KSE | Kyiv School of Economics

November 2024

Planned undergraduate programs in mathematics at KSE

Starting in the Fall of 2025, KSE will launch two new undergraduate programs: '**Physical Mathematics**' and '**Applied Mathematics**.' The latter is an updated version of the existing applied mathematics program at KSE. The duration of undergraduate programs at KSE is four years, after which students can enroll in graduate programs in mathematical sciences at KSE (also starting in 2025) or at other institutions.

This document presents a tentative description of the planned programs. Updates are scheduled for the spring of 2025.

Aims

To foster a competitive and innovative economy, it is crucial to connect basic research with practical applications and train a large number of young graduates to an advanced level. This is especially vital for mathematics, whose importance to the economy has grown due to its role in modeling, the widespread use of data, and artificial intelligence. Recent studies show the growing importance of numeracy skills and the significant productivity of workers in mathematical science occupations (see, for example, this global analysis of evidence from PIAAC — the Programme for the International Assessment of Adult Competencies, and this report by the Academy for the Mathematical Sciences in the UK.)

The aims of the new programs in mathematics at KSE are to:

- Offer innovative programs in mathematical sciences that maintain a core focus on foundational mathematics while providing training in its applications to other branches of science and technology, including physics and computer science;
- Provide an education that is suitable for both students aiming to pursue research and those preparing for careers in other sectors;
- Deliver a flexible, integrated teaching approach that meets the individual needs of students, supported by mentorship from KSE faculty;
- Foster in students the ability to learn independently, think logically, and solve complex problems;
- Attract outstanding students;
- Present a challenging and comprehensive curriculum in mathematics and its applications that meets the needs of a diverse group of students, including some of the most talented in the country;
- Produce high-caliber graduates in mathematics, ready to embark on postgraduate studies at KSE and other universities, or to be sought by employers in business and public services;
- Provide an intellectually stimulating environment in which students can develop their skills and enthusiasm to their full potential.

Undergraduate programs in mathematics at KSE

Global scope

Combined with intensive study of English in the first years, the undergraduate programs in mathematics at KSE will feature English-speaking lecturers and advertise faculty positions globally. Both programs will prepare students to pursue graduate studies abroad or in the global graduate program in mathematical sciences at KSE, where the working language will be English. Some of our graduates will advance to leadership positions in business and public services.

Contents of this document

- Course offerings by term and roadmaps
- Recommended course contents and literature

Working group on undergraduate programs in mathematics at KSE

KSE Faculty:	External Advisory Board:
Georgiy Shevchenko, Doctor of Physical and Mathematical Sciences, Academic Director for Undergraduate Education at KSE *Masha Vlasenko,* Associate Professor at KSE, Managing Director of the International Centre for Mathematics in Ukraine (ICMU)	*Alin Bostan*, Senior researcher at the French National Institute for Research in Digital Science and Technology (Inria) *Yuriy Drozd*, Senior researcher at the Institute of Mathematics of the National Academy of Sciences of Ukraine in Kyiv *Pavel Etingof*, Professor of mathematics, Massachusetts Institute of Technology *Kirill Krasnov*, Professor of Mathematical Physics, School of Mathematical Sciences, University of Nottingham *Maryna Viazovska*, Director of the Institute of Mathematics at École Polytechnique Fédérale de Lausanne, Member of the International Academic Board of KSE

Undergraduate programs in mathematics at KSE

Offer of courses by term

Basic courses which may be omitted by students with stronger preliminary preparation
Core courses for both programs in mathematics at KSE
'Applied Mathematics' core courses
'Physical Mathematics' core courses
Elective courses

	Fall: September-December	Spring: January-April	Summer: May-June
Year 1	* Introduction to Mathematics * Foundations of Mathematical Reasoning * Linear Algebra I * Real Analysis I * Introduction to Physics I	* Linear Algebra II * Real Analysis II * Programming Basics * Discrete Mathematics * Introduction to Physics II	* Differential Equations * Vector Calculus
Year 2	* Groups * Analysis and Topology * Discrete Applied Mathematics * Classical Mechanics * Numbers and Polynomials	* Complex Variables with Applications * Curves and Surfaces * Probability * Electromagnetism * Groups, Rings and Modules	* Variational Principles * Statistical Mechanics * Experimental Mathematics I * Fields and Galois Theory
Year 3	* Analysis of Differential Equations * Optimisation * Quantum Mechanics * Manifolds and Differential Forms * Introduction to Algebraic Geometry * Logic and Set Theory	* Numerical Methods * Statistics * Geometry of General Relativity * Introduction to Representation Theory * Measure-Theoretic Probability Theory	* Languages, Automata, and Complexity * Introduction to Quantum Fields * Introduction to Algebraic Topology * Applied Stochastic Processes
Year 4	* Introduction to Functional Analysis	* Experimental Mathematics II	
	Courses by choice depending on available lecturers: see the list of recommended topics. Every course can be offered at the core level or difficult level. Students with better marks are recommended to take more difficult courses at this stage.		

Objectives
Year 1: Transition to university mathematics, with a shift in learning style and pace from school mathematics. Introduction to basic concepts in higher mathematics and their applications, including the notions of proof, rigor, and axiomatic development. Intensive English classes.

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Students with sufficient preliminary preparation in mathematics are allowed to take courses from Year 2 under the guidance of mentors.

Years 2 and 3: Generalization of familiar mathematics to unfamiliar contexts and application of mathematics to problems outside the field. Laying the foundations—knowledge and understanding—of tools, facts, and techniques necessary for advanced courses in the final year. By the end of Year 3, students should have covered material in pure mathematics, statistics and operations research, applied mathematics, mathematical/theoretical physics, and computational mathematics, studying some of these topics in depth.

Year 4: Students study advanced material in the mathematical sciences, some of it in depth. They develop the ability to solve both abstract and concrete unseen problems, present concise and logical arguments, and (in some cases) use standard software to tackle mathematical problems.

Recommended further topics

Elective courses are offered in Year 4 based on the research interests of KSE faculty members and visiting professors. Each topic will be accompanied by a list of prerequisite courses from Years 1-3. Below is a list of some recommended topics:

Representation Theory Algebraic Topology Algebraic Geometry Homological Algebra Lie Groups and Lie Algebras Number Theory Fourier Analysis Functional Analysis Number Fields Riemann Surfaces Arithmetic Geometry Algebraic Combinatorics Differential Geometry Dynamical Systems Differential Equations of Mathematical Physics Supersymmetry String Theory Scattering Amplitudes Standard Model of Elementary Particles Cosmology Topological Quantum Matter Gravitational Waves and Numerical Relativity	Analytic Combinatorics and Algorithms Analysis Graphs and Network Analysis Coding and Cryptography Theory of Computation Automata and Formal Languages Quantum Information and Computation Game Theory Actuarial Mathematics Machine Learning Statistical Modeling Financial Mathematics Applied Integral Equations Design and Planning of Experiments Control Theory and System Dynamics Mathematical Methods in Engineering Environmental and Energy Systems Industrial Automation and Robotics Computational Fluid Dynamics Network Optimization and Supply Chain Management

Undergraduate programs in mathematics at KSE

Roadmaps

This section outlines the 'roadmaps' of course options for undergraduates interested in specific fields and applications of mathematics. Undergraduates should not view themselves as specializing in any one of these fields. A much better approach is to gain experience in several of them. Students should discuss their choices with their mentors.

