_{m+1})\}$$

throughout $m-k$ periods. The best reply against this sample will be $p*$, because there is no alternative price in this sample. Therefore, there is a positive probability that $p*$ will appear in all m past trades, and, after m periods, the history will be:

$h_{m+m} = \left((p*, \sigma_{2m}, \pi_{2m}), ..., (p*, \sigma_{2m-1}, \pi_{2m-1}), ..., (p*, \sigma_{m+1}, \pi_{m+1})\right)$,

that is a convention $s(p*)$ by definition.

Thus, a convention may be reached from any initial state within $N = k + (m-k) = m$ periods with some positive probability. As a consequence, it is reached with probability one on infinite horizon. This concludes the proof of Proposition 1.

Proof of Proposition 2

We need two lemmas for the proof of Proposition 2.

Lemma 1

A price that maximizes a firm's asymptotic utility exists and is unique.

Proof of Lemma 1

Existence of a price that maximizes a firm's asymptotic utility follows from the fact that the set of feasible prices $P(\Delta)$ is closed and bounded.

Now we show uniqueness of such a price. Suppose that a current convention is $s(p*)$ ($p*$ is the unique observable price). A history h of the convention $s(p*)$ is as follows:

$$h = \left((p*, \rho, \pi), (p*, \rho_{-1}, \pi_{-1}), (p*, \rho_{-2}, \pi_{-2}), ..., (p*, \rho_{-m+1}, \pi_{-m+1})\right)$$

A buyer draws a sample $\bar{\rho}$ (of size k_B) of observations about quality of the goods:

$$\bar{\rho} = (\rho^1, \rho^2, ..., \rho^{k_B})$$

A buyer's estimation function:

$$\hat{\pi}(x) = \frac{H(p*)}{H(p*) + L(p*)} = \frac{x}{x + (k_B - x)} = \frac{x}{k_B},$$

where x is the number of "peaches" in the sample $\bar{\rho}$.

A buyer's reservation price p_R is such that the following equality holds:

$$\hat{\pi}(x)u(p_H - p_R) + (1-\hat{\pi}(x))u(p_L - p_R) = 0$$

Let us rewrite this equality:

$$\hat{\pi}(x) = -\frac{u(p_L - p_R)}{u(p_H - p_R) - u(p_L - p_R)}$$

Now we substitute $\hat{\pi}(x)$ with x / k_B:

$$\frac{x}{k_B} = -\frac{u(p_L - p_R)}{u(p_H - p_R) - u(p_L - p_R)} \qquad \textbf{(A-1)}$$

In order to show that a reservation price positively depends on the number of "peaches" in the sample, we differentiate both sides of (A-1):

$$\frac{dx}{k_B} = \frac{u'(p_L - p_R)(u(p_H - p_R) - u(p_L - p_R)) + u(p_L - p_R)(u'(p_L - p_R) - u'(p_H - p_R))}{(u(p_H - p_R) - u(p_L - p_R))^2} dp_R$$

$u(\cdot)$ is strictly increasing, $p_H > p_L$, hence

(a) $\quad u'(p_L - p_R) > 0$

(b) $\quad u(p_H - p_R) - u(p_L - p_R) > 0$

$u(\cdot)$ is strictly concave, hence $u'(p_L - p_R) - u'(p_H - p_R) > 0$

Therefore, $\frac{dp_R}{dx} > 0$ for any $x \in [0, k_B]$. Besides, $p_R = p_L$ if $x = 0$, $p_R = p_H$ if $x = k_B$.

Thus, there is a unique one-to-one correspondence between p_R and x, $p_R(x)$ is a strictly increasing function. Since x can be $0,1,..., k_B$, a reservation price p_R can be $p_0, p_{1,} ..., p_{k_B}$, where $p_x = p_R(x)$, $x = 0,..., k_B$.

For a convention $s(p^*)$ the continuation condition (2.7) holds for k firms, but it does not hold for the other $n-k$ firms. Let π be the true probability that a buyer faced an arbitrary firm obtains a "peach":

$$\pi = \frac{k}{n}.$$

The probability that there are exactly x "peaches" in the sample $\bar{\rho}$ is $C_{k_B}^{x} \pi^x (1-\pi)^{k_B - x}$, where $C_{k_B}^x = \dfrac{k_B!}{x!(k_B - x)!}$.

