THE IMPACT OF THE DATA AGGREGATION OVER FIRMS ON THE ACCURACY OF EFFICIENCY MEASUREMENT

by

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ABSTRACT

The impact of the data aggregation over firms on the accuracy of efficiency measurement

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This paper evaluates the impact of the data aggregation over firms on the accuracy of efficiency measurement. On the base of previous studies was shown theoretical conditions under which data aggregation does not lead to bias. However, in the Monte Carlo simulations the presence of bias is detected. It was shown that the source of the bias is computational issues inherent to DEA technique.

Also the question of the impact of the data aggregation on the group ranking was considered. In the same series of Monte Carlo simulations the following result was established: data aggregation changes ranking of the group of firms with respect to their efficiency scores.

TABLE OF CONTENTS

List of figures and tables ii
List of appendices iii
Acknowledgments iv
Glossary v
Chapter 1: Introduction 1
Chapter 2: Literature review 4
Chapter 3: Methodology 13
Chapter 4: Results 20
Chapter 5: Conclusions and directions for further research 22
Bibliography 23

LIST OF FIGURES AND TABLES

Figure 1: Illustration of allocative and technical efficiency (input orientation). **Error! Bookmark not defined.**7

Table 1: Results of Monte Carlo simulations, 40 DMU's in the group **Error! Bookmark not defined.0**

Table 2: Results of Monte Carlo simulations, 20 DMU's in the group **Error! Bookmark not defined.4**

Table 3: Results of Monte Carlo simulations, 10 DMU's in the group Error! Bookmark not defined.**5**

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GLOSSARY

Constant returns to scale (CRS) Technology is said to exhibit CRS if an increase in all inputs by *t* leads to an increase in all outputs by *t*.

Decreasing returns to scale (DRS) Technology is said to exhibit DRS if an increase in all inputs by *t* leads to an increase in all outputs by less than *t*.

Increasing returns to scale (IRS) Technology is said to exhibit IRS if an increase in all inputs by *t* leads to an increase in all outputs by more than *t*.

Variable returns to scale (VRS) Technology exhibits VRS if it can exhibit CRS, DRS, and IRS in different regions.

Chapter 1

INTRODUCTION

The main task of the applied efficiency analysis is estimation of the technical efficiency of the single firm or the whole industry. There are two principal techniques to solve this task: Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA). In the last decades the DEA method has become especially popular because it permits to measure efficiency without assuming any particular functional form of the production function (in the single output case) or technology frontier (in the multi output case).

Obviously, the more detailed data the researcher posses the more precise the measurement is. It means that if somebody wants to estimate efficiency of some given industry she needs to begin analysis even not from the level of single firm but from the level of department if there is any. However, it's hard or even impossible to obtain such specified data in practice. So, a researcher is forced to use aggregated information.

Efficiency analysis is not unique area of economics that encounters this problem. Economists often meet aggregation in their researches. For example in the area of microeconomics, typical problem of utility maximization almost always requires aggregation of all variety of goods into some categories such as food, durable goods, services, *etc*. Usually this aggregation is carried out with the help of prices. Alternatively, the object of interest can be not some specific person (consumer) but a group of people. An obvious problem arises in this case; we need to be sure that solutions of two different problems (aggregated and unaggregated) coincide. The first type of aggregation (over goods) is thoroughly considered in Hicks (1956), while the second in Gorman (1953).

The natural problem which immediately arises is the extent to which studies based on the various levels of aggregation are consistent with one another, for example across plants to obtain a company-wide level. In the article by Blackorby and Russell (1999), the question of the aggregation of both input and output data over firms was considered. The results obtained are rather discouraging, *i.e.* very strong restrictions on the technology and\or the efficiency index are required.

It was shown in the article by Tauer (2000) that aggregation over inputs within one firm might lead to biased estimations even in the case of usage of the exact aggregator function. To show this result the Monte-Carlo technique was used. It's possible to prove necessary and sufficient conditions for a linear aggregator to give unbiased result (Fare and Zelenyuk (2002)). However, these conditions require the absence of allocative inefficiency in the sub-vector of aggregation. So, in general case using aggregated inputs information we may expect bias.