Pure Mathematics

- **Algebra / Algebraic Number Theory / Algebraic Topology**
  Groups; Groups, Rings and modules; Fields and Galois Theory; Introduction to Algebraic Geometry; Introduction to Algebraic Topology; Manifolds and Differential Forms; Introduction to Representation Theory
- **Analysis / Topology / Geometry**
  Analysis and Topology, Functions of Complex Variables, Curves and Surfaces; Probability; Manifolds and Differential Forms; Measure-Theoretic Probability theory; Introduction to Functional Analysis
- **Probability / Statistics**
  Probability; Discrete Applied Mathematics; Statistics; Measure-Theoretic Probability Theory; Introduction to Functional Analysis; Applied Stochastic Processes

Physical Mathematics

- Introduction to Physics; Classical Mechanics; Statistical Mechanics; Quantum Mechanics; Geometry and Relativity; Introduction to Quantum Fields

Applied Mathematics

- **Computer Science**
  Discrete Mathematics; Discrete Applied Mathematics; Logic and Set Theory; Languages, Automata, and Complexity; Coding and Cryptography; Graphs and Network Analysis; Analytic Combinatorics and Algorithms Analysis; Theory of Computation; Automata and Formal Languages
  Students may also take elective courses offered by the following undergraduate programs at KSE: Artificial Intelligence, Economics and Big Data, Cybersecurity, Software Engineering and Business Analysis
- **Economics and Finance**
  Probability; Statistics; Applied Stochastic Processes; Introduction to Functional Analysis; Machine Learning; Actuarial Mathematics; Financial Mathematics
  Students may also take elective courses offered by the following undergraduate programs at KSE: Business Economics, Economics and Big Data, Software Engineering and Business Analysis
- **Industrial Mathematics**
  Numerical Methods; Statistics; Applied Stochastic Processes; Applied Integral Equations; Design and Planning of Experiments; Control Theory and System Dynamics; Mathematical Methods in Engineering; Environmental and Energy Systems; Industrial Automation and Robotics; Computational Fluid Dynamics; Network Optimization and Supply Chain Management

TENTATIVE CONTENTS OF COURSES

INTRODUCTION TO MATHEMATICS - back to the list of all courses

This is a standard course at KSE which reviews high school topics with many practical exercises. At the beginning of their first year students write a test in math, and those who didn't pass it have to do this review.

FOUNDATIONS OF MATHEMATICAL REASONING - back to the list of all courses

24 hours of lectures

Set theory

Main notions of naïve set theory, set operations (intersection, union, difference, symmetric difference, complement), Cartesian product. Functions; injections, surjections and bijections. Indicator (characteristic) functions; their use in establishing set identities. Sequences and series.

Mathematical logic

Informal and formal propositional calculus: formulas, interpretations, main logical connectives and truth tables. Tautologies, satisfiable and contradictory formulas. Semantic consequence, its connection with tautologies. Equivalent formulas. DNF, CNF, perfect normal forms. Functionally complete sets of logical connectives. Boolean functions. Predicate-quantifier calculus: formulas, interpretations, free and bound variables. Equivalence of predicate formulas. Tautologies.

Mathematical proof techniques and combinatorics

Finite sets, sum and product rules, inclusion-exclusion principle for two and three sets. Permutations and combinations. Properties of binomial coefficients. Newton's binomial theorem. Polynomial formula and polynomial coefficients. General inclusion-exclusion principle and its applications. Stars and bars counting method. Ordered partition of a natural number. Direct, contrapositive and proof by contradiction. Proofs of biconditional statements. Proof by construction and non-constructive existence proofs. Uniqueness proof. Mathematical induction. Bijective proofs. Pigeonhole principle. Method of double-counting.

DISCRETE MATHEMATICS - back to the list of all courses

3 lectures per week

Basic concepts and orderings

Reminder on pigeonhole principle, mathematical induction, sets. Functions and relations, equivalence relations: equivalence classes, quotient sets and projections, partition of a set, equivalence relations induced by functions. Linear orders, partial orders, maximal and minimal elements.

Combinatorial counting

Reminder of binomial coefficients, partition problem, inclusion-exclusion principle; recurrence relations (including Fibonacci numbers), Catalan and Bell numbers.

Basic combinatorial structures

Graphs, adjacent vertices, vertex degree. Graph isomorphism, main classes of graphs. Handshaking lemma. Walks, paths and cycles in graphs. Connected and bipartite graphs, graph metric, adjacency matrix. Biconnected graphs, trees, spanning subgraphs, Matrix tree theorem, Cayley's formula for trees. Eulerian cycles and walks, criteria of Eulerian graphs, Hamiltonian graphs. Vertex and edge colorings, chromatic number and chromatic index, clique number and independence number.

Generating functions

Combinatorial applications of polynomials. Calculation with power series. Examples: Fibonacci numbers and the golden section, binary trees, rolling the dice, random walks, integer partitions.

Probabilistic methods

Proofs by counting, finite probability spaces, random variables and their expectation, several applications.

Applications of linear algebra

Block designs. Fisher's inequality. Covering by complete bipartite graphs. Cycle space of a graph. Circulations and cuts: cycle space revisited. Probabilistic checking.

Recommended books:

Invitation to discrete mathematics / Jiri Matousek, Jaroslav Nesetril. – 2nd ed., repr. – New York, 2011

Discrete Mathematics: Elementary and Beyond / L. Lovasz, J. Pelikan, K. Vesztergombi

Combinatorics: Set Systems etc. / B. Bollobas

Similar course: https://ocw.mit.edu/courses/18-314-combinatorial-analysis-fall-2014/

REAL ANALYSIS I & II - back to the [list of all courses]

3 classes per week, two terms

This course should contain in-class exams, e.g. four 50 minutes in-class exams per term, plus the final exam.

Limits and convergence

Sequences and series in R and C. Sums, products and quotients. Absolute convergence; absolute convergence implies convergence. The Bolzano-Weierstrass theorem and applications (the General Principle of Convergence). Comparison and ratio tests, alternating series test.

Continuity

Continuity of real- and complex-valued functions defined on subsets of R and C. The intermediate value theorem. A continuous function on a closed bounded interval is bounded and attains its bounds.

Differentiability

Differentiability of functions from R to R. Derivative of sums and products. The chain rule. Derivative of the inverse function. Rolle's theorem; the mean value theorem. One-dimensional version of the inverse function theorem. Taylor's theorem from R to R; Lagrange's form of the remainder. Complex differentiation.

Power series

Complex power series and radius of convergence. Exponential, trigonometric and hyperbolic functions, and relations between them. Direct proof of the differentiability of a power series within its circle of convergence.

Integration

Definition and basic properties of the Riemann integral. A non-integrable function. Integrability of monotonic functions. Integrability of piecewise-continuous functions. The fundamental theorem of calculus. Differentiation of indefinite integrals. Integration by parts. The integral form of the remainder in Taylor's theorem. Improper integrals.

Multivariable functions and partial derivatives

Level curves, partial derivatives, tangent plane approximation. Max-min problems, least squares. Second derivative test; boundaries and infinity. Differentials, chain rule. Gradient, directional derivative, tangent plane. Directional derivatives and the gradient vector. Statement of Taylor series for functions on R^n . Local extrema of real functions, classification using the Hessian matrix. Lagrange multipliers. Non-independent variables. *Partial differential equations.*

Similar courses:
https://ocw.mit.edu/courses/18-01-single-variable-calculus-fall-2006/
and section II of
https://ocw.mit.edu/courses/18-02-multivariable-calculus-fall-2007/pages/calendar/

LINEAR ALGEBRA I - back to the list of all courses

3 lectures per week

This course should contain in-class exams, e.g. four 50 minutes in-class exams per term, plus the final exam.

Complex numbers

Review of complex numbers, including complex conjugate, inverse, modulus and argument. Informal treatment of complex logarithm, n-th roots and complex powers, de Moivre's theorem.

Vectors

Review of elementary algebra of vectors in R^3 , including scalar product. Brief discussion of vectors in R^n and C^n ; scalar product and the Cauchy–Schwarz inequality. Concepts of linear span, linear independence, subspaces, basis and dimension.

Suffix notation: including summation convention, δij and ϵijk.Vector product and triple product: definition and geometrical interpretation. Solution of linear vector equations. Applications of vectors to geometry, including equations of lines, planes and spheres.

Matrices

Elementary algebra of 3 × 3 matrices, including determinants. Extension to n × n complex matrices. Trace, determinant, non-singular matrices and inverses. Matrices as linear transformations; examples of geometrical actions including rotations, reflections, dilations, shears; kernel and image, rank–nullity theorem (statement only).

Simultaneous linear equations: matrix formulation; existence and uniqueness of solutions, geometric interpretation; Gaussian elimination.