Some price $p*$ is accepted by a buyer if she observes at least $x*$ "peaches" in her sample $\bar{\rho}$, where $x*$ is such that $p_R(x*-1) < p* \le p_R(x*)$. Thus, an asymptotic (sample independent) probability that a price $p*$ is accepted is as follows:

$$\sigma(p*) = \sum_{x=x*}^{k_B} C_{k_B}^x \pi^x (1-\pi)^{k_B - x}$$

The probability function $\sigma(p)$ is non-increasing and (right-side) discontinuous at the points $p_0, p_1, ..., p_{k_B}$. $1-\sigma(p)$ represents binomial distribution with the mean $1-\pi$. Probability density function:

$$\Delta\sigma(p_R(x)) = \sigma(p_R(x+1)) - \sigma(p_R(x)) = C_{k_B}^x \pi^x (1-\pi)^{k_B - x}$$

A firm attains maximum expected utility at some point p_z:

$$p_z = \arg\max_{p \in P(\Delta)} \sigma(p) v(p-c) \qquad \textbf{(A-2)}$$

Due to discontinuity of $\sigma(p)$, there are only k_B candidates to p_z: $p'_1, p'_2, ..., p'_{k_B}$.

$p'_i = p_i - \varepsilon_i$ for all $i = 1,2,...,k_B$, where ε_i is the smallest non-negative number such that $p'_i \in P(\Delta)$. Besides, we can drop p'_i for which $p'_i - c \le 0$.

p'_i is a local maximum of $\sigma(p)v(p-c)$ if:

(a) $\sigma(p'_{i-1})v(p'_{i-1} - c) \le \sigma(p'_i)v(p'_i - c)$

(b) $\sigma(p'_i)v(p'_i - c) \ge \sigma(p'_{i+1})v(p'_{i+1} - c)$ **(A-3)**

Let us divide both sides of (A-3) by $\sigma(p'_{i+1})v(p'_i - c)$:

$$\frac{\sigma(p'_i)}{\sigma(p'_{i+1})} \ge \frac{v(p'_{i+1} - c)}{v(p'_i - c)}$$

Now let us subtract 1 from both sides of the obtained inequality:

$$\frac{\sigma(p'_i) - \sigma(p'_{i+1})}{\sigma(p'_{i+1})} \ge \frac{v(p'_{i+1} - c) - v(p'_i - c)}{v(p'_i - c)}$$

With respect to i:

(a) $\sigma(p'_{i+1})$ is decreasing

(b) $\sigma(p'_i)-\sigma(p'_{i+1})$ is increasing, as long as i is less than the mean ($i<\pi k_B$)

Hence the left side of the inequality is increasing before the mean.

With respect to i:

(a) $v(p'_i-c)$ is increasing

(b) $v(p'_{i+1}-c)-v(p'_i-c)$ is decreasing (due to concavity)

Hence the right side of the inequality is decreasing.

Therefore, there is a unique local maximum before the mean. Because firms are risk averse, none of possible local maxima after the mean can be the global one. So, we can conclude that there is a unique solution p_z of the problem (A-2) which is the local maximum before the mean. This concludes the proof of Lemma 1.

Lemma 2

A higher price that maximizes a firm's expected (asymptotic) utility corresponds to greater costs of production.

Proof of Lemma 2

Let us consider two firms with different costs of production: c_L, c_H ($c_L<c_H$).

Let p'_i be the price that maximizes expected utility of the firm with the lower costs of production c_L. As follows from Lemma 1, this price is such that the following condition holds:

$$\frac{\sigma(p'_i)-\sigma(p'_{i+1})}{\sigma(p'_{i+1})}\geq\frac{v(p'_{i+1}-c_L)-v(p'_i-c_L)}{v(p'_i-c_L)}$$

Now we show that

$$\frac{v(p'_{i+1}-c_H)-v(p'_i-c_H)}{v(p'_i-c_H)}-\frac{v(p'_{i+1}-c_L)-v(p'_i-c_L)}{v(p'_i-c_L)}>0\text{, or}$$

$$v(p'_i-c_L)\big(v(p'_{i+1}-c_H)-v(p'_i-c_H)\big)-v(p'_i-c_H)\big(v(p'_{i+1}-c_L)-v(p'_i-c_L)\big)>0$$

(a) $v(p'_i-c_L)>v(p'_i-c_H)$, since $c_L<c_H$ and $v(\cdot)$ is increasing;

(b) $\big(v(p'_{i+1}-c_H)-v(p'_i-c_H)\big)>\big(v(p'_{i+1}-c_L)-v(p'_i-c_L)\big)$, due to concavity of $v(\cdot)$.