There are two implications of these results. The first is that if you use data aggregated in different ways you may obtain different efficiency scores. The second is that it may happen that using differently aggregated data you will obtain different ranking of the firms. The first result is not very grave; indeed all efficiency measures are relative but not absolute. So, if your estimation is that some firm A is 50% efficient and firm B is fully efficient, it does not mean that if you remove all sources of inefficiency then output of firm A will be doubled. The only conclusion which can be made is that firm B is more efficient than firm A. But the second result implies that we should be very careful about DEA results because many researches use these scores for evaluating impact of environment on the production performance (market *vs* planned economy for example). Sometimes the scholars use these scores as dependent variables in their regressions but the problem of possible different ranking implies that even results of ordered probit\logit regressions should be taken with big carefulness. I believe that these reasons prove the importance of the research in this field of economics.

So far no articles have been devoted to simultaneous data aggregation over firms and especially impact of such aggregation on the ranking of the groups. Both questions of aggregation, *i.e.* whether this aggregation leads to the bias and whether it is possible to obtain unbiased aggregator, look very attractive and appealing. So, the thesis that covers this issue may fill the blank space in this field of productivity and efficiency analysis.

The method applied in the thesis to check existence of bias is similar to that used in the article by Tauer (2000), namely Monte-Carlo simulation method. Such technique is basic for the researches of this kind.

Chapter 2

LITERATURE REVIEW

The problem of aggregation is not new in the area of efficiency analysis. Generally, the measurement of the technical efficiency is carried out on the different levels of aggregation – at the individual branch, plant, division level, at the level of company or the whole industry, at the level of region or the country itself. The question of comparability of these results arises. Indeed, suppose you estimated efficiency on the level of the plants then what you can say about efficiency of the companies that own these plants. This question was explicitly posed in the article by Cook, *et al* (1998): "the necessity arises to combine multiple ratings on a level, and to evaluate 'groups' themselves DMUs (decision making units). This being the case, the extensions of DEA ideas to this more general setting would appear to be an important direction for future research". The authors considered the problem of DEA computations for various methods of grouping decision-making units in a hierarchical structure. The reason why we need to group (aggregate) DMUs is as following: "the ideal setting for this tool [DEA] is one in which the DMUs form a relatively homogenous set in regard to the scales on which the inputs and outputs are measured… In many problem settings that potentially lend themselves to analyses via DEA, there are identifiable groups or clusters of DMUs, whose impacts should be captured in the analysis." (Cook et al. (1998)). Later on, the authors developed idea by Banker et al. (1986) of categorical variables. This approach can be used when there is a natural nesting of the groups of DMUs and consists in the comparison of DMU to those from its own category and categories below. For example for warehouses such category can be the population of the cities where they operate. Then a given warehouse will be compared to those that operate in the cities with the same population or less. However, the size of population can be not the only factor that influences performance of DMUs. And different grouping can lead to different estimations of efficiency in absolute and relative values. In this case the authors propose the following rule of aggregating different efficiency indexes: $e_j = \sum_{i \in I} \alpha_i e_{ij}$

where e_j is aggregate efficiency of DMU j,

J is the number of DMUs.

e_{ij} are efficiency scores of j^{th} DMU with respect to i^{th} attribute,

α_i is either given weight or determined through the following procedure:

$$e^*_{jo} = \max e_{jo} = \sum_{i=1}^{I} \alpha_i e_{ijo}$$

subject to $\sum_{i=1}^{I} \alpha_i e_{ij} \leq 1, j \in J$,

$a_i > 0.$

One possible drawback of this approach is that a different set of $\{\alpha_i\}$ can arise in each of the J optimisations.