Symmetric, anti-symmetric, orthogonal, hermitian and unitary matrices. Decomposition of a general matrix into isotropic, symmetric trace-free and antisymmetric parts.

Eigenvalues and eigenvectors, change of basis

Eigenvalues and eigenvectors; examples and geometric significance. Proof that eigenvalues of hermitian matrix are real, and that distinct eigenvalues give an orthogonal basis of eigenvectors. The effect of a general change of basis (similarity transformations). Diagonalization of general matrices: sufficient conditions; examples of matrices that cannot be diagonalized. Canonical forms for 2 × 2 matrices.

*** Classification of conics, rotation matrices and applications in special relativity ***
Discussion of quadratic forms, including change of basis. Classification of conics, cartesian and polar forms. Rotations in 3-space. Lorentz transformations as transformation groups.

Appropriate book: G. Strang, Linear Algebra and Its Applications, 4th Edition
See also lecture notes by Alexandre Eremenko which follow this book and include applications: https://www.math.purdue.edu/~eremenko/lecturenotes.html

LINEAR ALGEBRA II - back to the list of all courses
3 hours of lectures per week
Prerequisite: Vectors and Matrices

Abstract linear algebra
Definition of a vector space (over R or C), subspaces, the space spanned by a subset. Linear independence, bases, dimension. Direct sums and complementary subspaces. Quotient spaces.
Linear maps, isomorphisms. Relation between rank and nullity. The space of linear maps from U to V, representation by matrices. Change of basis. Row rank and column rank.
Determinant and trace of a square matrix. Determinant of a product of two matrices and of the inverse matrix. Determinant of an endomorphism. The adjugate matrix.
Eigenvalues and eigenvectors. Diagonal and triangular forms. Characteristic and minimal polynomials.
Cayley–Hamilton Theorem over C. Algebraic and geometric multiplicity of eigenvalues. Statement and illustration of Jordan normal form.
Dual of a finite-dimensional vector space, dual bases and maps. Matrix representation, rank and determinant of dual map
Bilinear forms. Matrix representation, change of basis. Symmetric forms and their link with quadratic forms. Diagonalisation of quadratic forms. Law of inertia, classification by rank and signature. Complex Hermitian forms.
Inner product spaces, orthonormal sets, orthogonal projection, $V = W \oplus W^\perp$. Gram–Schmidt orthogonalisation. Adjoints. Diagonalisation of Hermitian matrices. Orthogonality of eigenvectors and properties of eigenvalues.
Markov chains and other applications
Definition and basic properties of Markov chains, the transition matrix. Calculation of n-step transition probabilities. Invariant distribution. Recurrence, irreducibility, ergodicity.
Other applications may include linear ordinary differential equations with constant coefficients and matrix exponentials.

Appropriate book: G. Strang, Linear Algebra and Its Applications, 4th Edition
See also lecture notes by Alexandre Eremenko which follow this book:
https://www.math.purdue.edu/~eremenko/lecturenotes.html
and lecture notes by Yaroslav Vorobets which include applications of linear algebra:
https://people.tamu.edu/~yvorobets/MATH304-2011C/MATH304-502.html

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NUMBERS AND POLYNOMIALS - back to the list of all courses
2 lectures per week
This course gives an overview of topics important for applications, such as prime numbers, congruences, Euclidean algorithm for numbers and polynomials. It also serves as a gentle introduction to the abstract algebra courses coming in the following years.

Introduction to number systems
Overview of the natural numbers, integers, real numbers, complex numbers; statement of the Fundamental Theorem of Algebra. Rational and irrational numbers, algebraic and transcendental numbers.
Elementary number theory
Prime numbers: existence and uniqueness of prime factorisation into primes; greatest common divisors and least common multiples. Euclid's proof of the infinity of primes. Euclid's algorithm. Solution in integers of $ax+by = c$.
Modular arithmetic (congruences). Units modulo n. Chinese Remainder Theorem. Wilson's Theorem; the Fermat-Euler Theorem. Public key cryptography and the RSA algorithm. Residues modulo prime powers and Hensel's lemma. Primitive elements modulo n. Quadratic reciprocity law.
Polynomials
Euclid's algorithm and factorisation of polynomials. Uniqueness of factorisation and analogy with integer numbers. Greatest common divisors and least common multiples of polynomials.
The real numbers
Least upper bounds; simple examples. Least upper bound axiom. Sequences and series; convergence of bounded monotonic sequences. Irrationality of $\sqrt{2}$ and e. Decimal expansions. Construction of a transcendental number.
Countability and uncountability
Definitions of finite, infinite, countable and uncountable sets. A countable union of countable sets is countable. Uncountability of R. Non-existence of a bijection from a set to its power set. Indirect proof of existence of transcendental numbers.

DIFFERENTIAL EQUATIONS - back to the list of all courses
3 lectures per week
First-order linear differential equations
Equations with constant coefficients: exponential growth, comparison with discrete equations, series solution; modeling examples including radioactive decay. Equations with non-constant coefficients: solution by integrating factor.
Linear equations with constant coefficients
The second order homogeneous equations and initial value problems for them. Linear dependence and independence, a formula for the Wronskian. The non-homogeneous equation of order two. Initial value problems for n-th order equations. The non-homogeneous equation of order n, a method of solving it. Algebra of constant coefficient operators.
Linear equations with variable coefficients
Initial value problems, solutions. Wronskian and linear independence, Abel's theorem. Reduction of the order of a homogeneous equation. The non-homogeneous equation. The power series method.

Linear equations with regular singular points
The Euler equation, examples of second order. The general case of second order equations with regular singular points. *A proof of convergence.* The Bessel equation. Regular singular points at infinity.
Existence and uniqueness of solutions to first order equations
First order equations, separated variables, exact equations. The method of successive approximations, the Lipschitz condition. Non-local existence of solutions. Equations with complex-valued functions.
Existence and uniqueness of solutions to systems and n-th order equations
An example of central forces and planetary motion, some special equations. Systems as vector equations. Existence and uniqueness of solutions to systems. Equations of order n.

Appropriate books:
An introduction to ordinary differential equations / Earl A. Coddington, Dover Publications, 1989
Introduction to the Theory of Linear Differential Equations / E. G. C. Poole, Clarendon Press, London, 1936
Ordinary Differential Equations/ E.L. Ince, Dover Publications, 1956

==== second variant ===

Review of basic calculus (optional)
Informal treatment of differentiation as a limit, the chain rule, Leibnitz's rule, Taylor series, informal treatment of O and o notation and l'Hôpital's rule; integration as an area, fundamental theorem of calculus, integration by substitution and parts.
Informal treatment of partial derivatives, geometrical interpretation, statement (only) of symmetry of mixed partial derivatives, chain rule, implicit differentiation. Informal treatment of differentials, including exact differentials. Differentiation of an integral with respect to a parameter.
First-order linear differential equations
Equations with constant coefficients: exponential growth, comparison with discrete equations, series solution; modeling examples including radioactive decay. Equations with non-constant coefficients: solution by integrating factor.
Existence and uniqueness of solutions: Picard's existence theorem and method of proof.
Nonlinear first-order equations
Separable equations. Exact equations. Sketching solution trajectories. Equilibrium solutions, stability by perturbation; examples, including logistic equation and chemical kinetics. Discrete equations: equilibrium solutions, stability; examples including the logistic map.
Higher-order linear differential equations
Complementary function and particular integral, linear independence, Wronskian (for second-order equations), Abel's theorem. Equations with constant coefficients and examples including radioactive sequences, comparison in simple cases with difference equations, reduction of order, resonance, transients, damping.
Homogeneous equations. Response to step and impulse function inputs; introduction to the notions of the Heaviside step-function and the Dirac delta-function. Series solutions including statement only of the need for the logarithmic solution.

Useful (but larger) course:
https://ocw.mit.edu/courses/18-03-differential-equations-spring-2010/
Recommended books:

An introduction to Differential Equations / J. Robinson, Cambridge University Press, 2004
Elementary Differential Equations and Boundary-Value Problems /W.E. Boyce and R.C. DiPrima, Wiley, 2004 (and associated web site: google Boyce DiPrima).