Therefore,

$$\frac{v(p'_{i+1}-c_H)-v(p'_i-c_H)}{v(p'_i-c_H)} > \frac{v(p'_{i+1}-c_L)-v(p'_i-c_L)}{v(p'_i-c_L)},$$

and the inequality

$$\frac{\sigma(p'_i)-\sigma(p'_{i+1})}{\sigma(p'_{i+1})} \geq \frac{v(p'_{i+1}-c_H)-v(p'_i-c_H)}{v(p'_i-c_H)}$$

might not hold. As a consequence, the price p'_i might not maximize expected utility of the firm with the greater costs of production c_H, and the maximizer is some price $p^* \geq p'_i$. Thus, a price that maximizes expected utility is non-decreasing function of costs of production. This concludes the proof of Lemma 2.

Proof of Proposition 2.

Accordingly to Young's Theorem 2 (1993), a stochastically stable state is a convention that has minimum stochastic potential among all conventions, in other words, a convention that requires the least number of mistakes to be driven out to another convention.

The proof includes two steps:

(1) We calculate the least number of mistakes for an arbitrary convention $s(p)$ such that there appears a positive probability of leaving this convention.

(a) We will fix some firm α and some price chosen by mistake and calculate what the least number of mistakes is so that the firm α prefers to *select* the new price;

(b) We will analyze how this number of mistakes depends on the characteristics of the firm (costs of production);

(c) We will vary a price chosen by mistake and figure out how the least number of mistakes depends on magnitude of the new price.

Thus, we will find the price among all prices and the firm among all firms so that the number of mistakes that requires for the convention $s(p)$ to be driven out is minimal. This number is a stochastic potential of the convention $s(p)$.

(2) We will compare all conventions and find ones with the largest stochastic potential.

Step 1.

Suppose that a currently played convention is $s(p)$. Let p' be a price played by mistake. Let z be the number of such mistakes. A firm draws a sample (of size k_A) of previous trades and estimates the probability $\hat{\sigma}(p')$ that the new price p' will be accepted.

$$\hat{\sigma}(p') = \frac{A(p')}{A(p') + R(p')},$$

where $A(p')$ is the number of accepted offers at the price p', $R(p')$ is the number of rejected offers at both prices p' and p.

Let $\theta_\alpha(p') = \dfrac{v(p' - c_\alpha)}{v(p - c_\alpha)}$, where α specifies the firm with costs of production c_α, and $v(\cdot)$ is its utility function.

Property 1.

The greater costs of production c_α, the higher $\theta_\alpha(p')$ if $p' > p$, and the lower $\theta_\alpha(p')$ if $p' < p$.

Proof:

$$\frac{d\left(\dfrac{v(p' - c_\alpha)}{v(p - c_\alpha)}\right)}{dc_\alpha} = \frac{v'(p - c_\alpha)v(p' - c_\alpha) - v'(p' - c_\alpha)v(p - c_\alpha)}{(v(p - c_\alpha))^2} > 0, \text{ provided } p' > p$$

because:

(a) $v(p' - c_\alpha) > v(p - c_\alpha)$, since $p' > p$ and $v(\cdot)$ is increasing;

(b) $v'(p - c_\alpha) > v'(p' - c_\alpha)$, since $p' > p$ and $v(\cdot)$ is concave.

The opposite holds for $p' < p$.

Property 2.

The greater a price p', the higher $\theta_\alpha(p')$ ($\theta_\alpha(p')$ monotonically increases with respect to p')

$$\frac{d\theta_\alpha(p')}{dp'} = \frac{v'(p'-c_\alpha)}{v(p-c_\alpha)} > 0$$

Case 1. $p' > p$

$$A(p') = z \cdot \sigma(p')$$

$$R(p') = z \cdot (1-\sigma(p')) + (k_A - z)(1-\sigma(p))$$

So, $$\hat{\sigma}(p') = \frac{z \cdot \sigma(p')}{z \cdot \sigma(p) + k_A(1-\sigma(p))} \qquad \textbf{(A-4)}$$

Let us fix some firm α with costs of production c_α. The firm selects p' if it yield higher expected utility:

$$\hat{\sigma}(p') \cdot v(p'-c_\alpha) > \sigma(p) \cdot v(p-c_\alpha) \qquad \textbf{(A-5)}$$

$p - c_\alpha > 0$, otherwise the firm would not trade. Then, we can rewrite (A-5) as follows:

$$\hat{\sigma}(p') \cdot \theta_\alpha(p') > \sigma(p)$$

Now we substitute $\hat{\sigma}(p')$ from (A-4):

$$\frac{z \cdot \sigma(p')}{z \cdot \sigma(p) + k_A(1-\sigma(p))} \cdot \theta_\alpha(p') > \sigma(p)$$

After some transformations, we obtain:

$$z > k_A \frac{1-\sigma(p)}{\frac{\sigma(p')}{\sigma(p)}\theta_\alpha(p') - \sigma(p)} \qquad \textbf{(A-6)}$$

$p' > p$, $\theta_\alpha(p')$ is increasing with respect to costs of production c_α, therefore the least number of mistakes z is for the firm with the greatest costs of production c_H.

