# Kyiv School of Economics

July 4, 2012

Admission exam in Business Mathematics (Both versions together)

# Answer Sheet for Questions 1 - 15.

- 1. \_\_\_\_\_
- 2. \_\_\_\_\_
- 4. \_\_\_\_\_

3. \_\_\_\_\_

- 5. \_\_\_\_\_
- 6. \_\_\_\_\_
- 7. \_\_\_\_\_
- 8. \_\_\_\_\_
- 9. \_\_\_\_\_
- 10. \_\_\_\_\_
- 11. \_\_\_\_\_
- 12. \_\_\_\_\_
- 13. \_\_\_\_\_
- 14. \_\_\_\_\_
- 15. \_\_\_\_\_

# 1. A

If the average of three numbers (a, b, 7) is 4, the average of (a, c, 8) is 5, and the average of (b, c, 3) is 3, what is the average of (a, b, c)?

В

If the average of three numbers (x, y, 3) is 6, the average of (y, z, 5) is 7, and the average of (x, z, 4) is 9, what is the average of (x, y, z)?

### 2. A

A student has to pass four exams. Her average score on three exams is 85. If the student wants to increase her average by 2 points, what score must she earn on the fourth exam?

### В

A student has to pass four exams. His average score on three exams is 90. If the student wants to increase her average by 2 points, what score must he earn on the fourth exam?

### 3. A

An ophthalmologist charges 300 UAH for an eye examination, frames, and glass lenses, but 420 UAH for an eye examination, frames, and plastic lenses. It plastic lenses cost four times as much as the glass lenses, how much do the glass lenses cost?

### В

To get from Kyiv to his summer house, Stepan needs to take metro, suburban train and a bus, and this would cost him 35 UAH. If he misses the bus, he will need to take a taxi instead, and the whole trip would cost him 60 UAH. If taxi costs six times more compared to the bus, what is the price of one bus ticket?

### 4. A

An entrepreneur bought 10 boxes of oranges for 800 UAH total, 5 kilos of oranges in every box. 20% of the oranges got lost during storage and transportation. At what price per kilo should he sell the remaining oranges in order to earn a profit of 25% of the total cost?

В

A butcher has bought 300 kg of pork for 3600 UAH. He predicts that 20% will be lost during the storage and transportation. At what price per kilo should he sell the remaining part of pork in order to earn a profit of 30%?

# 5. A

Every client of a firm has a four-digit code (e.g. 7531). If the digits in the code must appear in descending numerical order (from the largest down to the smallest), and no digit can be used more than once, what is the difference between largest and smallest possible client codes?

В

Every client of a firm has a four-digit code (e.g. 2459). If the digits in the code must appear in ascending numerical order (from the smallest up to the largest), and no digit can be used more than twice, what is the difference between largest and smallest possible client codes?



#### State budget, Country X



For the next two questions, use Figure 1. The figure shows the distribution of selected articles of the state budget of a country X for fiscal years 2010 and 2011. All expenditures are shown in billions of TZP, the local currency in country X.

6. Using this figure, determine, which of the articles experienced the greatest percent increase in spending in fiscal year 2011 compared to fiscal year 2010?

7. A

In 2010, the budget deficit was estimated at the level of 2.5%. What was the approximate amount of the budget deficit in billions TZP?

В

In 2011, the budget deficit was estimated at the level of 4.5%. What was the approximate amount of the budget deficit in billions TZP?

For the next question, use Figures 2 and 3. The figures show the dynamics of the car sales of a certain car dealership, in thousands, in 2001-2009, and the regional distribution of sales in 2007.

8. A

28% of sales in Central region in 2007 were made in the Capital city, the main city of that region. How many cars have been sold in the Capital city in 2007?

В

20% of sales in Eastern region in 2007 were made in the city of Moreville, the largest city of that region. How many cars have been sold in Moreville in 2007?



Total Car Sales, Thousands

Figure 2: Total Car Sales, 2001-2009



Figure 3: Car Sales by Regions, 2007

### 9. A

The surface area of a sphere with radius r is  $4\pi r^2$ . By what factor will the surface area increase if the radius is trippled?

В

The surface area of a sphere with radius r is  $4\pi r^2$ . By what factor will the surface area increase if the radius is doubled?