GROUPS - back to the [list of all courses]
2 lectures per week
Examples of groups
Axioms for groups. Examples from geometry: symmetry groups of regular polygons, cube, tetrahedron. Permutations on a set; the symmetric group. Subgroups and homomorphisms. Symmetry groups as subgroups of general permutation groups. The Möbius group; cross-ratios, preservation of circles, the point at infinity. Conjugation. Fixed points of Möbius maps and iteration.
Lagrange's theorem
Cosets. Lagrange's theorem. Groups of small order (up to order 8). QuateRnions. Fermat-Euler theorem from the group-theoretic point of view.
Group actions
Group actions; orbits and stabilizers. Orbit-stabilizer theorem. Cayley's theorem (every group is isomorphic to a subgroup of a permutation group). Conjugacy classes. Cauchy's theorem.
Quotient groups
Normal subgroups, quotient groups and the isomorphism theorem.
Matrix groups
The general and special linear groups; relation with the Möbius group. The orthogonal and special orthogonal groups. Proof (in R^3) that every element of the orthogonal group is the product of reflections and every rotation in R^3 has an axis. Basis change as an example of conjugation.
Permutations
Permutations, cycles and transpositions. The sign of a permutation. Conjugacy in S_n and in A_n . Simple groups; simplicity of A_5 .

VECTOR CALCULUS - back to the [list of all courses]
3 lectures per week
Prerequisites: [Real Analysis], [Vectors and Matrices], [Linear Algebra]
Curves in R^3
Parameterised curves and arc length, tangents and normals to curves in R^3; curvature and torsion.
Integration in R^2, R^3, and R^n
Line integrals. Surface and volume integrals: definitions, examples using Cartesian, cylindrical and spherical coordinates; change of variables.
Vector operators
Review of functions of several variables, continuity, differentiability, partial derivatives. Directional derivatives. The gradient of a real-valued function: definition; interpretation as normal to level surfaces; examples including the use of cylindrical, spherical and general orthogonal curvilinear coordinates.
Vector fields. Gradient curl and divergence in Cartesian coordinates, examples; formulae for these operators (statement only) in cylindrical, spherical ∗ and general orthogonal curvilinear∗ coordinates. Solenoidal fields, irrotational fields and conservative fields; scalar potentials. Vector derivative identities.

Differential forms and integration theorems
Antisymmetric differential forms. Divergence theorem, Green's theorem, Stokes' theorem, Green's second theorem: statements; informal proofs; examples; application to fluid dynamics, and to electromagnetism including statement of Maxwell's equations.
*** Cartesian tensors in R^3 ***
Tensor transformation laws, addition, multiplication, contraction, with emphasis on tensors of second rank. Isotropic second and third rank tensors. Symmetric and antisymmetric tensors. Revision of principal axes and diagonalization. Quotient theorem. Examples including inertia and conductivity.

A variant: if topics of conics and rotations weren't covered in the courses 'Vectors and Matrices' or 'Linear algebra', they can be included in this course. Cartesian tensors can be moved to 'Classical Mechanics'.

Similar courses: https://ocw.mit.edu/courses/18-022-calculus-of-several-variables-fall-2010/
https://ocw.mit.edu/courses/18-02-multivariable-calculus-fall-2007/pages/calendar/

COMPLEX VARIABLES WITH APPLICATIONS - back to the list of all courses
2 lectures per week
Analytic functions
Complex differentiation and the Cauchy-Riemann equations. Examples. Conformal mappings. Informal discussion of branch points, examples of log z and z^c.
Contour integration and Cauchy's theorem
Contour integration (for piecewise continuously differentiable curves). Statement and proof of Cauchy's theorem for star domains. Cauchy's integral formula, maximum modulus theorem, Liouville's theorem, fundamental theorem of algebra. Morera's theorem.
Expansions and singularities
Uniform convergence of analytic functions; local uniform convergence. Differentiability of a power series. Taylor and Laurent expansions. Principle of isolated zeros. Residue at an isolated singularity. Classification of isolated singularities.
The residue theorem
Winding numbers. Residue theorem. Jordan's lemma. Evaluation of definite integrals by contour integration. Rouche's theorem, principle of the argument. Open mapping theorem.
Conformal mappings with applications
Geometric definition of conformal mappings. Proof that analytic functions are conformal. Riemann mapping theorem (statement only). Fractional linear transformations, reflections and symmetries. Flows around cylinders. Applications in electrostatics.
Analytic continuation and the gamma function
Uniqueness of analytic continuation; definition and properties of the Gamma function

Recommended books:
H.A. Priestley Introduction to Complex Analysis. Clarendon 1990
K. F. Riley, M. P. Hobson, and S.J. Bence Mathematical Methods for Physics and Engineering: a Comprehensive Guide. Cambridge University Press 2002
Similar course:
https://ocw.mit.edu/courses/18-04-complex-variables-with-applications-spring-2018/
(see also lecture notes and literature given there)

ANALYSIS AND TOPOLOGY - back to the list of all courses
2 lectures per week
Prerequisite: Real Analysis I & II
Uniform convergence and uniform continuity
The general principle of uniform convergence. A uniform limit of continuous functions is continuous. Uniform convergence and termwise integration and differentiation of series of real-valued functions. Local uniform convergence of power series. Uniform continuity. Continuous functions on closed bounded intervals are uniformly continuous; Riemann integrability of continuous functions from Real Analysis revisited.
Metric spaces
Definition and examples. Limits and continuity. Open sets and neighborhoods. Characterizing limits and continuity using neighborhoods and open sets. Completeness. The contraction mapping theorem. Applications including Picard's solution of differential equations.
Topological spaces
Definition and examples. Metric topologies. Further examples. Neighborhoods, closed sets, convergence and continuity. Hausdorff spaces. Homeomorphisms. Topological and non-topological properties. Subspace, product and quotient topologies.
Connectedness
Definition using open sets and integer-valued functions. Examples. Components. The continuous image of a connected space is connected. Path-connectedness and its relation to connectedness. Connected open sets in Euclidean space are path-connected. Connectedness of products.
Compactness
Definition using open covers. Examples including [0, 1]. Closed subsets of compact spaces are compact. Compact subsets of Hausdorff spaces are closed. The compact subsets of the real line. Continuous images of compact sets are compact. Continuous real-valued functions on a compact space are bounded and attain their bounds. Compactness of products. Compactness of quotient spaces. Sequential compactness.
Differentiation from R^m to R^n
Definition of derivative as a linear map. Elementary properties; the chain rule. Partial derivatives; continuous partial derivatives imply differentiability. Second derivatives; symmetry of mixed partial derivatives. Relationship between the Hessian and local extrema. The mean-value inequality. A function having zero derivative on a path-connected set is constant. The inverse function theorem (proof of continuity of inverse function; statement of differentiability). The implicit function theorem *with proof*.

PROBABILITY - back to the list of all courses
2 lectures per week
Prerequisites: Real Analysis I&II, Discrete Mathematics

Axiomatic approach
Discrete probability spaces. Probability spaces. Inclusion-exclusion formula. Continuity and subadditivity of probability measures. Conditional probability, Bayes's formula. Examples, including Simpson's paradox. Independence. Binomial, Poisson and geometric distributions. Poisson limit theorem. De Moivre–Laplace theorem.
Discrete random variables
Expectation. Functions of a random variable, indicator function, variance, standard deviation. Covariance, independence of random variables. Generating functions: sums of independent

random variables, random sum formula, moments.
Conditional expectation. **Continuous random variables**
Distributions and density functions. Expectations; expectation of a function of a random variable. Uniform, normal and exponential random variables. Memoryless property of exponential distribution. Joint distributions: transformation of random variables (including Jacobians), examples. Simulation: generating continuous random variables, Box–Muller transform, rejection sampling. Geometrical probability: Bertrand's paradox, Buffon's needle. Correlation coefficient, bivariate normal random variables.
Inequalities and limits
Markov's inequality, Chebyshev's inequality. Weak law of large numbers. Convexity: Jensen's inequality for general random variables, AM/GM inequality.
Moment generating functions and statement of continuity theorem. Statement of central limit theorem and sketch of proof. Examples, including sampling. Basics of random walks and Poisson process.