The problem of aggregating of efficiency indexes in more general fashion was considered in the article by Blackorby and Russell (1999). In this paper the authors explored the conditions under which aggregation of efficiency indexes gives consistent results. It should be clarified here what is meant by "consistent aggregation" here, since the notion of consistency in efficiency analysis is rather different from that in econometrics. Blackorby and Russell proposed the following definition of consistent aggregation:

$E^0(u,x)=F(E^1(u^1, x^1),\dots, E^k(u^k, x^k))$, (1)

where

E^r , r=1,…, K is efficiency index for r^th DMU

E^0 is efficiency index for the aggregate DMU, comprising K disaggregated DMUs,

u^r is a vector of outputs of r^th DMU,

x^r is a vector of inputs of r^th DMU,

$u = \sum_{r=1}^{K} u^r$ and $x = \sum_{r=1}^{K} x^r$

F is a continuous, increasing function.

The intuition behind this formula is pretty simple: efficiency index based on the aggregated information should have possibility to be recomputed through efficiency indexes calculated on the unaggregated levels.

Using this definition the following theorem was formulated and proven:

The efficiency indexes E^r , r=0,…, K satisfy (1) if and only if they are functionally equivalent to each other and to a linear function; that is,

$$E^r(u^r, x^r) = \varepsilon_r(\sum_{i=1}^{M} \alpha_i u_i^r + \sum_{i=1}^{N} \beta_i x_i^r)$$

and

$$E^o(u,x) = \varepsilon(\sum_{r=1}^{K} \varepsilon_r^{-1}(E^r(u^r, x^{r0}))) = \varepsilon(\sum_{i=1}^{M} a_i u_i + \sum_{i=1}^{N} \beta_i x_i),$$

where ε and ε_r, r=1,…, K, are continuous, increasing functions and the α_i and β_i are arbitrary parameters.

This theorem has the following implication:

No efficiency index that is homogeneous of degree minus one on input quantities or homogeneous of degree one in output quantities can satisfy the aggregation condition (1).

An immediate implication of this statement is that there does not exist a technology set such that Farrell-type measure of technical efficiency can be aggregated (since it is homogeneous for all possible technologies). It means that if we use input- or output-based distance functions for measuring efficiency; then we cannot predict the value of these scores on the differently aggregated level.

This result is rather discouraging; however in real life analysis besides "straight" aggregation thoroughly considered by Blackorby and Russell (1998), the aggregation with the use of prices as weights is frequently employed. The last type of aggregation is more applicable in the real life analysis since such kind of data is easier to find. Indeed, the data on total expenditures (revenues) of a firm are more likely to be obtained than data on the exact number of all possible inputs (outputs). Nevertheless, such kind of aggregation also tends to give biased results. As was mentioned before in the field of efficiency analysis the notion of bias (inconsistency) differs from that in econometrics. The following definition is due to Fare and Zelenyuk (2002). Further on the next definition (input based version) of unbiased aggregation is used:

$$SF_i(u^k, x^r) = K(y^k, c_{kN}, x_{kN+1}, \ldots, x_{kN})$$

where

$$SF_i(y^k, x^k) = \min \lambda$$

$$s.t.$$

$$\sum_{k=1}^{K} z_k y_{km} \geq y_{km}$$

$$\sum_{k=1}^{K} z_k x_{kn} \leq \lambda x_{kn}, n = 1, \ldots \overline{N}$$

$$\sum_{k=1}^{K} z_k x_{kn} \leq \lambda x_{kn}, n = \overline{N} + 1, \ldots N$$

$$z_k \geq 0$$

and

$$K(y_k, c_{k\overline{N}}, x_{k\overline{N}+1}, \ldots, x_{kN}) = \min \lambda$$

$$s.t.$$

$$\sum_{k=1}^{K} z_k y_{km} \geq y_{km}$$

$$\sum_{k=1}^{K} z_k c_{kn} \leq \lambda c_{kn},$$

$$\sum_{k=1}^{K} z_k x_{kn} \leq \lambda x_{kn}, n = \overline{N} + 1, \ldots N$$

$$z_k \geq 0$$

$$c_{k\overline{N}} = \sum_{n=1}^{\overline{N}} w_n x_{kn}$$

x^k is a vector of inputs of the k^{th} DMU,

y^k is a vector of outputs of the k^{th} DMU,

w_n is a input price vector.