Substituting $\theta_\alpha(p')$, we can rewrite (A-6) as follows:

$$z > k_A \frac{1-\sigma(p)}{\frac{\sigma(p')v(p'-c_\alpha)}{\sigma(p)v(p-c_\alpha)} - \sigma(p)}$$

The expected utility of playing p' (instead of p) $\sigma(p')v(p'-c_\alpha)$ attains its maximum for some price p^* (Lemma 1). For the firm with the greatest costs of production this price is the highest among all such maxima for the other firms (Lemma 2). Therefore, the number of mistakes z achieves its minimum if the price p^* is played by mistake and if the firm with the greatest costs of production makes decision.

Conclusion for Case 1.

Suppose p^* is the price that maximizes the expected utility of the firm with the greatest costs of production, provided a convention $s(p)$ is currently played. Then, as long as $p^* > p$, the number of mistakes required for the convention to be broken down attains it minimum at the price p^* and for the firm with the greatest costs of production c_H. Let us denote the least number of mistakes z^* as a function of the current convention:

$$z^*(p) = k_A \frac{1-\sigma(p)}{\dfrac{\sigma(p^*)v(p^*-c_H)}{\sigma(p)v(p-c_H)} - \sigma(p)} \quad \textbf{(A-7)}$$

Case 2. $p' < p$

$$A(p') = z \cdot \sigma(p') + (k_A - z)\sigma(p)$$

$$R(p') = z \cdot (1 - \sigma(p'))$$

So, $\hat{\sigma}(p') = \dfrac{z \cdot \sigma(p') + (k_A - z)\sigma(p)}{z + (k_A - z)\sigma(p)} = 1 - \dfrac{z \cdot (1-\sigma(p'))}{k_A \cdot \sigma(p) + z(1-\sigma(p))}$

The firm selects p' if it yield higher expected utility:

$$\hat{\sigma}(p') \cdot v(p' - c_\alpha) > \sigma(p) \cdot v(p - c_\alpha)$$

$p' - c_\alpha > 0$, otherwise the firm would not trade. Then, we can rewrite this as follows:

$$1 - \frac{z \cdot (1-\sigma(p'))}{k_A \cdot \sigma(p) + z(1-\sigma(p))} > \frac{\sigma(p)}{\theta_\alpha(p')}$$

After some transformations, we obtain:

$$z < \frac{(\theta_\alpha(p') - \sigma(p))k_A \sigma(p)}{\theta_\alpha(p')(\sigma(p) - \sigma(p')) + \sigma(p)(1-\sigma(p))}$$

Similarly to Case 1, we obtain the following:

Suppose p^{**} is the price that maximizes the expected utility of the firm with the lowest costs of production c_L, provided a convention $s(p)$ is currently played. Then, as long as

$p^{**} < p$, the number of mistakes required for the convention to be broken down attains it minimum at the price p^{**} and for the firm with the least costs of production. Let us denote the least number of mistakes z^{**} as a function of the current convention:

$$z^{**}(p) = \frac{(\theta_\alpha(p^{**}) - \sigma(p))k_A\sigma(p)}{\theta_\alpha(p^{**})(\sigma(p) - \sigma(p^*)) + \sigma(p)(1-\sigma(p))} \qquad \textbf{(A-8)}$$

Step 2.

The stochastic potential $z(p)$ of a convention $s(p)$ is the least number of mistakes:

$$z(p) = \min\{z^*(p), z^{**}(p)\}$$

A stochastically stable state, a state with the greatest stochastic potential, is such that maximizes $z(p)$:

$$\max_{p \in P(\Delta)} z(p)$$

Since $z^*(p)$ is increasing with respect to p, $z^{**}(p)$ is decreasing with respect to p, $z(p)$ is a pseudo-concave function, and the maximum is achieved at the price $\tilde{p}$ such that the equality $z^*(\tilde{p}) = z^{**}(\tilde{p})$ holds.

The set of prices $P(\Delta)$ is discrete, so there are two candidates to a stochastically stable convention: The price that next greater than $\tilde{p}$, and the price that next lower than $\tilde{p}$. One of them is stochastically stable, and if they correspond to the same stochastic potential, both are stochastically stable. As $\Delta \to 0$, the set of feasible prices becomes more dense, and these candidates converge to $\tilde{p}$, so $s(\tilde{p})$ is a unique stochastically stable state if $\Delta = 0$.

This concludes Proof of Proposition 2.