CURVES AND SURFACES - back to the list of all courses
24 lecture hours
Prerequisites: Analysis and topology
This course is an introduction to Differential geometry.
Surfaces
Topological surfaces via charts and atlases. Examples including the sphere via stereographic projection, the real projective plane and polygons with side identifications.
Smooth surfaces and smooth parametrizations. Orientability. The implicit-function theorem. Surfaces in R^3.
Informal discussion of triangulations; Euler characteristic and genus.
Surfaces in 3-space
The first fundamental form of an embedded surface in R^3. Length and area. Examples including surfaces of revolution. Change of parametrisation.
The second fundamental form. Gauss curvature of an embedded surface. The Gauss map; Gauss curvature and area. Statement of theorema egregium.
Geodesics
Length and energy. Geodesics as critical points for the energy functional; Euler–Lagrange equations for energy. Review of Picard's theorem and existence of geodesics. Examples: geodesics on spheres, flat tori, surfaces of revolution.
Hyperbolic surfaces
Abstract Riemannian metrics on a disc; isometries. Moebius group of the sphere, disc and half-plane. The hyperbolic metric on the disc and half-plane; geodesics and isometries. Gauss–Bonnet theorem for hyperbolic triangles. Hyperbolic hexagons; hyperbolic structures on closed surfaces.
Further topics
Statement of Gauss–Bonnet for geodesic polygons and closed surfaces. Action of SL(2, Z) on the torus; elliptic, parabolic and hyperbolic elements. Moduli space of flat metrics on the torus.

GROUPS, RINGS AND MODULES - back to the list of all courses
2 lectures per week

Groups, characters and representations
Basic concepts of group theory recalled from Groups course. Normal subgroups, quotient groups and isomorphism theorems. Permutation groups. Groups acting on sets, permutation representations. Conjugacy classes, centralizers and normalizers. The centre of a group. Elementary properties of finite p-groups.
Examples of finite linear groups and groups arising from geometry. Simplicity of An.
Sylow subgroups and Sylow theorems. Applications, groups of small order.
Group representations, unitary representations, characters, one-dimensional characters the regular representation.
Rings
Definition and examples of rings (commutative, with 1). Ideals, homomorphisms, quotient rings, isomorphism theorems. Prime and maximal ideals. Fields. The characteristic of a field. Field of fractions of an integral domain.
Factorization in rings; units, primes and irreducibles. Unique factorization in principal ideal domains, and in polynomial rings. Gauss' Lemma and Eisenstein's irreducibility criterion.
Rings Z[α] of algebraic integers as subsets of C and quotients of Z[x]. Examples of Euclidean domains and uniqueness and non-uniqueness of factorization. Factorization in the ring of Gaussian integers; representation of integers as sums of two squares.
Ideals in polynomial rings. Hilbert basis theorem.
Modules
Definitions, examples of vector spaces, abelian groups and vector spaces with an endomorphism. Submodules, homomorphisms, quotient modules and direct sums. Equivalence of matrices, canonical form. Structure of finitely generated modules over Euclidean domains, applications to abelian groups and Jordan normal form.

FIELDS AND GALOIS THEORY - back to the list of all courses
2 lectures per week

Characteristic of a field. Field extensions; the degree of a field extension and the tower theorem. Algebraic elements. Constructions with ruler and compass. Symbolic adjunction of roots of polynomials, multiple roots.
Finite fields: existence, uniqueness, primitive elements, number of irreducible polynomials over the field of size p, subfields of finite fields.
Existence and uniqueness of algebraic closure.
Separability. Theorem of primitive element. Trace and norm.
Normal and Galois extensions, automorphism groups. Fundamental theorem of Galois theory.
Galois theory of finite fields. Reduction mod p.
Cyclotomic polynomials, Kummer theory, cyclic extensions. Symmetric functions. Galois theory of cubics and quartics.
Solubility by radicals. Insolubility of general quintic equations and other classical problems.
Artin's theorem on the subfield fixed by a finite group of automorphisms. Polynomial invariants of a finite group; examples.

INTRODUCTION TO ALGEBRAIC GEOMETRY - back to the list of all courses
2 lectures per week
Prerequisite: Groups, Rings, and Modules

Affine varieties and coordinate rings. Projective space, projective varieties and homogenous coordinates.
Rational and regular maps.
Discussion of basic commutative algebra. Dimension, singularities and smoothness.
Conics and plane cubics. Quadric surfaces and their lines. Segre and Veronese embeddings.
Curves, differentials, genus. Divisors, linear systems and maps to projective space. The canonical class.
Statement of the Riemann-Roch theorem, with applications.

Appropriate books
K. Hulek Elementary Algebraic Geometry. American Mathematical Society, 2003
F. Kirwan Complex Algebraic Curves. Cambridge University Press, 1992
M. Reid Undergraduate Algebraic Geometry. Cambridge University Press 1989
B. Hassett Introduction to Algebraic Geometry. Cambridge University Press, 2007
K. Ueno An Introduction to Algebraic Geometry. American Mathematical Society 1977
R. Hartshorne Algebraic Geometry, chapters 1 and 4. Springer 1997

INTRODUCTION TO REPRESENTATION THEORY - back to the list of all courses
2 lectures per week
Prerequisites: Linear Algebra; Groups, Rings and Modules
Representations of finite groups
Representations of groups on vector spaces, matrix representations. Equivalence of representations. Invariant subspaces and submodules. Irreducibility and Schur's Lemma. Complete reducibility for finite groups. Irreducible representations of Abelian groups.
Character theory
Determination of a representation by its character. The group algebra, conjugacy classes, and orthogonality relations. Regular representation. Permutation representations and their characters. Induced representations and the Frobenius reciprocity theorem. Mackey's theorem. Frobenius's Theorem.
Arithmetic properties of characters
Divisibility of the order of the group by the degrees of its irreducible characters. Burnside's pa q b theorem.
Tensor products
Tensor products of representations and products of characters. The character ring. Tensor, symmetric and exterior algebras.
Representations of S^1 and SU2
The groups S^1 , SU2 and SO(3), their irreducible representations, complete reducibility. The Clebsch-Gordan formula. Compact groups.
Further worked examples
The characters of one of GL2 (Fq), Sn or the Heisenberg group.

MEASURE-THEORETIC PROBABILITY THEORY - back to the list of all courses
2 lectures per week
Prerequisites: Real Analysis, Probability, Analysis and Topology
Foundations of measure theory

Sigma-algebras, measures, measurable functions, Borel functions. Lebesgue measure. Probability measures, uniqueness, Dynkin–Doob lemma.

Integration and convergence

Integration with respect to a measure. Dominated and monotone convergence theorems. Standard machinery. Product measures, Fubini and Tonelli theorems. Absolute continuity of measures, Radon–Nikodym theorem.

Probability measures

Concepts of probability spaces in the measure-theoretic framework, conditional expectation, independence. Uniform integrability and the la Vallée Poussin theorem. 0-1 laws. Convergence of random series (Kolmogorov theorems). Strong law of large numbers. Birkhoff ergodic theorem.

Information divergence and entropy

Definitions and properties of Kullback-Leibler (KL) divergence, mutual information, and applications to information theory. Concepts of entropy, divergence measures in machine learning and statistical inference.

INTRODUCTION TO FUNCTIONAL ANALYSIS - back to the list of all courses

2 lectures per week

Spaces of integrable functions

Review of integration: simple functions, monotone and dominated convergence; existence of Lebesgue measure; definition of Lp spaces and their completeness. The Lebesgue differentiation theorem. Egorov's theorem, Lusin's theorem. Mollification by convolution, continuity of translation and separability of L^p when p ≠ ∞.

Banach and Hilbert space analysis

Strong, weak and weak-* topologies; reflexive spaces. Review of the Riesz representation theorem for Hilbert spaces; the Radon–Nikodym theorem; the dual of Lp . Compactness: review of the Ascoli–Arzelà theorem; weak-* compactness of the unit ball for separable Banach spaces. The Riesz representation theorem for spaces of continuous functions. The Hahn–Banach theorem and its consequences: separation theorems; Mazur's theorem.