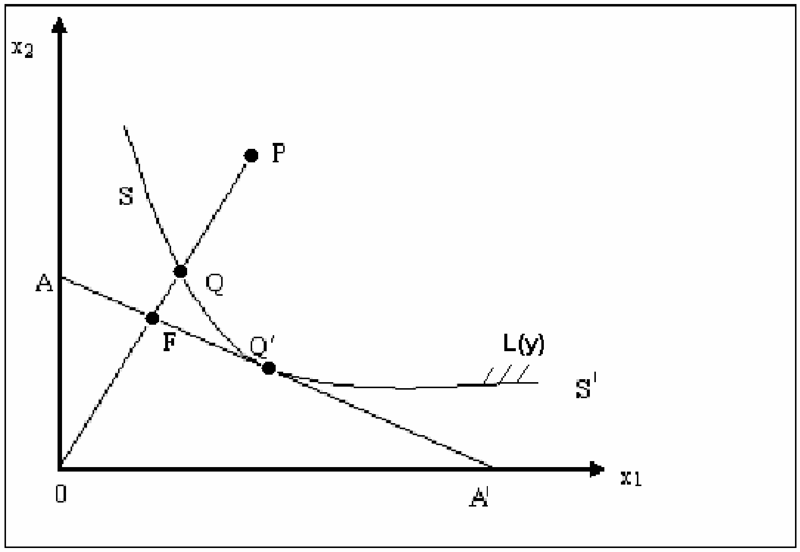
The first problem is the classical input oriented Farrell (1957) measure of technical efficiency and the second is the measure of technical efficiency with some inputs being aggregated with the use of prices as weights.

Tauer (2001) has shown that such aggregation leads to the biased results. The technique used in this article is the following: the author assumed functional form of production function (Cobb-Douglas), derived demand functions from condition of cost minimization, using Monte-Carlo simulation generated prices, inefficiency and parameters of production function. After that, the author did efficiency estimations using DEA technique. The results showed that aggregation gives biased outcomes comparing to the base model with no aggregation and this bias increases with the increase in the level of aggregation.

The question when this type of aggregation gives unbiased results was explored in the paper by Fare and Zelenyuk (2002). The authors showed that condition of unbiasedness holds if and only if there is no allocative inefficiency.

Allocative efficiency (AE) is defined as follows. Consider output possibilities set $L(y)=\{X: y \le f(X)\}$, where X – set of inputs, f(.) – production function.

Fig.1 Illustration of allocative and technical efficiency (input orientation).



SS' –input isoquant; AA' – ratio of input prices

Thus, firm P is technically inefficient (it can increase output without increase of inputs). Measure of technical efficiency – distance QP. Firm Q is technically efficient, but not allocatively efficient, because being in the point Q' provides the same output but with less costs. Measure of allocative efficiency for firms Q or P – distance QF. And firm Q' is both technically and allocatively efficient.

The problem of direction and bounds of aggregation bias was considered in the paper by Fare, Grosskopf and Zelenyuk (2002). Their findings are that the bias due to aggregation has downward direction and bounded by the size of allocative inefficiency. Probably the most important conclusion is "…although the bias for every firm is bonded in the same fashion, its size may (most likely will) vary from firm to firm – depending on the firm's allocative inefficiency, implying that the ranking of firms by their efficiency may be different at different levels of aggregation."

However, considered above papers are devoted to the aggregation within firms, i.e. aggregation of inputs into sub-costs or, alternatively, outputs into sub-revenues.

Now, let me briefly describe the paper by Li and Ng (1995). In the paper authors introduce efficiency measure that can be used to find the efficiency of a group of firms.

The authors define group technology in the similar to individual technology way:

$T^{g} \equiv \{(X, Y): \text{"Y can be produced from X"}\}$,

where Y – total outputs of the group and X – total inputs of the group. $Y = \sum_{k=1}^{K} y^{k}, X = \sum_{k=1}^{K} x^{k}$.

Similarly to the individual technical efficiency the authors define technical efficiency of the group (output orientation): $TE^{g} \equiv \max_{\theta}\{\theta : (X, \theta Y) \in T\}$.

