Version B

General Instructions:

- You should not open the exam before your proctor says so.
- Write ONLY in the space provided.
- Read the assignment carefully and do not do more than it is explicitly required.
- Make sure to show all your work.
- Please, write accurately. I may reduce your grade if I fail to read your writing.
- **Cheating on any exam automatically invalidates all your admission tests!**
- You have **75** minutes for this exam.
- The exam has **7** pages.
- You can use a back of any page for your draft notes. They are **NOT** going to be graded.
- GOOD LUCK!!!

1. (15 points) Let $\mathbf{X}$ be an $n \times k$ matrix with rank k, and let
$$\mathbf{Q} = \mathbf{X}'\mathbf{X}, \quad \mathbf{A} = \mathbf{Q}^{-1}\mathbf{X}', \quad \mathbf{N} = \mathbf{X}\mathbf{A}, \quad \mathbf{M} = \mathbf{I} - \mathbf{N}.$$

   (a) Find $\mathbf{S} = \mathbf{N}\mathbf{M}$

   (b) What is the dimension of $\mathbf{S}$?

2. (10 points) For this question you should clearly explain your choice from the mathematical point of view and motivate your answer. **No grade will be given without explanation!**

Olena is 32 years old. She was one of the best students since the very beginning of her studies at KSE. She majored in Financial Economics. In the summer after the first year she was an Intern in PriceWaterhouseCoopers. Next year her team won the second price in the Global Management Challenge competition, having outrun 58 other teams from Ukraine. She has also received the "Best Thesis Award" after the graduation.

Which is more probable?

(a) Olena is a mother of three children.

(b) Olena is a mother of three children and a senior economist in an investment bank.

3. (10 points) If you randomly and without replacement pick six letters from the word "**ECONOMICS**", what is the probability that you compose a word "**COMICS**" using them?

4. (15 points) If $y = x^{e^x}$, find y'.

5. (15 points) Let $f(z)$ be given as

$$f(z) = \int\limits_{0}^{z} \left(e^{-x} - e^{x}\right) dx$$

Find the maximum and minimum values of z on the interval $[-1; 2]$.

6. (15 points) Let's t is your age. A matrix $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$.

    (a) What is t?

    (b) Find $\mathbf{A}^t$. Prove your answer.

7. (20 points) A factory manager is considering whether to replace a machine. A review of past records indicates the following probability distribution for the number of hours per day this machine is idle (cannot work) due to breakdowns.

Number of idle hours per day	0	1	2	3
Probability	0.2	0.3	0.4	a

If the machine does not break down it normally operates 8 hours per day. One hour of normal operation of this machine costs the company $10. If the machine breaks down it costs $20 per hour to repair it. The machine should be supervised by a specially trained worker whose wage is $15 per hour. Even if the machine breaks down the worker still gets paid due to trade unions requirement. The machine produces $(50H - 100)$ of output per day, where H is the number of hours the machine is operating.

(a) Find a.

(b) Find the mean idle time per day.

(c) Find the expected **daily labor cost** of the company.

(d) Find the expected **daily capital cost** of the company.

(e) Find the expected **daily revenue** of the company.

(f) Find the expected **daily profit** of the company. Given this distribution of breakdown time, does it make sense to continue operating this machine?