Fourier analysis

Definition of Fourier transform in L^1 ; the Riemann–Lebesgue lemma. Fourier inversion theorem. Extension to L^2 by density and Plancherel's isometry. Duality between regularity in real variable and decay in Fourier variable.

Generalized derivatives and function spaces

Definition of generalized derivatives and of the basic spaces in the theory of distributions: D/D′ and S/S ′ . The Fourier transform on S ′ . Periodic distributions; Fourier series; the Poisson summation formula. Definition of the Sobolev spaces H^s in R^d . Sobolev embedding. The Rellich–Kondrashov theorem. The trace theorem.

Applications

Construction and regularity of solutions for elliptic PDEs with constant coefficients on R^n . Construction and regularity of solutions for the Dirichlet problem of Laplace's equation. The spectral theorem for the Laplacian on a bounded domain. The direct method of the Calculus of Variations.

Recommended course:
https://ocw.mit.edu/courses/18-102-introduction-to-functional-analysis-spring-2021/

INTRODUCTION TO ALGEBRAIC TOPOLOGY- back to the list of all courses
2 lectures per week
The fundamental group
Homotopy of continuous functions and homotopy equivalence between topological spaces. The fundamental group of a space, homomorphisms induced by maps of spaces, change of base point, invariance under homotopy equivalence.
Covering spaces
Covering spaces and covering maps. Path-lifting and homotopy-lifting properties, and their application to the calculation of fundamental groups. The fundamental group of the circle; topological proof of the fundamental theorem of algebra. Construction of the universal covering of a path-connected, locally simply connected space. The correspondence between connected coverings of X and conjugacy classes of subgroups of the fundamental group of X.
The Seifert–Van Kampen theorem
Free groups, generators and relations for groups, free products with amalgamation. Statement and proof of the Seifert–Van Kampen theorem. Applications to the calculation of fundamental groups.
Simplicial complexes
Finite simplicial complexes and subdivisions; the simplicial approximation theorem.
Homology
Simplicial homology, the homology groups of a simplex and its boundary. Functorial properties for simplicial maps. Proof of functoriality for continuous maps, and of homotopy invariance.
Homology calculations
The homology groups of S^n , applications including Brouwer's fixed-point theorem. The Mayer-Vietoris theorem. Sketch of the classification of closed combinatorial surfaces; determination of their homology groups. Rational homology groups; the Euler–Poincaré characteristic and the Lefschetz fixed-point theorem

OPTIMISATION - back to the list of all courses
2 lectures per week

Elements of convex optimisation
Convex sets and functions in R^n, global and constrained optimality. Algorithms for unconstrained convex optimisation: gradient descent, Newton's algorithm. Introduction to convex optimisation on a convex set, the barrier method. Examples.
Lagrangian methods and duality
General formulation of constrained problems; the Lagrangian sufficiency theorem. Interpretation of Lagrange multipliers as shadow prices. The dual linear problem, duality theorem in a standardized case, complementary slackness, dual variables and their interpretation as shadow prices. Relationship of the primal simplex algorithm to dual problem. Examples.
Linear programming in the nondegenerate case
Convexity of feasible region; sufficiency of extreme points. Standardization of problems, slack variables, equivalence of extreme points and basic solutions. The primal simplex algorithm and the tableau. Examples.
Applications of linear programming
Two person zero-sum games. Network flows; the max-flow min-cut theorem; the Ford–Fulkerson algorithm, the rational case. Network flows with costs, the transportation

algorithm, relationship of dual variables with nodes. Examples. Conditions for optimality in more general networks. The formulation of simple practical and combinatorial problems as linear programming or network problems.

VARIATIONAL PRINCIPLES - back to the list of all courses
2 lectures per week

Stationary points for functions on R^n. Necessary and sufficient conditions for minima and maxima. Importance of convexity. Variational problems with constraints; method of Lagrange multipliers. The Legendre Transform; need for convexity to ensure invertibility; illustrations from thermodynamics.
The idea of a functional and a functional derivative. First variation for functionals, Euler-Lagrange equations, for both ordinary and partial differential equations. Use of Lagrange multipliers and multiplier functions.
Fermat’s principle; geodesics; least action principles, Lagrange’s and Hamilton’s equations for particles and fields. Noether theorems and first integrals, including two forms of Noether’s theorem for ordinary differential equations (energy and momentum, for example). Interpretation in terms of conservation laws.
Second variation for functionals; associated eigenvalue problem.

MANIFOLDS AND DIFFERENTIAL FORMS - back to the list of all courses
2 lectures per week
Introduction to the theory of manifolds: vector fields and densities on manifolds, integral calculus in the manifold setting and the manifold version of the divergence theorem.
Multilinear algebra: tensors and exterior forms. Differential forms on R^n: exterior differentiation, the pull-back operation and the Poincaré lemma. Applications to physics: Maxwell's equations from the differential form perspective. Integration of forms on open sets of R^n. The change of variables formula revisited. The degree of a differentiable mapping. Differential forms on manifolds and De Rham theory. Integration of forms on manifolds and Stokes' theorem.

EXPERIMENTAL MATHEMATICS I & II - back to the list of all courses
Prerequisites: Programming Basics
A course in numerical and symbolic methods of experimental mathematics involving extensive practical sessions using MATLAB and Maple (or Sage) software.

DISCRETE APPLIED MATHEMATICS - back to the list of all courses
3 lectures per week
Prerequisites: Linear algebra I & II, Discrete Mathematics
Study of illustrative topics in discrete applied mathematics, including probability theory, information theory, coding theory, secret codes, generating functions, and linear programming. It is recommended to practice written communication with this course. This component requires teaching assistance; it may take the form of writing assignments (often proofs) in homework problems; one of the assignments may be a relatively long term paper whose writing would take a few iterations.

Probability and Combinatorics

Pigeonhole principle and sample spaces; independence and conditioning; inclusion-exclusion; random variables, expectation, and variance; counting and bijections; Catalan numbers and Dyck paths; generating functions; recurrences and Pascal’s triangle.

Probability Theory and Bounds

Markov chains, Chebyshev's inequality, and weak law of large numbers (WLLN); Chernoff bound

Number Theory and Modular Arithmetic

Modular arithmetic, groups, and Euclid’s algorithm; Chinese remainder theorem; Lagrange's theorem and Fermat’s little theorem

Linear Programming

Introduction to linear programming and duality; weak duality, strong duality, and complementary slackness; more linear programming applications

Information Theory and Coding

Shannon’s noiseless compression theorem; Huffman coding; Shannon’s noisy coding theorem; linear codes and Reed-Solomon codes

Combinatorics and Cryptography

Van der Waerden theorem and Ramsey theory; cryptography and RSA; Lagrange interpolation and secret sharing; zero-sum games and the minimax theorem; applications of the Chinese remainder theorem; applications of Markov chains and probability in cryptography

Similar course:
https://ocw.mit.edu/courses/18-310-principles-of-discrete-applied-mathematics-fall-2013/

LANGUAGES, AUTOMATA, AND COMPLEXITY - back to the list of all courses

2 lectures per week

Prerequisite: Discrete Applied Mathematics

Automata and languages

Finite automata, regular expressions, push-down automata, context-free grammars, pumping lemmas.

Computability theory

Turing machines, the Church-Turing thesis, decidability, the halting problem, reducibility, the recursion theorem.

Complexity theory

Time and space measures of complexity, complexity classes P, NP, L, NL, PSPACE, BPP and IP, complete problems, the P versus NP conjecture, quantifiers and games, hierarchy theorems, provably hard problems, relativized computation and oracles, probabilistic computation, interactive proof systems.