Further, Li and Ng show that under quite general assumptions: technologies of the group members are identical and technologies are convex, the technical efficiency of the group is theoretically equal to the technical efficiency of the imaginary "average" firm. $TE^{g}(X, Y) = TE(\tilde{x}, \tilde{y}) \equiv \max_{\theta}\{\theta : (\tilde{x}, \tilde{y}) \in T\}$, where $\tilde{y} = \frac{1}{K}\sum_{k=1}^{K} y^{k}, \tilde{x} = \frac{1}{K}\sum_{k=1}^{K} x^{k}$.

It is worth mentioning that above stated assumptions from the paper by Li and Ng coincide with the assumptions of DEA set-up. However, estimation of the efficiency scores in the DEA framework besides theoretical assumptions also involves computational issues. One of them is the effect of number of DMU's in the sample on the DEA efficiency scores. In the paper by Zhang and Bartels (1998) the authors show that the mean efficiency of the sample is related to the sample size. By employing Monte Carlo simulation the authors demonstrate that the mean efficiency increases while number of DMU's in the sample decreases. The same conclusion can be made for the individual efficiency. With the increase in the number of DMU's in the sample technical efficiency of any single firm tend to decrease (Diewert, 1993). The intuition behind this result is simple. As the number of DMU's increases, the chance that some DMU will be close to the theoretical frontier also increases and then the distance to DEA frontier for any given DMU tend to rise. Thus, individual efficiency decreases on average.

So, because estimation of individual efficiencies and group ones involves different number of observations we may expect that estimations on the group level will tend to over evaluate group efficiency. Further in the thesis, this result will be demonstrated.

C h a p t e r   3

METHODOLOGY

In this section the scheme of Monte Carlo experiment and some theoretical findings are provided.

As a real world example of aggregation we can consider multi-plant firm or different firms located in the one region. Suppose that there are two researchers A and B respectively. First of them possesses unaggregated data set; other one only aggregated data set. Their research question is to find efficiency scores of the group of DMU's and rank them on the base of technical efficiency obtained from DEA procedure. Researcher A to solve the problem can find individual efficiency scores of every DMU and then find average of individual efficiency scores. Alternatively, she can refer to the paper Li and Ng (1995) and find efficiency of the group as efficiency of the average firm within this group. There is only one way to solve study question for researcher B. She can only treat every group as if it is single DMU and calculate efficiency from ordinary DEA procedure. In the thesis the problem of comparability of their results examined. To answer this problem all approaches were employed and obtained results compared. The scheme of Monte Carlo simulations is the following:

1) obtain group efficiency score using unaggregated information through averaging of efficiency scores

$$(x^1, y^1) \Rightarrow \hat{TE}(x^1, y^1)$$

$$(x^2, y^2) \qquad \hat{TE}(x^2, y^2) \qquad \bar{TE}^1$$

$$\ldots \qquad \ldots \qquad \ldots$$

$$\ldots \Rightarrow \ldots \qquad \ldots$$

$(x^m, y^m) \qquad \hat{TE}(x^m, y^m) \qquad \bar{TE}^K$

where $\bar{TE}^i = \sum_{i=1}^{L} \hat{TE}^i \cdot s^i$. s^i is either output share if weighted average is used or just $\frac{1}{L}$ in case of non-weighted average.

2) obtain group efficiency score through estimation of the efficiency of the 'average' DMU

$$
\left.\begin{array}{l}(x^1, y^1)\\(x^2, y^2)\end{array}\right\} (\tilde{x}^1, \tilde{y}^1) \Rightarrow TE^1
$$

$$
\begin{array}{lll}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{array}
$$

$$
(x^m, y^m) \;\Big\}\; (\tilde{x}^k, \tilde{y}^k) \Rightarrow TE^K
$$

where $\tilde{y}^i = \frac{1}{L}\sum_{l=1}^{L} y_l^1, \tilde{x}^i = \frac{1}{L}\sum_{l=1}^{L} x^1{}_l$. Here $x^i{}_l$ and y^i_1 denotes the vector of inputs and outputs of the l^{th} DMU from the I^{th} group.

