

Kyiv School of Economics  
Admission Exam in Mathematics  
Version A

General instructions (read carefully!):

- You should NOT open the exam before your proctor says so.
- The exam has 10 problems. All problems will be weighted equally.
- You have 70 minutes for this exam.
- The answer to each problem is a number or a short expression. Write down your answers in the Answer sheet. However, please, provide, in the exam book, detailed explanations of how the answers have been attained.
  - In the case of a wrong answer, a partial credit may be given based on your explanations.
  - Please, write legibly (readably).
  - Cheating on any exam automatically invalidates all your admission tests!
- You can use the back of any page for your draft notes.

YOUR NAME \_\_\_\_\_

## Answer Sheet

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

5. \_\_\_\_\_

6. \_\_\_\_\_

7. \_\_\_\_\_

8. \_\_\_\_\_

9. \_\_\_\_\_

10. \_\_\_\_\_

1. A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy?

2. Calculate the cumulative distribution function of a random variable uniformly distributed over  $[1, 3]$ .

3. Evaluate the following integral

$$\int_0^{\infty} (x+1)e^{-x} dx$$

4. Compute  $\lim_{x \rightarrow 0} \frac{(x+1) \cos x - \sqrt{x+1}}{x}$

5. Determine the inverse of the matrix

$$M = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

6. Solve the following inequality

$$\ln x + \ln(x - 3) \leq \ln 4$$

7. A fair coin is tossed until a head appears or until the coin has been tossed five times. If a head does not occur on the first two tosses, what is the probability that the coin will be tossed five times?

8. Find the maximum value of the objective function  $f(x, y) = 2x + 3y$  over the following admissible set:

$$\{(x, y) \in \mathbb{R}^1 \times \mathbb{R}^1 : 1 \leq x \leq 5, y \geq 2, y \leq -x + 10\}$$

9. The inverse demand function for a commodity produced by a monopolist is given by  $p(x) = -x^2 + 49$ . The monopolist's total cost function is given by  $c(x) = 2x^3 + 13x + 40$ , where  $p(x)$  and  $c(x)$  are measured in thousands of dollars and  $x$  is measured in thousands of units. Determine the amount of production which will maximize the monopolist's profits.

10. Find the value of

$$\int_0^1 \int_0^{\sqrt{x}} x \, dy dx$$