Similar course: https://math.mit.edu/~sipser/18404/
https://ocw.mit.edu/courses/18-404j-theory-of-computation-fall-2020/

NUMERICAL METHODS - back to the list of all courses

2 lectures per week

This course introduces numerical techniques for solving mathematical problems that may be difficult or impossible to address analytically. Emphasis is placed on the development, analysis, and implementation of algorithms to approximate solutions. Key topics include:

- **Error Analysis:** Understanding approximation errors and their propagation.
- **Root-Finding Methods**: Techniques such as the bisection method, Newton's method, and secant method.
- **Interpolation and Polynomial Approximation**: Lagrange and spline interpolation, least-squares approximation.
- **Numerical Differentiation and Integration**: Techniques for approximating derivatives and integrals, including trapezoidal and Simpson's rules.
- **Solving Systems of Linear Equations**: Gaussian elimination, LU decomposition, iterative methods.
- **Numerical Solutions to Differential Equations**: Euler’s method, Runge-Kutta methods, stability and conv**ergence.**

STATISTICS - back to the list of all courses

2 lectures per week

Prerequisites: Probability

This course provides a foundation in statistical inference, essential for analyzing and interpreting data. Students will learn both theoretical concepts and practical techniques for statistical analysis. Key topics include:

- **Descriptive Statistics**: Measures of central tendency, variability, data visualization, and summarization techniques.
- **Estimation**: Concepts of population vs. sample, sampling distributions.Point estimation, properties of estimators, and interval estimation.
- **Hypothesis Testing**: Formulating and testing hypotheses, p-values, and significance levels, types of errors, and power analysis. Caveats and limitations of hypothesis testing.
- **Regression and Correlation**: Linear regression, least squares estimation, one-way ANOVA, correlation analysis, and model fitting.

APPLIED STOCHASTIC PROCESSES - back to the list of all courses

2 lectures per week

Prerequisites: Linear Algebra, Probability, Analysis and Topology

This course provides a comprehensive introduction to stochastic processes, with a focus on applications in areas like finance, biology, and engineering. Topics include:

- **Introduction to Stochastic Processes**: Basic definitions and classifications of stochastic processes, discrete and continuous time, and examples of common applications. Basics of martingales and stopping times.

- **Renewal Processes**: Poisson processes, renewal theorem, and applications to queuing theory and reliability.
- **Branching Processes**: Galton-Watson process, extinction probabilities, and applications in biology and population dynamics.
- **Markov Chains and Processes**: A recap of discrete-time Markov chains, classification of states, long-term behavior, ergodicity, and applications. Continuous-time Markov processes, transition rates, generator matrices, Kolmogorov forward and backward equations, and applications to queueing models.
- **Markov Chain Monte Carlo (MCMC)**: Introduction to MCMC methods, Metropolis-Hastings, Gibbs sampling, and applications in Bayesian statistics and machine learning.
- **Brownian Motion**: Definition, properties, path properties, and applications in physics and finance.
- **Stochastic Integration and Stochastic Differential Equations (SDEs)**: Introduction to Itô calculus, Itô's lemma, SDEs, and applications in financial modeling and physical sciences.

ANALYSIS OF DIFFERENTIAL EQUATIONS - back to the list of all courses
2 lectures per week
Self-adjoint ODEs
Periodic functions. Fourier series: definition and simple properties; Parseval's theorem. Equations of second order. Self-adjoint differential operators. The Sturm–Liouville equation; eigenfunctions and eigen-values; reality of eigenvalues and orthogonality of eigenfunctions; eigenfunction expansions (Fourier series as prototype), approximation in mean square, statement of completeness.
PDEs on bounded domains: separation of variables
Physical basis of Laplace's equation, the wave equation and the diffusion equation. General method of separation of variables in Cartesian, cylindrical and spherical coordinates. Legendre's equation: derivation, solutions including explicit forms of P_0, P_1 and P_2, orthogonality. Bessel's equation of integer order as an example of a self-adjoint eigenvalue problem with non-trivial weight.
Examples including potentials on rectangular and circular domains and on a spherical domain (axisymmetric case only), waves on a finite string and heat flow down a semi-infinite rod.
Inhomogeneous ODEs: Green's functions
Properties of the Dirac delta function. Initial value problems and forced problems with two fixed end points; solution using Green's functions. Eigenfunction expansions of the delta function and Green's functions.
Fourier transforms
Fourier transforms: definition and simple properties; inversion and convolution theorems. The discrete Fourier transform. Examples of application to linear systems. Relationship of transfer function to Green's function for initial value problems.
PDEs on unbounded domains
Classification of PDEs in two independent variables. Well posedness. Solution by the method of characteristics. Green's functions for PDEs in 1, 2 and 3 independent variables; fundamental solutions of the wave equation, Laplace's equation and the diffusion equation. The method of images. Application to the forced wave equation, Poisson's equation and forced diffusion equation. Transient solutions of diffusion problems: the error function.

Recommended books:
Mathematical Methods for Physicists / G.B. Arfken, H.J. Weber & F.E. Harris, Elsevier 2013
Mathematical Methods in the Physical Sciences / M.L. Boas, Wiley 2005
Parts of this course are also relevant:
https://legacy-www.math.harvard.edu/archive/115_fall_06/index.html

LOGIC AND SET THEORY - back to the list of all courses
2 lectures per week
Ordinals and cardinals
Well-orderings and order-types. Examples of countable ordinals. Uncountable ordinals and Hartogs' lemma. Induction and recursion for ordinals. Ordinal arithmetic. Cardinals; the hierarchy of alephs. Cardinal arithmetic.
Posets and Zorn's lemma
Partially ordered sets; Hasse diagrams, chains, maximal elements. Lattices and Boolean algebras. Complete and chain-complete posets; fixed-point theorems. The axiom of choice and Zorn's lemma. Applications of Zorn's lemma in mathematics. The well-ordering principle.
Propositional logic
The propositional calculus. Semantic and syntactic entailment. The deduction and completeness theorems. Applications: compactness and decidability.
Predicate logic
The predicate calculus with equality. Examples of first-order languages and theories. Statement of the completeness theorem; sketch of proof. The compactness theorem and the Löwenheim-Skolem theorems. Limitations of first-order logic. Model theory.
Set theory
Set theory as a first-order theory; the axioms of ZF set theory. Transitive closures, epsilon-induction and epsilon-recursion. Well-founded relations. Mostowski's collapsing theorem. The rank function and the von Neumann hierarchy.
Consistency
Problems of consistency and independence.

INTRODUCTION TO PHYSICS - back to the list of all courses
3 lectures per week
This first year course provides a foundation in physics for the 'Mathematics with Physics' program.

Physics is concerned with the fundamental laws which govern the behaviour of all forms of matter and therefore underlie all science. In exploring these laws, the first year course has several aims. It is designed to bridge the gap between school and university physics. It aims to consolidate school physics by providing a more logical and analytical framework for classical physics. It introduces non-classical topics such as special relativity and quantum physics which foreshadow major themes of the physics course in later years. Finally the course aims to broaden your perspective, so that you can begin to appreciate the flexibility and generality of the laws of physics, which allow us to apply them to topics ranging from the extremely remote and theoretical, such as the behavior of matter near black holes, to matters of everyday and technical application.

The lectures cover mechanics, relativity, oscillating systems, waves (including quantum waves) and fields, including Maxwell's theory of electricity and magnetism. The course assumes that students will be taking foundational courses in mathematics concurrently.

CLASSICAL MECHANICS - back to the list of all courses
2 lectures per week
Prerequisites: Differential Equations

Classical mechanics deals with the mathematical description of the motion of bodies, or point-like objects. By understanding the forces that are exerted on a body we can construct Newton's equation that describes the motion of the object in question. There are, however, other mathematical approaches to this class of problems, known as the Lagrangian and Hamiltonian descriptions of classical mechanics. This course will introduce these various perspectives, and in the process cover the subject of the calculus of variations. Furthermore this course is the first in a series of mathematical physics courses, such as Quantum Mechanics, Introduction to Quantum Fields and Geometry of General Relativity.

This course is an introduction to the subject of classical mechanics. It will cover Newton's equation, the motion of point particles, including planetary motion, and an introduction to the notion of variational calculus for point particles. In particular the course will cover Hamilton's principle of least action, Lagrangians for systems with conservative forces, and Noether's theorem. The latter provides a conserved quantity whenever there exists a continuous symmetry. Finally, an introduction to the Hamiltonian formalism will be given which prepares the ground for the follow-up course on quantum mechanics for mathematicians. The classical mechanics for mathematicians course is a great opportunity to leaRn about many classical differential equations and physical problems that helped shape many developments in mathematics. It is also a nice arena to practice one's knowledge of several variable calculus and differential equations.