3) obtain group efficiency score using aggregated information

$$
\left.\begin{array}{l}(x^1, y^1)\\(x^2, y^2)\end{array}\right\} (X^1, Y^1) \Rightarrow \hat{TE}^1
$$

$$
\begin{array}{lll}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{array}
$$

$$
(x^m, y^m) \;\Big\}\; (X^k, Y^k) \Rightarrow \hat{TE}^K
$$

4) compare results

$$
\begin{array}{lll}\hat{TE}^1 \Leftrightarrow TE^1 & & \hat{TE}^1 \Leftrightarrow \bar{TE}^1\\ \quad\cdots & \text{and} & \quad\cdots\\ \quad\ldots & & \quad\cdots\\ \hat{TE}^K \Leftrightarrow TE^K & & \hat{TE}^K \Leftrightarrow \bar{TE}^K\end{array}
$$

As Monte Carlo technique suggests a large number of simulations should be run. In these simulations assumption was made that all firms face the same technology. Two types of production function were considered. The first one is Cobb-Douglas production function with constant returns $y = x_1^{1/3} x_2^{2/3}$ and the second is the VRS production function $y = \begin{Bmatrix} x^2 \\ x \\ \sqrt{x} \end{Bmatrix}$. The choice of functional form of the first function is made for the sake of convenience, since the function with the same returns to scale was used in the paper by Tauer (2000). The choice of the second production is made to check the impact of various returns to scale on the ranking of DMU's. Note, that technology set defined by the second production function is not convex, thus we cannot use results of Li and Ng to obtain group efficiency from the efficiency of the average firm.

Inputs will be generated from uniform distribution [0, 1]. In the work the assumption is made that the number of inputs is equal to 2 or 1.

So, $y_{theor} = x_1^{1/3} x_2^{2/3}$. ($y_{theor}$ stands for theoretically maximum output). Output obtained through such procedure is the output of technically efficient firm. Thus, next step should be generating of inefficiency. In the work the following assumption on the distribution of the technical efficiency was made: technical efficiency (TE) is half-normally distributed and inefficient output is generated as follows: $y^i = \frac{y_{theor}{}^i}{TE^i}$. To avoid unreasonable situation when TE^i<1 and 'inefficient' output is more that 'efficient' one assumption was made that

$TE^i \sim N^+(1,0.5)+1$. This set up is consistent with output oriented DEA model. $TE_o \in [1,\infty)$, the higher value of TE_o the more inefficient the firm is (o stands for output orientation).

Aggregated data set needs to be constructed. This can be done by dividing unaggregated data into sub-groups and summing over all DMU's inside every given sub-group, *i.e.* output of i[th] group is $Y^i = \sum_{j=1}^{l} y^i{}_j$, input set of i[th] is $X_k{}^i = \sum_{j=1}^{l} x^i_{k,j}, k = 1,2$, l is a number of DMU's in the i[th] group. As a result we have the same number of inputs and outputs but decreased number of DMU's. I[th] group technical efficiency ('true' one) is $TE^i = \frac{Y_{theor}{}^i}{Y^i}$, where $Y^i_{theor} = X_1^{1/3} X_2^{2/3}$. Technical efficiency of the group is estimated as if all groups were ordinary DMU's through DEA procedure. Group TE estimations obtained through different methodologies are compared them with 'true' ones on the base of the following criteria:

1) how far is the estimation of efficiency from the 'true' one;
2) what is the correlation between estimation of efficiency and 'true' one;
3) are these measures of TE statistically different;
4) whether aggregation over DMU's changes ranking of the groups

The distance is measured using mean squared difference.

$\sqrt{\sum_{i=1}^{K} (\frac{1}{K}(TE^i - \hat{TE}^i)^2)}$, where TE_i is 'true' technical efficiency of i[th] group, $\hat{TE}_i$ is estimation of i[th] group TE. Under CRS assumption it is possible to prove that all measures are theoretically the same. Recall that group technical efficiency is defined as follows:

$$TE^g \equiv \max_\theta\{\theta : (X, \theta Y) \in T\} = \max_\theta\{\theta : (\sum_{l=1}^{L} x^l, \theta \sum_{l=1}^{L} y^l) \in T\} =$$

$$\max_\theta\{\theta : \theta \sum_{l=1}^{L} y^l \leq y_{theor}\} \Rightarrow \theta = \frac{y_{theor}}{\sum_{l=1}^{L} y^l}$$

