

Kyiv School of Economics
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Admission Exam in Mathematics
Version A

General instructions (read carefully!):

- You should NOT open the exam before your proctor says so.
- The exam has 10 problems. All problems will be weighted equally.
- You have 80 minutes for this exam.
- The answer to each problem is a number or a short expression. Write down your answers in the Answer sheet. However, please, provide, in the exam book, detailed explanations of how the answers have been attained.
 - In the case of a wrong answer, a partial credit may be given based on your explanations.
 - Please, write legibly (readably).
 - Cheating on any exam automatically invalidates all your admission tests!
- You can use a back of any page for your draft notes.

Answer Sheet

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

1. A bin contains 5 defective (that immediately fail when put in use), 10 partially defective (that fail after a couple of hours of use), and 25 acceptable transistors. A transistor is chosen at random from the bin and put into use. If it does not immediately fail, what is the probability it is acceptable?

2. Suppose the random variable X has distribution function

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 - e^{-x} & \text{if } x > 0 \end{cases}$$

What is the probability that X exceeds $\ln 3$?

3. The joint density function of random variables X and Y is given by

$$f(x, y) = \begin{cases} 2e^{-x}e^{-2y} & \text{if } 0 < x < \infty \text{ and } 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Compute $P\{X < Y\}$ (the probability that the random variable X takes on a value that is less than the value taken on by Y).

4. When will average variable cost be at its minimum value for the following total cost function:

$$TC = 40 + 82q - 6q^2 + 0.2q^3?$$

5. Compute the determinant of the product $A \times A$ where

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

6. A company uses inputs K and L to manufacture goods A and B . It has available 200 units of K and 180 units of L and the input requirements are

10 units of K plus 30 units of L for each unit of A

25 units of K plus 15 units of L for each unit of B

If the per-unit profit is \$80 for A and \$30 for B , what combination of A and B should it produce to maximize profit?

7. Find the inverse of the following matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

8. Evaluate the following integral

$$\int_0^{\infty} x e^{-x} dx$$

9. Find $\lim_{x \rightarrow 2} \frac{5x-10}{\ln(x-1)}$.

10. In a competitive market where the supply price (in hryvnias) is $p = 3 + 0.25q$ and the demand price (in hryvnias) is $p = 15 - 0.75q$ the government imposes a per-unit tax of 4 hryvnias. What will be the tax revenue raised?