#### 10. A

Simplify the following expression so that it contains only a single exponent of a:  $\frac{\left(((3a)^{-1})^{-2}(2a^{-2})^{-1}\right)}{a^{-3}}.$ 

В

Simplify the following expression so that it contains only a single exponent of a:  $\frac{\sqrt[3]{a}a^{1/12}\sqrt[4]{a^3}}{a^{5/12}\sqrt{a}}.$ 

11. A

Find the largest real root of the following equation:  $z^{-2} - 2z^{-1} - 8 = 0$ В

Find the largest real root of the following equation:  $y^{-2} + y^{-1} - 6 = 0$ 

12. A

In order to summarize how much the prices of different goods within a country have risen or fallen, a number of different price indexes have been suggested. One of such indexes is Laspeyres price

index, which is calculated as:  $PL_t = \frac{\sum_{i=1}^{n} p_{it}q_{i0}}{\sum_{i=1}^{n} p_{i0}q_{i0}}$ , where the subscript *i*0 refers to the **base year** 

value for good i, and t refers to the current year.

Imagine the economy consisting of only two goods, guns and roses. Given the data in the table below, calculate the value of the index for the year 2011, using 2009 as the base year.

	Quantity of Guns	Price of Guns	Quantity of Roses	Price of Roses
2009	100	2.00	50	1.00
2010	120	3.00	75	1.50
2011	125	4.00	25	3.00

В

In order to summarize how much the prices of different goods within a country have risen or fallen, a number of different price indexes have been suggested. One of such indexes is Paasche price index,

which is calculated as:  $PP_t = \frac{\sum_{i=1}^{n} p_{it}q_{it}}{\sum_{i=1}^{n} p_{i0}q_{it}}$ , where the subscript *i*0 refers to the **base year** value for

good i, and t refers to the current year.

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2011	125	4.00	25	3.00

13. A

Determine a relationship between the Centigrade (C) and Fahrenheit (F) temperature scales when you know that (i) the relation is linear; (ii) water freezes at  $0^{\circ}C$  and  $32^{\circ}F$ ; and (iii) water boils at  $100^{\circ}C$  and  $212^{\circ}F$ . As Ray Bradbury claimed in his famous novel, the paper spontaneously catches fire and burns at  $451^{\circ}F$ . How much is it in Celsius Centigrades?

# В

Determine a relationship between the Centigrade (C) and Fahrenheit (F) temperature scales when you know that (i) the relation is linear; (ii) water freezes at  $0^{\circ}C$  and  $32^{\circ}F$ ; and (iii) water boils at  $100^{\circ}C$  and  $212^{\circ}F$ . As Ray Bradbury claimed in his famous novel, the paper spontaneously catches fire and burns at  $451^{\circ}F$ . Apparently, he was wrong: the paper starts burning at  $451^{\circ}C$ , not  $451^{\circ}F$ . How much is  $451^{\circ}C$  in Fahrenheit?

### 14. A

If you toss two dies simultaneously, what is the probability that you will get 13? B

If you toss two dies simultaneously, what is the probability that you will get 14?

#### $15. \ \mathrm{A}$

Find the values of  $\alpha$  for which the determinant  $\begin{vmatrix} 3 & \alpha^{-1} \\ 2 & 8 \end{vmatrix}$  is zero.

В

Find the values of  $\alpha$  for which the determinant  $\begin{vmatrix} \sqrt{\alpha} & 4 \\ 3 & 2 \end{vmatrix}$  is zero.

For the following problems, provide full solution and give the answer in the suggested place.

16. A

Find the equation of a parabola  $y = ax^2 + bx + c$  that passes through points (-2, 11), (1, 2), and (2, 3).

В

Find the equation of a parabola  $y = ax^2 + bx + c$  that passes through points (-1, -7), (-2, -7), and (2, 5).

- 17. A square iron sheet with edges of 18 cm long is to be made into an open square box of depth x cm by cutting out equally sized squares of width x in each corner and then folding the edges.
  - (a) Draw a figure
  - (b) Derive the formula for the volume V(x) of the box
  - (c) Find for which value of  $x \in [1, 9]$  the volume V(x) is maximized.