To test whether estimation of technical efficiency is statistically different from 'true' measure I use Pearson test of rank correlation.
In technical terms the algorithm of my work can be formulated as follows:

First, obtain efficiency scores using unaggregated data. For this I need to solve the ordinary problem of linear programming:

$$\begin{aligned} &\min \theta \\ &s.t. \\ &\sum_{k=1}^{K} z_k y_k \geq \theta y_k \\ &\sum_{k=1}^{K} z_k x_k \leq x_k, \\ &z_k \geq 0 \end{aligned} \qquad (2)$$

where, x_k is a vector of inputs of the k^{th} DMU,
y_k is a vector of outputs of the k^{th} DMU,

λ is efficiency score of the k^{th} DMU.

After that make "straight" aggregation both over inputs and outputs and find efficiency scores on the base of aggregated data set:

$$\begin{aligned} &\max \theta \\ &s.t. \\ &\sum_{k=1}^{K} z_k Y_k \geq \theta Y_k \\ &\sum_{k=1}^{K} z_k X_k \leq X_k, \\ &z_k \geq 0 \end{aligned} \qquad (3)$$

$$Y_k = \sum_{m=1}^{M} y_m \,, \quad X_k = \sum_{k=1}^{M} x_k$$

To find group efficiency through the efficiency of the 'average' firm the following problem needs to be solved:

$$\begin{aligned}
&\max \theta \\
&s.t. \\
&\sum_{k=1}^{K} z_k \tilde{y}_k \geq \theta \tilde{y}_k \\
&\sum_{k=1}^{K} z_k \tilde{x}_k \leq \tilde{x}_k \,, \\
&z_k \geq 0
\end{aligned}$$

Two types of averaging of individual DMU's technical efficiency into group TE were considered, the first is the simple average of all TE scores of the member of each particular group, and the second is the weighted average with output share of each DMU in the output of its group used as weights. Any of these approaches cannot provide us with estimations of TE scores obtained using aggregated information according to Blackorby and Russell (1998); however the second way (weighted average) is intuitively more appealing. Indeed, we should expect that the DMU with larger output should influence the efficiency of the group more than that with small output. Consider simple example

DMU	**Output share**	**TE**
1	90%	1
2	10%	2

This type of averaging is theoretically justified in the paper by Fare and Zelenyuk ()

Non-weighted average efficiency = 1.5; weighted average efficiency = 1.1, which seems to be more plausible.

To check whether the results of the simulation is 'robust' this procedure was done for three different of grouping structure (number of DMU's 10, 20 or 40); number of DMU's on the unaggregated level is 2000.

The results of the simulations are considered in the next section.

C h a p t e r 4

RESULTS

The following results were obtained through Monte Carlo simulation procedure.

Table 1. Results of Monte Carlo simulations, 40 DMU's in the group

$y_{theor} = x_1^{1/3} x_2^{2/3}$, DEA model with CRS assumption, 300 simulations, 40 DMU's in the group. Average figures are provided.

Estimation of group TE **($\hat{TE}$)**	**Mean squared difference from** $\sqrt{\sum_{i=1}^{K} (\frac{1}{K}(TE_i - \hat{TE}_i)^2)}$	**Correlation coefficient,**	**Direction of bias**
Non-weighted average of individual TE's	0.3129	0.61254	Indefinite
Weighted average of individual TE's	0.26176	0.7293	Indefinite
TE obtained on the base of 'average' firm	0.41296	0.91428	Downward
True efficiency	0.41838	0.91255	Downward

Note, that besides the fact that theoretically under CRS assumption TE obtained on the base of 'average' firm should be equal to efficiency estimated on the base of aggregated data set the distance is rather big. However, if we decrease the number of the DMU's in each single group this distance reduces. See appendices. This finding is consistent with the results of Zhang and Bartels (1998).

The direction of the bias is also consistent with their results.

Spearman rank correlation test in all simulations shows that we are able to conclude that our estimates are not statistically different at all levels of confidence.

