Kyiv School of Economics

Admission Exam in Mathematics

Version A

General instructions (read carefully!):

• You should NOT open the exam before your proctor says so.

• The exam has ten problems on five pages (two problems per page). All problems will be weighted equally.

• You have 75 minutes for this exam.

• The answer to each problem is a number or a short expression. Write down your answers in the Answer Sheet. Please, provide, in the exam book, detailed explanations of how the answers have been attained. Answers without explanations will not be taken into account.

• Please, write legibly (readably).

• Cheating during the exam automatically invalidates all your admission tests!

• You can use the back of any page for your draft notes.

Answer Sheet

1._____

2._____

3._____

4._____

5._____

6._____

7._____

8._____

9._____

10._____

1. Suppose a fair coin is flipped five times. What is the probability of getting at least one head?

2. Ivan has 300 hryvnias to spend on potatoes and tomatoes. The price of a kilo of tomatoes is 20 hryvnias and the price of a kilo of potatoes is 4 hryvnias. How many kilograms of potatoes (P) and tomatoes (T) Ivan will buy if he wants to maximize his utility that is described by the following utility function $U(T, P) = T^{\frac{2}{3}} P^{\frac{1}{3}}$.

3. Evaluate the following integral: $\int_0^1 x\sqrt{1-x}dx$

4. A monopolistic producer can produce recorders at a cost of \$20 apiece. It is estimated that if the recorders are sold for $x$ dollars apiece, consumers will buy $200 - 4x$. Use calculus to determine the price at which the manufacturer's total revenue will be the greatest.

5. Find the inverse of the following matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

6. A card is chosen at random from a standard deck of 52 cards. Without replacing it, a second card is chosen. What is the probability that both cards chosen will be a king?

7. A farmer can plant up to 8 acres of land with wheat and barley. He can earn \$7,000 for every acre he plants with wheat and \$3,000 for every acre he plants with barley. His use of a necessary pesticide is limited by federal regulations to 10 gallons for his entire 8 acres. Wheat requires 2 gallons of pesticide for every acre planted and barley requires just 1 gallon per acre. What is the maximum profit the farmer can make?

8. Find the intervals of monotonicity of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x(x^2 + 2x + 1)$.

9. Compute the area enclosed by the graphs of the fwo functions $f_1 : \mathbb{R} \to \mathbb{R}$ and $f_2 : \mathbb{R} \to \mathbb{R}$ given by

$$f_1(x) = x^2 - 4 \text{ and } f_2(x) = 2x - x^2.$$

10. Solve the system of linear equations

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 4 \\ x_1 - x_2 + x_3 &= 5 \\ 2x_1 + 3x_2 - x_3 &= 1 \end{aligned}$$