The course will include the following topics:

- Newton's equations for simple mechanical systems
- Celestial mechanics
- Lagrangians and Euler-Lagrange equations
- Noether's theorem and continuous symmetries
- Hamiltonians and Hamilton's equations
- Poisson brackets

Recommended books:
Classical Mechanics / Herbert Goldstein

ELECTROMAGNETISM - back to the list of all courses
2 lectures per week
Prerequisites: Classical Mechanics

Electrostatics Currents and the conservation of charge. Lorentz force law and Maxwell's equations. Gauss's law. Application to spherically symmetric and cylindrically symmetric charge distributions. Point, line and surface charges. Electrostatic potentials; general charge distributions, dipoles. Electrostatic energy. Conductors.

Magnetostatics Magnetic fields due to steady currents. Ampere's law. Simple examples. Vector potentials and the Biot– Savart law for general current distributions. Magnetic dipoles. Lorentz force on current distributions and force between current-carrying wires.

Electrodynamics Faraday's law of induction for fixed and moving circuits. Ohm's law. Plane electromagnetic waves in vacuum, polarization. Electromagnetic energy and Poynting vector.

Electromagnetism and relativity Review of special relativity; tensors and index notation. Charge conservation. 4-vector potential, gauge transformations. Electromagnetic tensor. Lorentz transformations of electric and magnetic fields. Maxwell's equations in relativistic form. Lorentz force law.

Appropriate books:
D.J. Griffiths Introduction to Electrodynamics. Cambridge University Press 2017
E.M. Purcell and D.J. Morin Electricity and Magnetism. Cambridge University Press 2013
A. Zangwill Modern Electromagnetism. Cambridge University Press 2013
J.D. Jackson Classical Electrodynamics. Wiley 1999
P. Lorrain and D. Corson Electromagnetism, Principles and Applications. Freeman 1990
R. Feynman, R. Leighton and M. Sands The Feynman Lectures on Physics, Vol 2. Basic Books 2011

STATISTICAL MECHANICS - back to the list of all courses
2 lectures per week
Prerequisites: Classical Mechanics, Probability and Measure

This is an introduction to the subject of statistical mechanics, as well as statistical approach to thermodynamics.

Syllabus:

- Thermodynamics
- Microcanonical and canonical ensembles, the classical ideal gas, existence of the thermodynamic limit
- Spin systems, Ising model in one dimensions, Curie-Weiss model

Recommended books:
A. Bovier, Statistical mechanics of disordered systems: A mathematical perspective, Cambridge 2006. (Part I)
A.I. Khinchin, Mathematical foundations of statistical mechanics, Dover 1960.

QUANTUM MECHANICS - back to the list of all courses

2 lectures per week
Prerequisites: Classical Mechanics

The discovery of the quantum theory of nature is arguably the most far-reaching scientific revolution of the twentieth century. In the first quarter of that century it became apparent that everyday assumptions about the nature of the world begin to break down when objects the size of atoms are involved.

It is the mathematics of the quantum world which is the main subject of this course. The basic ideas are now readily accessible at the undergraduate level and provide a marvellous illustration of the importance of linear spaces, linear operators, eigenvalues and differential equations in a central context in modeRn mathematics. (It is not intended to go into technical detail about infinite-dimensional spaces etc). This course does, however, motivate the need for a precise theory of self-adjoint operators in infinite-dimensional spaces.

Apart from its intrinsic interest, a further justification for this course is that quantum ideas are now becoming important in many areas of pure mathematics: quantum groups (algebra), quantum cohomology (topology/geometry), quantum cryptography, quantum information.

The course will include (some of) the following topics:

- Basic postulates of quantum theory, wave function, probabilistic interpretation, Dirac notation
- Schrödinger equation and examples: potentials, bound states, tunnelling.
- Operators, Heisenberg uncertainty principle, correspondence principle. Harmonic oscillator, wave packets, dispersion.
- Symmetries in quantum theory: hydrogenic atoms and their spectrum
- Scattering, WKB approximation

Recommended in addition to materials provided:
Brian Hall, Quantum Theory for Mathematicians, Springer 2013 (Selected chapters)
Keith Hannabuss, An introduction to Quantum Theory, OUP 1997

GEOMETRY OF GENERAL RELATIVITY - back to the list of all courses
2 lectures per week
Prerequisites: Classical Mechanics, Curves and Surfaces
This course assumes familiarity with the language of differentiable manifolds, but develops the theory of affine connections and enough pseudo-riemannian geometry (metric tensor, curvature) in order to describe the theory of General Relativity. This is done via the postulates of General Relativity and the Einstein field equations. The course then explores solutions of the Einstein field equations,including the famous Schwarzschild black hole and the cosmological solutions, which introduces the geometric notions of homogeneity anisotropy.

Syllabus:

- Affine connections: covariant derivative, torsion, curvature, parallel transport, geodesics, geodesic deviation.

- Riemannian geometry: metric tensors, Lorentzian metrics, Levi-Civita connection, curvature tensors, moving frames, Cartan structure equations, isometries, Killing vector fields.
- General Relativity: special relativity and Minkowski spacetime, Maxwell's equations, postulates of General Relativity, spacetime, general covariance, energy-momentum tensor, Einstein equations.
- Causal structure and Penrose diagram for Minkowski spacetime.
- Schwarzschild solution: static and spherically symmetric spacetimes, black hole, Kruskal extension, causal structure and Penrose diagram.
- Cosmological models: homogeneity and isotropy, the Friedmann-Lemaître-Robertson-Walker metric.

Recommended books:
An Introduction to General Relativity, L.P Hughston and K.P. Tod (LMS, CUP, 1990)
General Relativity, R. M. Wald, University of Chicago Press (1984)

INTRODUCTION TO QUANTUM FIELDS - back to the list of all courses
2 lectures per week

Prerequisites: Familiarity with analysis in several variables, complex analysis, linear algebra and basic functional analysis, and basic differential geometry. For some parts: Riemann surfaces and fundamentals of Lie groups. Knowledge of advanced physics is not required.

The development of quantum field theory (QFT) and string theory in the last four decades led to an unprecedented level of interaction between physics and mathematics, incorporating into physics such “pure” areas of mathematics as algebraic topology, algebraic geometry, representation theory, combinatorics, and even number theory. This interaction has been highly fruitful in both directions, and led to a necessity for physicists to know basic mathematics and for mathematicians to know basic physics. Physicists have been quick to leaRn, and nowadays good physicists know relevant areas of mathematics as deeply as professional mathematicians. On the other hand, many mathematicians have been slower, intimidated by the absence of rigor in physical texts, and, more importantly, by a different manner of presentation. In particular, even the basic setting of quantum field theory, necessary for understanding its more advanced (and mathematically exciting) parts, is already largely unfamiliar to mathematicians. Nevertheless, many of the basic ideas of quantum field theory can in fact be presented in a completely rigorous and mathematical way. Doing so will be the main goal of this course.

Topics:

- Generalities on classical and quantum mechanics and field theory.
- 0-dimensional QFT: Stationary phase (steepest descent) formula. Calculus of Feynman diagrams with applications to combinatorics. Matrix models, large N limits. Applications to moduli space of curves and planar graphs.
- 1-dimensional QFT: Formalism of classical mechanics. Lagrangians, Hamiltonians, and the least action principle. The path integral approach to quantum mechanics. Correlation functions. Perturbative expansion using Feynman diagrams. The Hamiltonian approach and operator formalism. The Feynman-Kac formula.

- d-dimensional QFT for d>1. Formalism of classical field theory. Gauge theories. Path integral and Hamiltonian approaches to QFT. Wightman axioms. Free field theories. Perturbative expansion. Divergences. Renormalization theory.
- Supergeometry and field theories with fermions.
- Introduction to 2-dimensional conformal field theory.

Textbook and Lecture Notes

A recommended textbook: *Quantum Fields and Strings: A Course for Mathematicians*, AMS, 1998 Also, Pavel Etingoff lecture notes.