But perhaps the most striking result is that under presence of aggregation even such that theoretically should not influence the estimation of the efficiency scores the ranking of the DMU's changes. This result also holds for the case of VRS function specification. Indeed, this should be the case because under this assumption different measures of the group efficiency are no longer equal to the true one.

Chapter 5

CONCLUSIONS AND DIRECTIONS FOR FURTHER RESEARCH

On the base of Monte Carlo simulations the following results were obtained.

Estimation of the group efficiency on the basis of aggregated data set leads to over evaluating of the group efficiency. There two potential sources of the bias: due to aggregation and computational one.

The less aggregated data set is the less the magnitude of the bias.

Ranking of the groups according to their efficiency can be different from the true one even in the case if theoretically they are identical.

If the researcher uses wrong specification of the DEA model (in the thesis it is shown for the case CRS specification instead of the VRS one) the group ranking is also affected.

As the direction for further research the author can propose study of the impact of the bias correction procedure (bootstrap technique) on the accuracy of group technical efficiency measurement.

BIBLIOGRAPHY

Blackorby, C., Russell, R.R. (1999) "*Aggregation of efficiency indices*", Journal of Productivity Analysis 12 (1), 5-20.

Coelli, T., Prasado Rao, D. S. and Battese G.E. (1998) "*An introduction to efficiency and productivity analysis".*

Cook, W. D., Chai, D., Doyle, J. and Green R. (1998). "*Hierarchies and groups in DEA",* Journal of Productivity Analysis, 10, 177-198.

Fare, R., Grosskopf, S. and Zelenyuk V. (2003) *"Aggregation bias and its bounds in measuring technical efficiency",* Applied Economics Letters, forthcoming.

Fare, R., Zelenyuk V. (2002) *"Input aggregation and technical efficiency*", Applied Economics Letters, 9, 635.

Farrell, M. J. (1957) "*The measurement of productive efficiency",* Journal of Royal Statistical Society, 120, 253-81.

Li, S.K. and Ng, Y.C. (1995) "*Measuring the productive efficiency of a group of firms*" Journal of Productivity Analysis, vol. 1, #4.

Tauer, L. (2000) *"Input aggregation and computed technical efficiency*", Applied Economics Letters, 8, 295-7.

Zhang, Y., Bartels, R. (1998) *"The effect of sample size on the mean efficiency in DEA with an application to electricity distribution in Australia, Sweden and New Zealand",* Journal of Productivity Analysis 9, 187-204.

Zelenyuk, V. *"Efficiency analysis of economic systems*", book in progress

Table 2. Results of Monte Carlo simulations, 20 DMU's in the group

$y_{theor} = x_1^{1/3} x_2^{2/3}$, DEA model with CRS assumption, 300 simulations, 20 DMU's in the group. Average figures are provided.

Estimation of group TE **($\hat{TE}$)**	**Mean squared difference from** $\sqrt{\sum_{i=1}^{K} (\frac{1}{K}(TE_i - \hat{TE}_i)^2)}$	**Correlation coefficient,**	**Direction of bias**
Non-weighted average of individual TE's	0.26916	0.62318	Indefinite
Weighted average of individual TE's	0.21916	0.73894	Indefinite
TE obtained on the base of 'average' firm	0.36685	0.92703	Downward
True efficiency	0.37256	0.92494	Downward

Table 3. Results of Monte Carlo simulations, 10 DMU's in the group

$y_{theor} = x_1^{1/3} x_2^{2/3}$, DEA model with CRS assumption, 300 simulations, 10 DMU's in the group. Average figures are provided.

Estimation of group TE ($\hat{TE}$)	**Mean squared difference from** $\sqrt{\sum_{i=1}^{K} (\frac{1}{K}(TE_i - \hat{TE}_i)^2)}$	**Correlation coefficient,**	**Direction of bias**
Non-weighted average of individual TE's	0.21112	0.636	Indefinite
Weighted average of individual TE's	0.16342	0.75177	Indefinite
TE obtained on the base of 'average' firm	0.29893	0.94841	Downward
True efficiency	0.30501	0.94646	Downward